Penalizing Atrocities*
Appendix

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This appendix contains derivations for some of the results discussed in the text.

To find the equilibrium level of atrocities, we maximize the war payoff with respect to α. The first-order condition is solved for α as follows:

$$\begin{aligned}
\frac{\partial p_g}{\partial \alpha} &= k_g \\
\frac{(m_g + \epsilon\alpha + m_r)\,\epsilon - (m_g + \epsilon\alpha)\epsilon}{(m_g + \epsilon\alpha + m_r)^2} &= k_g \\
\frac{m_r\epsilon}{(m_g + \epsilon\alpha + m_r)^2} &= k_g \\
(m_g + \epsilon\alpha + m_r)^2 &= \frac{\epsilon m_r}{k_g} \\
m_g + \epsilon\alpha + m_r &= \sqrt{\frac{\epsilon m_r}{k_g}}
\end{aligned}$$

Solving for the equilibrium level of α produces:

$$\alpha^* = \sqrt{\frac{m_r}{\epsilon k_g}} - \frac{m_g + m_r}{\epsilon} \tag{1}$$

To see how this varies as a function of the strength of the rebels, we take the derivative of α^* with respect to m_r as follows:

$$\frac{\partial \alpha^*}{\partial m_r} = \frac{1}{\sqrt{\epsilon k_g}}\frac{1}{2}m_r^{-\frac{1}{2}} - \frac{1}{\epsilon}$$

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$$
\begin{aligned}
&= \frac{1}{2}\left(\frac{1}{m_r \epsilon k_g}\right)^{\frac{1}{2}} - \frac{1}{\epsilon} \\
&= \frac{1}{2\sqrt{m_r \epsilon k_g}} - \frac{1}{\epsilon}
\end{aligned}
$$

It will be positive so long as the following holds:

$$
\begin{aligned}
2\sqrt{m_r \epsilon k_g} &< \epsilon \\
\sqrt{m_r \epsilon k_g} &< \frac{\epsilon}{2} \\
m_r \epsilon k_g &< \frac{\epsilon^2}{4} \\
m_r k_g &< \frac{\epsilon}{4} \\
m_r &< \frac{\epsilon}{4k_g}
\end{aligned}
$$

This condition is equivalent to the rebels being weaker than the government side, as can be seen below. The government's strength is $m_g + \epsilon\alpha^*$. The government is stronger than the rebels when the following holds.

$$
\begin{aligned}
m_g + \epsilon\alpha^* &> m_r \\
m_g + \epsilon\sqrt{\frac{m_r}{\epsilon k_g}} - \epsilon\frac{m_g + m_r}{\epsilon} &> m_r \\
\sqrt{\frac{\epsilon m_r}{k_g}} &> 2m_r \\
\frac{\epsilon m_r}{k_g} &> 4m_r^2 \\
\frac{\epsilon}{4k_g} &> m_r
\end{aligned}
$$

This is the previous condition, so the result holds. So if the rebels are weaker than the government side, then strengthening the rebels leads to an increase in atrocities, as the government side tries to compensate.

The derivative of α^* with respect to ϵ is the following:

$$\frac{\partial \alpha^*}{\partial \epsilon} = \frac{1}{2}\left(\frac{m_r}{\epsilon k_g}\right)^{-\frac{1}{2}} \frac{m_r}{k_g}(-1)\epsilon^{-2} - (m_g + m_r)(-1)\epsilon^{-2} \tag{2}$$

This is positive when

$$\begin{aligned} 0 &< -\sqrt{\frac{m_r \epsilon}{4k_g}} + m_g + m_r \\ \epsilon &< \frac{4k_g}{m_r}(m_g + m_r)^2 \end{aligned}$$

Rearranging, this implies that if

$$\epsilon \alpha^* < m_g + m_r \tag{3}$$

then the slope is positive, so decreasing ϵ will decrease α^*.