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Corporate Governance, Finance, and the Real Sector

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Online Appendix. Proofs

Proof of Proposition 1. Taking as given n^* and $\widetilde{p}^*$, the 1st-order condition to expression (3) leads to expression (14). This implies that the equilibrium level of cash flow to a firm i is

$$\text{(A-1)} \qquad X_i^{T*} \quad = \quad X^{T*} \quad = \quad (p_i^* - c_i)\, q_i^* \quad = \quad \left(\frac{\alpha}{n^*}\right)^2 .$$

Substituting the constraints (10), (11), and (12) into equation (9), we obtain that equation (9) can be written as

$$\text{(A-2)} \qquad \max_{B_i} \mathbf{E}_0 \left[X_i^T\left(p^*, \tau_i(B_i)\right) - F_{H,i} - \beta(1-\mu) \max\{X_i^T\left(p^*, \tau_i(B_i)\right) - B_i; 0\}\right],$$

$$\text{s.t.} \quad \tau_i(B_i) = \arg\max_{\tau_i \in \{H,L\}} \mathbf{E}_1 \; X_i^E(p^*, \tau_i, \kappa_i).$$

Since low-quality technology is not sustainable, in equilibrium only firms that are expected (and have the incentive) to choose high-quality technology enter the market. This leads to the incentive-compatibility condition (20). From expression (A-2) it is easy to see that entrepreneurs first issue debt up to debt capacity $\bar{D}$, after which they will issue equity. Given expression (21), the maximum amount of equity that the marginal entrepreneur with cash flow X^{T*} can issue is $S^*_{n^*} = (1-\beta)\eta$. This implies that n^* is determined by

$$\text{(A-3)} \qquad \overline{D} + S^*_{n^*} \quad = \quad \left(\frac{\alpha}{n^*}\right)^2 - \beta\eta \quad = \quad F_{H,n^*} \quad = \quad F_H + \theta n^*,$$

giving equation (13). Inframarginal entrepreneurs will issue an amount of equity that is just sufficient to cover the fixed cost $F_{H,i}$ giving equation (15). Thus, the fraction of equity sold to outside investors, κ_i, is $S_i^*/(1-\beta)\eta$, giving equation (17). The payoff to the marginal entrepreneur, who given expression (A-3) sells all his shares to obtain entry, is $\mu\beta\eta$. The payoff to inframarginal entrepreneurs is thus equation (18). Finally, from equation (17), it is easy to see that $1-\kappa_i < \mu$ for all $i < n^*$ if

$$\text{(A-4)} \qquad \mu \quad \geq \quad \mu_c \quad \equiv \quad \frac{\theta n^*}{(1-\beta)\eta}.$$

In addition, note that no additional entrepreneur with $i > n^*$ can enter when $\phi(\frac{\alpha}{n^*}) < F_L + \theta n^*$, that is, when

$$\text{(A-5)} \qquad \phi \quad \leq \quad \phi_c \quad \equiv \quad \frac{F_L + \theta n^*}{(\frac{\alpha}{n^*})^2}.$$

The proof is concluded by noting that expression (A-5) implies that

$$\text{(A-6)} \qquad V_i \quad = \quad \mu\beta\eta + \theta(n^* - i) \quad > \quad \phi\left(\frac{\alpha}{n^*}\right)^* - F_L - \theta i$$

and, thus, all entrepreneurs that enter the market prefer to adopt high-quality technology rather than low-quality technology.

Proof of Proposition 2. The 1st result follows immediately from Proposition 1 and implicit function differentiation of equation (13), obtaining

$$\text{(A-7)} \qquad \frac{\partial n^*}{\partial \beta} \quad = \quad -\frac{\eta}{\frac{2\alpha^2}{n^{*3}} + \theta} \quad < \quad 0.$$

The sign of $\frac{\partial \bar{D}}{\partial \beta}$ follows from direct differentiation of $\bar{D}$ in expression (21) and from expression (A-7). The sign of $\frac{\partial S_i^*}{\partial \beta}$ follows from the 1st equality in expression (15) and the previous result that $\frac{\partial \bar{D}}{\partial \beta} > 0$. The sign of $\frac{\partial E_i^{M*}}{\partial \beta}$ follows from direct differentiation of $E_i^{M*} = (1-\beta)\,\eta$. By differentiation of

$$\text{(A-8)} \qquad \omega_i \quad = \quad 1 - \frac{S_i^*}{E_i^{M*}} \quad = \quad \frac{\theta(n^* - i)}{(1-\beta)\,\eta},$$

using expression (A-7), we obtain that

$$\text{(A-9)} \qquad \frac{\partial \omega_i^*}{\partial \beta} \quad = \quad \theta \frac{\left[\left(\frac{2\alpha^2}{n^{*3}} + \theta\right)(n^* - i) - (1-\beta)\eta\right]}{\left(\frac{2\alpha^2}{n^{*3}} + \theta\right)(1-\beta)^2\eta} \quad > \quad 0$$

iff $i < i_c(\beta, \eta) \equiv n^* - \frac{(1-\beta)\eta}{\frac{2\alpha^2}{n^{*3}} + \theta}$. The inefficiency of low-quality technology implies that $n^* > i_c(\beta, \eta) > 0$. To see this, note that $\phi F_H < F_L$ implies

$$\text{(A-10)} \qquad \frac{2\alpha^2}{n^{*2}} \quad = \quad 2\,(F_H + \theta n^* + \eta\beta) \quad > \quad F_L \quad > \quad \frac{\phi\,(F_H - F_L)}{(1-\phi)} \quad = \quad \eta.$$

Finally, expression (24) is obtained by substituting expression (A-7) into $\varepsilon = \left|\frac{\beta}{n^*}\frac{\partial n^*}{\partial \beta}\right|$, giving

$$\text{(A-11)} \qquad \varepsilon \quad = \quad \frac{\eta\beta}{\frac{2\alpha^2}{n^{*2}} + \theta n^*} \quad = \quad \frac{\eta\beta}{2\,(F_H + \theta n^* + \eta\beta) + \theta n^*} \quad = \quad \frac{1}{\frac{2F_H + 3\theta n^*}{\eta\beta} + 2},$$

which is increasing in η (since, in the Proof of Proposition 3, we will show that n^* is decreasing in η).

Proof of Proposition 3. The 1st result that $\frac{\partial n^*}{\partial \eta} < 0$ follows immediately from Proposition 1 and implicit function differentiation of equation (13). The sign of $\frac{\partial S_i^*}{\partial \eta}$ follows from direct differentiation of S_i^* in expression (15) and the result that $\frac{\partial n^*}{\partial \eta} < 0$. The sign of $\frac{\partial \bar{D}}{\partial \eta}$ then follows from the 1st equality in expression (15). The sign of $\frac{\partial E_i^{M*}}{\partial \eta}$ follows from direct differentiation of $E_i^{M*} = (1-\beta)\,\eta$. The result that $\frac{\partial \omega_i}{\partial \eta} < 0$ follows from expression (A-8) and $\frac{\partial n^*}{\partial \eta} < 0$.

Proof of Proposition 4. Entrepreneurs maximize their expected profits, that is,

$$\text{(A-12)} \qquad \max_{B_i,\tau_i,e_i} \mathbf{E}_0 \left[X_i^{T*}(\tau_i) - F_{H,i} - (1-e_i)\beta(1-\mu)\max\{X_i^{T*}(\tau_i) - B_i; 0\}\right] - C\left(k, e_i\right),$$

subject to

$$\text{(A-13)} \qquad \tau_i = \arg\max_{\tau_i \in \{H,L\}} \mathbf{E}_1[\mu\beta + (1-\kappa_i)(1-\beta)]\max\{X_i^{T*}(\tau_i) - B_i; 0\}.$$

With the given cost function for effort, assuming that Assumptions 1 and 2 hold, we can rewrite the entrepreneurs' objective function, (A-12), using our previous results, regarding B_i^*, as

$$\text{(A-14)} \qquad \max_{e_i} \mathbf{E}_0 \left[\left(\frac{\alpha}{n}\right)^2 - F_H - \theta i - (1-e_i)\beta(1-\mu)\eta - ke\left(1-e_i\right)^{-1}\right].$$

Let

$$\text{(A-15)} \qquad k_1 \equiv \frac{\left(1-2\mu\right)^2}{1-\mu}\beta\eta.$$

Under our assumption that $k \leq k_1$, the 1st-order condition with respect to e_i gives the optimal level of effort for all entrepreneurs i:

$$\text{(A-16)} \qquad e_i^{**} = 1 - \sqrt{\frac{k}{\beta(1-\mu)\eta}}.$$

Entry to an industry occurs until the marginal entrepreneur's payoff equals 0. Hence, n^{**} satisfies

$$\text{(A-17)} \qquad \left(\frac{\alpha}{n^{**}}\right)^2 - F_H - \theta n^{**} - (1-e_i^{**})\beta(1-\mu)\eta - ke_i^{**}\left(1-e_i^{**}\right)^{-1} = \left(\frac{\alpha}{n^{**}}\right)^2 - F_H - \theta n^{**} - 2\sqrt{k\beta(1-\mu)\eta} + k = 0,$$

implying that n^{**} is implicitly determined by

$$\text{(A-18)} \qquad n^{**} = \frac{\alpha}{\sqrt{F_H + \theta n^{**} + 2\sqrt{k\beta(1-\mu)\eta} - k}} > n^*.$$

To see that $n^{**} > n^*$, note that

$$\beta\eta \quad > \quad 2\sqrt{k\beta\eta} - k \quad > \quad 2\sqrt{k\beta(1-\mu)\eta} - k, \tag{A-19}$$

since

$$\beta\eta - 2\sqrt{k\beta\eta} + k \quad = \quad \left(\sqrt{k} - \sqrt{\beta\eta}\right)^2 \quad > \quad 0. \tag{A-20}$$

We now need to show that, by exerting effort e^{**}, the marginal entrepreneur is able to raise financing, that is

$$\left(\frac{\alpha}{n^{**}}\right)^2 - F_H - \theta n^{**} - (1-e^{**})\beta\eta \ \geq \ 0. \tag{A-21}$$

Using expression (A-17), it is easy to check that expression (A-21) is verified when

$$ke^{**}(1-e^{**})^{-1} \ \geq \ (1-e^{**})\beta\mu\eta, \tag{A-22}$$

that is, from equation (A-16), when

$$k \quad \leq \quad k_1 \quad \equiv \quad \frac{(1-2\mu)^2}{1-\mu}\beta\eta \quad \leq \quad (1-\mu)\beta\eta. \tag{A-23}$$

The proof is concluded by noting that Assumption 1 holds with the previous definition of ϕ_c and redefining μ_c as $\mu_c = \frac{\theta n^{**}}{\left(\eta - \sqrt{\frac{k\beta\eta}{(1-\mu)}}\right)}$.

Proof of Proposition 5. In this case, the financing constraint (A-21) fails with n^{**} firms in the market. Hence, fewer firms enter, and at the effort level e^{**} all entering firms would have strictly positive payoffs. This implies that for some marginal firms (which otherwise would be left out), it pays to exert an amount of effort $\hat{e}_i > e^{**}$ in order to obtain entry. For these firms, $\hat{e}_i$ is set sufficiently high to raise the necessary funds to successfully enter the market, that is,

$$\left(\frac{\alpha}{\widehat{n}}\right)^2 - F_H - \theta i - (1-\hat{e}_i)\beta\eta \quad = \quad 0. \tag{A-24}$$

The number of firms in this equilibrium, $\hat{n}$, is again determined by the condition that the marginal entrepreneur earns zero expected profits. That is, by

$$\left(\frac{\alpha}{\hat{n}}\right)^2 - F_H - \theta\hat{n} - (1-\hat{e}_{\widehat{n}})(1-\mu)\beta\eta - k\hat{e}_{\widehat{n}}(1-\hat{e}_{\widehat{n}})^{-1} \quad = \quad 0. \tag{A-25}$$

Substituting equation (A-24) to equation (A-25) gives

$$(1-\hat{e}_{\hat{n}})^2\mu\beta\eta - k\hat{e}_{\widehat{n}} \quad = \quad 0 \tag{A-26}$$

$$\Longrightarrow$$

$$1 + \hat{e}_{\hat{n}}^2 - \left(2 + \frac{k}{\mu\beta\eta}\right)\hat{e}_{\hat{n}} = 0 \tag{A-27}$$

$$\text{or} \quad \hat{e}_{\hat{n}} = \frac{1 + 2\mu\beta\eta/k - \sqrt{4\mu\beta\eta/k + 1}}{2\mu\beta\eta/k} \in (0,1). \tag{A-28}$$

From equation (A-24) and the 1st-order condition for effort (A-16), it is easy to see that for other firms,

$$\hat{e}_i = \max\left\{\hat{e}_{\hat{n}} - \frac{\theta(\hat{n} - i)}{\beta\eta}, e^{**}\right\}. \tag{A-29}$$

Taking the derivatives with respect to β and η gives

$$\frac{\partial \hat{e}_{\hat{n}}}{\partial \beta} = \left(\sqrt{1 + \frac{1}{\left(\frac{k}{\mu\beta\eta}\right) + \left(\frac{k}{2\mu\beta\eta}\right)^2}} - 1\right)\frac{k}{2\mu\eta\beta^2} > 0, \tag{A-30}$$

$$\frac{\partial \hat{e}_{\hat{n}}}{\partial \eta} = \left(\sqrt{1 + \frac{1}{\left(\frac{k}{\mu\beta\eta}\right) + \left(\frac{k}{2\mu\beta\eta}\right)^2}} - 1\right)\frac{k}{2\mu\beta\eta^2} > 0, \tag{A-31}$$

which implies, given our previous results for e^{**}, and the fact that $\frac{\partial \hat{n}}{\partial \beta} < 0$ and $\frac{\partial \hat{n}}{\partial \eta} < 0$, as can be verified using equation (A-25), that these derivatives are positive also for other firms.

Proof of Proposition 6. Low-quality technology is sustainable in equilibrium if

$$\phi > \phi_c \equiv \frac{F_L + \theta n^*}{(\frac{\alpha}{n^*})^2} \iff \phi\left(\frac{\alpha}{n^*}\right)^2 - F_L - \theta n^* > 0. \tag{A-32}$$

When expression (A-32) holds, if the first n^* firms choose high-quality technology, some additional marginal firms can enter the market by adopting low-quality technology. Let $\{n', n''\}$ be a candidate equilibrium in which n' is the total number of firms in the industry and $n'' \in [0, n')$ is the number of firms that choose high-quality technology. Note first that, in the candidate equilibrium, firms with high-quality technology produce $\tilde{q}_i^* = \frac{\alpha}{n'' + \phi(n' - n'')}$, and sell their production at a price $\tilde{p}_i^* = c + \frac{\alpha}{n'' + \phi(n' - n'')}$. This results in cash flow

$$X_i^T = \left(\frac{\alpha}{n'' + \phi(n' - n'')}\right)^2. \tag{A-33}$$

Thus, debt capacity for firms selecting high-quality technology is now equal to

$$\text{(A-34)} \qquad \overline{D} = \left(\frac{\alpha}{n'' + \phi(n' - n'')}\right)^2 - \eta.$$

In equilibrium, firms selecting high-quality technology finance $\overline{D}$ with debt and $F_{H,i} - \overline{D}$ with equity. The remaining $n' - n'' > 0$ entrepreneurs who enter the market produce with low-quality technology, and with probability ϕ can produce superior quality goods in the quantity $\tilde{q}_i^*$. Furthermore, these firms can be financed entirely with debt; thus, they borrow $D_i^* = F_L + \theta n'$ of debt with a face value $B_i = \frac{F_L + \theta n'}{\phi}$, and repurchase shares for $D_i^* - F_{L,i}$.

Equilibrium is determined by 3 conditions: (31), (32), and the entry condition for the n':th low-quality producer

$$\text{(A-35)} \qquad \phi\left(\frac{\alpha}{n'' + \phi(n' - n'')}\right)^2 = F_L + \theta n'.$$

Furthermore, 2 of the 3 conditions bind, equation(A-35) and either expression (31) or (32). Consider 2 cases: First, if $\mu \geq \phi$, it is easy to verify that expression (31) implies expression (32) for all $i \geq 0$ if

$$\text{(A-36)} \qquad (1 - \phi)\theta n'' + \beta\eta(\mu - \phi) + F_L - \phi F_H \geq 0,$$

which holds for all β. In this case, using equation (A-35) and expression (31) as equalities gives

$$\text{(A-37)} \qquad n'' = \frac{n'}{\phi} - \frac{\phi F_H - F_L + \phi\beta\eta}{\theta\phi}.$$

This can be used in equation (A-35) or expression (31) to substitute for either n' or n'' to verify that n'' is decreasing in β, while n' is increasing in β. Substituting for n' from equation (A-37) into expression (31) and setting $n'' = 0$ gives that $n'' \geq 0$ if and only if $\beta \leq \beta_1$, where β_1 is defined implicitly by

$$\text{(A-38)} \qquad \left(\frac{\alpha\theta}{\phi\left(\phi F_H - F_L + \phi\beta_1\eta\right)}\right)^2 = F_H + \beta_1\eta.$$

Second, if $\mu < \phi$, expression (A-36) holds for $\beta \leq \beta_2$, where β_2 is defined by

$$\text{(A-39)} \qquad \beta_2 = \frac{F_L - \phi F_H}{\eta(\phi - \mu)}.$$

Let $\bar{\beta} = I_{\mu > \phi}\beta_1 + I_{\mu < \phi}\min(\beta_1, \beta_2)$. Note that our assumption that $F_L > \phi F_H$ implies that $\bar{\beta} > 0$.

Proof of Proposition 7. When $\mu \geq \phi$, or when $\mu < \phi$, but $\beta_1 \leq \beta_2$, let $\beta' = \bar{\beta}$. When $\mu < \phi$, but $\beta_1 > \beta_2$, equation (A-35) and expression (32) hold as an equality for small enough n''. Solving for n'' using equation (A-35) and expression (32), we can verify that n'' is decreasing in β. Thus, $n''=0$ whenever $\beta>\beta_3$, where β_3 solves

$$(1-\phi)\left(\frac{\alpha\theta}{\phi\left(\frac{\phi F_H-F_L+(1-\mu)\phi\beta_3\eta}{(1-\phi)}\right)}\right)^2-(F_H-F_L)-(1-\mu)\,\beta_3\eta \;=\; 0. \tag{A-40}$$

Let $\beta'=I_{\mu>\phi}\beta_1+I_{\mu<\phi}\left(I_{\beta_1<\beta_2}\beta_1+I_{\beta_1>\beta_2}\beta_3\right)$. The result regarding the limit when $\theta\to 0$ follows from (A-38), since in the limit $\beta_1<\beta_2$ when $\mu<\phi$.

Proof of Proposition 8. The proof is similar to the Proof of Proposition 1, and is only sketched. Taking again n° and $\tilde{p}$ as given, entrepreneurs choosing high-quality technology set $p_i=\frac{\alpha'}{\tilde{p}n^\circ}$, which gives $p_i^\circ=\sqrt{\frac{\alpha'}{n^\circ}}$ and $q_i^\circ=\sqrt{\frac{\alpha'}{n^\circ}}$; thus, firm profits are now equal to $X^{T\circ}=\frac{\alpha'}{n^\circ}$. This implies that debt capacity now is $\bar{D}^\circ=\frac{\alpha'}{n^\circ}-\eta$, where η is defined as before. Given that the marginal entrepreneur now issues $S_{n^\circ}^\circ=(1-\delta)(1-\beta)\eta$ of equity, using a similar line of reasoning as the one in the Proof of Proposition 1, we obtain that n° firms producing all with high-quality technology can enter the market, where n° is the positive root of

$$\theta n^2+(F_H+\eta\xi)n-\alpha' \;=\; 0, \tag{A-41}$$

giving (34). Defining $\phi_c^\circ\equiv\frac{F_L+\theta n^\circ}{\frac{\alpha'}{n^\circ}}$, it is easy to show (along the lines in the Proof of Proposition 1) that all incumbents prefer to use high-quality technology, and that there cannot be any entry of firms that use low-quality technology when $\phi\le\phi_c^\circ$. Similarly, $1-\kappa_i\le\mu$ for all firms when $\mu\ge\mu_c^\circ\equiv\frac{\theta n^\circ}{(1-\delta)(1-\beta)\eta}$. Direct calculation now gives that

$$\varepsilon(n^\circ,\delta)\;=\;\left|\frac{\frac{\partial n^\circ}{\partial\delta}}{\frac{n^\circ}{\delta}}\right|\;=\;\frac{\eta\,(1-\beta)\,\delta}{F_H+2\theta n^\circ+\eta\xi}\;=\;\frac{(1-\beta)\,\delta}{\frac{F_H+2\theta n^\circ}{\eta}+\xi}. \tag{A-42}$$

Thus,

$$\frac{\partial\varepsilon(n^\circ,\delta)}{\partial\alpha'}\;=\;-\frac{2\eta\delta\theta\,(1-\beta)\,\frac{\partial n^\circ}{\partial\alpha'}}{(F_H+2\theta n^\circ+\eta\xi)^2}\;<\;0, \tag{A-43}$$

and

$$\frac{\partial\varepsilon(n^\circ,\delta)}{\partial\eta}\;=\;\delta\,(1-\beta)\,\frac{\frac{F_H+2\theta n^\circ}{\eta^2}-\frac{2\theta\frac{\partial n^\circ}{\partial\eta}}{\eta}}{\left(\frac{F_H+2\theta n^\circ}{\eta}+\xi\right)^2}\;>\;0, \tag{A-44}$$

and

$$\text{(A-45)} \quad \frac{\partial \varepsilon(n^\circ, \delta)}{\partial \beta} = -\frac{\eta\delta}{F_H + 2\theta n^\circ + \eta\xi} - \frac{\eta(1-\beta)\delta\left[2\theta\frac{\partial n^\circ}{\partial \beta} + \eta(1-\delta)\right]}{(F_H + 2\theta n^\circ + \eta\xi)^2}$$

$$= -\frac{\eta\delta(F_H + 2\theta n^\circ + \eta\xi) + \eta(1-\beta)\delta\left[2\theta\frac{\partial n^\circ}{\partial \beta} + \eta(1-\delta)\right]}{(F_H + 2\theta n^\circ + \eta\xi)^2}$$

$$= -\frac{\eta\delta(F_H + 2\theta n^\circ + \eta\xi) + \eta(1-\beta)\delta\left[\eta(1-\delta) - 2\theta\left(\frac{\eta n^\circ(1-\delta)}{F_H + 2\theta n^\circ + \eta\xi}\right)\right]}{(F_H + 2\theta n^\circ + \eta\xi)^2}$$

$$= -\frac{\eta\delta(F_H + 2\theta n^\circ + \eta\xi) + \eta(1-\beta)\delta\eta(1-\delta)\left[\frac{F_H + \eta\xi}{F_H + 2\theta n^\circ + \eta\xi}\right]}{(F_H + 2\theta n^\circ + \eta\xi)^2} < 0.$$

Proof of Proposition 9. Low-quality technology is sustainable in equilibrium if

$$\text{(A-46)} \quad \phi > \phi_c^\circ \equiv \frac{F_L + \theta n^\circ}{\frac{\alpha'}{n^\circ}} \iff \phi\frac{\alpha'}{n^\circ} - F_L - \theta n^\circ > 0.$$

When expression (A-46) holds, as in the limiting case where $\delta = 0$, the 1st $n^{\circ\prime\prime}$ firms choose high-quality technology and $n^{\circ\prime} - n^{\circ\prime\prime}$ select low-quality technology. Equilibrium is determined by 3 conditions: The entry condition for the $n^{\circ\prime\prime}$:th entrepreneur,

$$\text{(A-47)} \quad \left(\frac{\alpha'}{n^{\circ\prime\prime} + \phi(n^{\circ\prime} - n^{\circ\prime\prime})}\right) - \left(F_H + \theta n^{\circ\prime\prime}\right) - \xi\eta \geq 0;$$

the condition that entrepreneurs prefer to raise $F_{H,n''}$, and select high-quality technology, rather than to raise $F_{L,n''}$ and select low-quality technology, that is,

$$\text{(A-48)} \quad (1-\phi)\left(\frac{\alpha'}{n^{\circ\prime\prime} + \phi(n^{\circ\prime} - n^{\circ\prime\prime})}\right) - (F_H - F_L) - (\xi - \mu\beta)\eta \geq 0;$$

and the entry condition for the $n^{\circ\prime}$:th low-quality producer,

$$\text{(A-49)} \quad \phi\left(\frac{\alpha'}{n^{\circ\prime\prime} + \phi(n^{\circ\prime} - n^{\circ\prime\prime})}\right) = F_L + \theta n^{\circ\prime}.$$

Furthermore, 2 of the 3 conditions bind, equation (A-49) and either expression (A-47) or (A-48). Expression (A-48) is implied by expression (A-47) when

$$\text{(A-50)} \quad (1-\phi)\theta n^{\circ\prime\prime} + (\mu\beta - \phi\xi)\eta + F_L - \phi F_H > 0.$$

This is satisfied when

$$\text{(A-51)} \quad \frac{(\mu-\phi)\beta\eta + F_L - \phi F_H}{\phi(1-\beta)\eta} \equiv \overline{\delta}^\circ \geq \delta.$$

Now $\overline{\delta}^{\circ} > 0$ when $(\mu-\phi)\beta\eta + F_L - \phi F_H > 0$. As $F_L - \phi F_H > 0$, there exists $\overline{\beta}^{\circ} > 0$ such that this holds for all $\beta < \overline{\beta}^{\circ}$.

In this case, using expressions (A-47) and (A-49) as equalities gives

$$\text{(A-52)} \qquad n'' = \frac{n'}{\phi} - \frac{\phi F_H - F_L + \phi\xi\eta}{\theta\phi}.$$

This can be used in expression (A-47) or (A-49) to substitute for either $n^{\circ\prime}$ or $n^{\circ\prime\prime}$ to verify that $n^{\circ\prime\prime}$ is decreasing in δ, while $n^{\circ\prime}$ is increasing in δ. The claim on total production can now be verified, as an increase in δ must lead to a decrease in total output $\alpha'/\widetilde{p}$ as $\widetilde{p} = \sqrt{\frac{\alpha'}{n^{\circ\prime\prime}+\phi(n^{\circ\prime}-n^{\circ\prime\prime})}}$, which increases by equation (A-49), given the result that $n^{\circ\prime}$ increases in δ.

Proof of the Claims Related to Table 1.

The comparative statics results for n^* follow from the results in Propositions 2 and 3 and equation (13). The comparative statics results in Table 1 related to the partial derivatives with respect to η follow from the results in Propositions 2 and 3 given that the ratios are

$$\text{(A-53)} \qquad \left(\frac{D_i^*}{S_i^*}\right)^{\text{ind}} = \frac{\int_i D_i^* di}{\int_i S_i^* di} = \frac{\int_i \left[\left(\frac{\alpha}{n^*}\right)^2 - \eta\right] di}{\int_i \left[(1-\beta)\eta - \theta(n^*-i)\right] di} = \frac{F_H + \theta n^* + \eta(\beta-1)}{\left[(1-\beta)\eta - \frac{\theta n^*}{2}\right]},$$

$$\text{(A-54)} \qquad \left(\frac{S_i^*}{E_i^{M*}}\right)^{\text{ind}} = \frac{\int_i S_i^* di}{\int_i E_i^{M*} di} = \frac{\int_i \left[(1-\beta)\eta - \theta(n^*-i)\right] di}{\int_i \left[(1-\beta)\eta\right] di} = 1 - \frac{\frac{\theta n^*}{2}}{\left[(1-\beta)\eta\right]},$$

$$\text{(A-55)} \qquad (\omega_i)^{\text{ind}} = \frac{\int_i \left[E_i^{M*} - S_i^*\right] di}{\int_i E_i^{M*} di} = \frac{\int_i \left[(1-\beta)\eta - (1-\beta)\eta + \theta(n^*-i)\right] di}{\int_i (1-\beta)\eta di} = \frac{\frac{\theta n^*}{2}}{(1-\beta)\eta},$$

$$\text{(A-56)} \qquad (\text{ROA}_i^*)^{\text{ind}} = \frac{\int_i X_i^T di}{\int_i F_i^* di} = \frac{\int_i \left(\frac{\alpha}{n^*}\right)^2 di}{\int_i \left[F_H + \theta i\right] di} = \frac{\left(\frac{\alpha}{n^*}\right)^2}{F_H + \frac{\theta n^*}{2}} - 1.$$

The comparative statics results for the partial derivative with respect to β also follow from the results in Propositions 2 and 3. First note that

$$\text{(A-57)} \qquad \left(\frac{D_i^*}{S_i^*}\right)^{\text{ind}} = \frac{\int_i D_i^* di}{\int_i S_i^* di} = \frac{\int_i \left[\left(\frac{\alpha}{n^*}\right)^2 - \eta\right] di}{\int_i \left[(1-\beta)\eta - \theta(n^*-i)\right] di} = \frac{\left(\frac{\alpha}{n^*}\right)^2 - \eta}{\left[\eta - \beta\eta - \frac{1}{2}\theta n^*\right]}$$

increases in β. This result follows as the fact that $\left(\frac{\alpha}{n^*}\right)^2 = F_H + \theta n^* + \eta\beta$ increases in β implies that $\eta - \beta\eta - \frac{1}{2}\theta n^*$ decreases in β.

Next note that

$$\text{(A-58)} \qquad \left(\frac{S_i^*}{E_i^{M*}}\right)^{\text{ind}} = \frac{\int_i S_i^* di}{\int_i E_i^{M*} di} = \frac{\int_i \left[(1-\beta)\eta - \theta(n^* - i)\right] di}{\int_i \left[(1-\beta)\eta\right] di} = 1 - \frac{\frac{\theta n^*}{2}}{\left[(1-\beta)\eta\right]}$$

decreases and

$$\text{(A-59)} \qquad (\omega_i)^{\text{ind}} = \frac{\int_i \left[E_i^{M*} - S_i^*\right] di}{\int_i E_i^{M*} di} = \frac{\int_i \left[(1-\beta)\eta - (1-\beta)\eta + \theta(n^* - i)\right]}{\int_i (1-\beta)\eta} = \frac{\frac{\theta n^*}{2}}{(1-\beta)\eta}$$

increases in β as

$$\text{(A-60)} \qquad \frac{\frac{\theta n^*}{2}}{(1-\beta)\eta} = \frac{\theta}{2}\frac{\alpha}{\sqrt{(F_H + \theta n^* + \eta\beta)(1-\beta)^2\eta^2}}$$

increases in β when $(F_H + \eta\beta)(1-\beta)^2\eta^2$ decreases in β. This, in turn, occurs as taking derivatives

$$\text{(A-61)} \qquad \begin{aligned} \frac{\partial (F_H + \eta\beta)(1-\beta)^2\eta^2}{\partial\beta} &= \eta(1-\beta)^2\eta^2 - 2(F_H + \eta\beta)(1-\beta)\eta^2 \\ &= \left[\eta(1-\beta) - 2(F_H + \eta\beta)\right]\eta^2(1-\beta) < 0 \end{aligned}$$

under the assumption that $F_H > \eta$, as is implied by our assumption that $F_L > \phi F_H$. Also,

$$\text{(A-62)} \qquad (\text{ROA}_i^*)^{\text{ind}} = \frac{\int_i X_i^T di}{\int_i F_i^* di} = \frac{\int_i \left(\frac{\alpha}{n^*}\right)^2 di}{\int_i \left[F_H + \theta i\right] di} = \frac{\left(\frac{\alpha}{n^*}\right)^2}{F_H + \frac{\theta n^*}{2}}$$

increases in β.

The comparative statics results for the partial derivatives with respect to θ follow from the results in Propositions 2 and 3 and the fact that θn^* is increasing in while n^* is decreasing in θ given equation(13), as the relevant ratios can be written as

$$\text{(A-63)} \qquad \left(\frac{D_i^*}{S_i^*}\right)^{\text{ind}} = \frac{\int_i D_i^* di}{\int_i S_i^* di} = \frac{\int_i \left[\left(\frac{\alpha}{n^*}\right)^2 - \eta\right] di}{\int_i \left[(1-\beta)\eta - \theta(n^* - i)\right] di} = \frac{\left(\frac{\alpha}{n^*}\right)^2 - \eta}{\left[(1-\beta)\eta - \frac{\theta n^*}{2}\right]},$$

$$\text{(A-64)} \qquad \left(\frac{S_i^*}{E_i^{M*}}\right)^{\text{ind}} = \frac{\int\limits_i S_i^* di}{\int\limits_i E_i^{M*} di} = \frac{\int\limits_i [(1-\beta)\eta - \theta(n^* - i)]\, di}{\int\limits_i [(1-\beta)\eta]\, di} = 1 - \frac{\frac{\theta n^*}{2}}{[(1-\beta)\eta]},$$

$$\text{(A-65)} \qquad (\omega_i)^{\text{ind}} = \frac{\int\limits_i \left[E_i^{M*} - S_i^*\right] di}{\int\limits_i E_i^{M*} di} = \frac{\int\limits_i [(1-\beta)\eta - (1-\beta)\eta + \theta(n^* - i)]\, di}{\int\limits_i (1-\beta)\eta di} = \frac{\frac{\theta n^*}{2}}{(1-\beta)\eta},$$

$$\text{(A-66)} \qquad (\text{ROA}_i^*)^{\text{ind}} = \frac{\int\limits_i X_i^T di}{\int\limits_i F_i^* di} = \frac{\int\limits_i \left(\frac{\alpha}{n^*}\right)^2 di}{\int\limits_i [F_H + \theta i]\, di} - 1 = \frac{\left(\frac{\alpha}{n^*}\right)^2}{F_H + \frac{\theta n^*}{2}} - 1 = \frac{\beta\eta + \frac{\theta n^*}{2}}{F_H + \frac{\theta n^*}{2}}.$$

The last result follows as $\beta\eta < F_H$ by our assumption that $F_L > \phi F_H$.