Online Appendix
Option-Based Estimation of the Price of Co-Skewness and Co-Kurtosis Risk

Overview

In this Online Appendix, we report on various empirical results that complement the analysis in the paper. Section A.1 discusses computation of the option-implied moments. Section A.2 discusses our approach for bootstrapping the measures of fit, ΔMSE and ΔR^2, and the confidence intervals. Section A.3 presents additional empirical results on the price of co-skewness risk. Section A.4 presents our estimates of the price of co-kurtosis risk and discusses the empirical results.

A.1. Extracting Option-Implied Moments

We implement estimation of the risk-neutral second and third moments using the method of Bakshi and Madan (2000). We use data on S&P500 index options from OptionMetrics for the period January 1996 to December 2012. We use the implied volatility estimates reported in OptionMetrics to approximate a continuum of implied volatilities, which are in turn converted to a continuum of prices. For strike prices outside the available range, we simply use the implied volatility of the lowest or highest available strike price.

Following standard practice, we filter out options that (i) violate no-arbitrage conditions; (ii) have missing or extreme implied volatility (larger than 200% or lower than 0.01%); (iii) with open-interest or bid price equal to zero; and (iv) have a bid-ask spread lower than the minimum tick size, i.e., bid-ask spread below \$0.05 for options with prices lower than \$3 and bid-ask spread below \$0.10 for option with prices equal or higher than \$3.

Let S_t denote the value of the market index and $R_{m,t+\tau} = \ln S_{t+\tau} - \ln S_t$ its return over the horizon τ. We can get the risk-neutral second moment via

$$\begin{aligned} E_t^Q \left[R_{m,t+\tau}^2 \right] &= e^{r\tau} \int_{S_t}^{\infty} \frac{2\left(1 - \ln\left[K/S_t\right]\right)}{K^2} C_t\left(\tau, K\right) dK \\ &\quad + e^{r\tau} \int_{0}^{S_t} \frac{2\left(1 + \ln\left[S_t/K\right]\right)}{K^2} P_t\left(\tau, K\right) dK. \end{aligned}$$

where $C_t(\tau, K)$ and $P_t(\tau, K)$ are call and put options quoted at time t with maturity τ and strike price K. We can get the option-implied third moment via

$$\begin{aligned} E_t^Q \left[R_{m,t+\tau}^3 \right] &= e^{r\tau} \int_{S_t}^{\infty} \frac{6 \ln\left[K/S_t\right] - 3\left(\ln\left[K/S_t\right]\right)^2}{K^2} C_t\left(\tau, K\right) dK \\ &\quad - e^{r\tau} \int_{0}^{S_t} \frac{6 \ln\left[S_t/K\right] + 3\left(\ln\left[S_t/K\right]\right)^2}{K^2} P_t\left(\tau, K\right) dK. \end{aligned}$$

When computing these moments, we eliminate put options with strike prices of more than 105% of the underlying asset price ($K/S > 1.05$) and call options with strike prices of less than 95% of the underlying asset price ($K/S < 0.95$). We only estimate the moments for days that have at least two OTM call prices and two OTM put prices available.

Since we do not have a continuum of strike prices, we calculate the integrals using cubic splines. For each maturity, we interpolate implied volatilities using a cubic spline across moneyness levels (K/S) to obtain a continuum of implied volatilities. For moneyness levels below or above the available moneyness level in the market, we use the implied volatility of the lowest or highest available strike price. After implementing this interpolation-extrapolation technique, we obtain a fine grid of implied volatilities for moneyness levels between 1% and 300%. We then convert these implied volatilities into call and put prices using the following rule: moneyness levels smaller than 100% ($K/S < 1$) are used to generate put prices and moneyness levels larger than 100% ($K/S > 1$) are used to generate call prices using trapezoidal numerical integration. Linear interpolation between maturities is used to calculate the moments for a fixed 30-day horizon.

A.2 Bootstrapping Measures of Fit and Confidence Intervals

A.2.1 Bootstrapping ΔMSE and ΔR^2

To assess the statistical significance of ΔMSE and ΔR^2, we rely on bootstrapping. Our null hypothesis is that the option and regression predictions perform equally well, that is $\Delta MSE = 0$ and $\Delta R^2 = 0$, when forecasting the one-month ahead portfolio returns for a given set of test assets. This will be the case when the sample average of $\left(\varepsilon_t^{p,OI}\right)^2$ is equal to that of $\left(\varepsilon_t^{p,RB}\right)^2$ for each portfolio in the set considered. To see this, let us consider ΔMSE first. Given equations (23) and (24), we can write

$$\begin{aligned}\Delta MSE &= \frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{25}\sum_{p=1}^{25}\left(\varepsilon_t^{p,RB}\right)^2-\left(\varepsilon_t^{p,OI}\right)^2\right)\cdot 12\cdot 100 \\ &= \frac{12\cdot 100}{25}\sum_{p=1}^{25}\left(\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^{p,RB}\right)^2-\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^{p,OI}\right)^2\right). \qquad (1)\end{aligned}$$

Thus, $\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^{p,RB}\right)^2=\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^{p,OI}\right)^2$ implies $\Delta MSE = 0$. A similar argument applies for ΔR^2 since from equations (25) and (26), we have

$$\begin{aligned}\Delta R^2 &= \frac{100}{25}\sum_{p=1}^{25}\left(1-\frac{\sum_{t=1}^{T}\left(\varepsilon_t^{p,OI}\right)^2}{\sum_{t=1}^{T}\left(\varepsilon_t^{p,RB}\right)^2}\right) \\ &= \frac{100}{25}\sum_{p=1}^{25}\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^{p,RB}\right)^2-\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^{p,OI}\right)^2}{\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^{p,RB}\right)^2}\right). \qquad (2)\end{aligned}$$

Therefore $\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^{p,RB}\right)^2=\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^{p,OI}\right)^2$ also results in $\Delta R^2 = 0$.

In order to draw statistical inference on the significance level of ΔMSE and ΔR^2, we must obtain an estimate of their respective distribution under H_0. To this end, we first adjust the sample average of $\left(\varepsilon_t^{p,OI}\right)^2$ to match that of $\left(\varepsilon_t^{p,RB}\right)^2$ for each portfolio in a given set of test assets. This is done by recentering $\left(\tilde{\varepsilon}_t^{p,OI}\right)^2$ according to

$$\left(\tilde{\varepsilon}_t^{p,OI}\right)^2 = \left(\varepsilon_t^{p,OI}\right)^2 - \frac{1}{T}\sum_{u=1}^{T}\left(\varepsilon_u^{p,OI}\right)^2 + \frac{1}{T}\sum_{u=1}^{T}\left(\varepsilon_u^{p,RB}\right)^2 \text{ for all } t \text{ and } p. \tag{3}$$

Note that based on the time-series of adjusted squared errors $\left\{\left(\tilde{\varepsilon}_t^{p,OI}\right)^2, \left(\varepsilon_t^{p,RB}\right)^2\right\}_{t=1}^{T}$, we have $\Delta MSE = 0$ and $\Delta R^2 = 0$ by construction.[1]

The bootstrap simulations for ΔMSE and ΔR^2 now proceed as follows:

Step 1: We draw with replacement from $\left\{\left(\tilde{\varepsilon}_u^{p,OI}\right)^2, \left(\varepsilon_u^{p,RB}\right)^2\right\}_{u=1}^{T}$ to obtain a bootstrapped sample of the squared errors under H_0 of size T. We denote it $\left\{\left(\tilde{\varepsilon}_{u,b}^{p,OI}\right)^2, \left(\varepsilon_{u,b}^{p,RB}\right)^2\right\}_{u=1}^{T}$ where b refers to a particular bootstrap sample.

Step 2: Using the bootstrapped sample $\left\{\left(\tilde{\varepsilon}_{u,b}^{p,OI}\right)^2, \left(\varepsilon_{u,b}^{p,RB}\right)^2\right\}_{u=1}^{T}$, we then compute ΔMSE_b and ΔR_b^2 according to equations (23), (24), (25), and (26).

Step 3: We repeat Steps 1 and 2 B times to obtain $\{\Delta MSE_b, \Delta R_b^2\}_{b=1}^{B}$ which can now be used to estimate the distributions of ΔMSE and ΔR^2 under the null hypothesis.

Step 4: Finally, we calculate the one-sided p-value of the ΔMSE and ΔR^2 obtained for a given set of test assets by computing

$$p\text{-}value\left(\Delta MSE\right) = \frac{1}{B}\sum_{b=1}^{B}\mathbf{1}\left\{\Delta MSE_b > \Delta MSE\right\} \tag{4}$$

$$p\text{-}value\left(\Delta R^2\right) = \frac{1}{B}\sum_{b=1}^{B}\mathbf{1}\left\{\Delta R_b^2 > \Delta R^2\right\} \tag{5}$$

where $\mathbf{1}\{*\}$ is the indicator function.

We are also interested in estimating the significance level of ΔMSE and ΔR^2 when the 25 portfolios of the four test assets are combined together. In this case, we have a total of 100 portfolios instead of 25 when calculating ΔMSE and ΔR^2. We use a similar methodology as the one described above. We first bootstrap $\left\{\left(\tilde{\varepsilon}_u^{p,OI}\right)^2, \left(\varepsilon_u^{p,RB}\right)^2\right\}_{u=1}^{T}$ for each of the 100 portfolios, compute the corresponding ΔMSE_b and ΔR_b^2, and then infer the one-sided p-values of these statistics.

[1] Adjusting the regression-based squared errors instead of the option-implied errors does not change their respective variance and thus has no impact on the results.

A.2.2 Bootstrapping Confidence Intervals

Given the time-series of the estimators' (unadjusted) squared errors, $\left\{ \left(\varepsilon_t^{p,OI}\right)^2, \left(\varepsilon_t^{p,RB}\right)^2 \right\}_{t=1}^T$, the bootstrap simulations for the confidence intervals proceed as follows:

Step 1: We draw with replacement from $\left\{ \left(\varepsilon_t^{p,OI}\right)^2, \left(\varepsilon_t^{p,RB}\right)^2 \right\}_{t=1}^T$ to obtain a bootstrapped sample of size T of the squared errors. We denote it $\left\{ \left(\varepsilon_{u,b}^{p,OI}\right)^2, \left(\varepsilon_{u,b}^{p,RB}\right)^2 \right\}_{u=1}^T$ where b denotes a particular bootstrap sample.

Step 2: Using the bootstrapped sample $\left\{ \left(\varepsilon_{u,b}^{p,OI}\right)^2, \left(\varepsilon_{u,b}^{p,RB}\right)^2 \right\}_{u=1}^T$, we then compute ΔMSE_b and ΔR_b^2 according to equations (23), (24), (25) and (26).

Step 3: We repeat Steps 1 and 2 B times to obtain $\{\Delta MSE_b, \Delta R_b^2\}_{b=1}^B$ which can now be used to estimate the empirical distributions of ΔMSE and ΔR^2 including their .05 and .95 percentiles.

A.3 The Price of Co-Skewness Risk: Additional Empirical Results

In this section we present additional results on the option-implied and regression-based estimates of the price of co-skewness risk.

Figure A1 reports on the bivariate model that includes the co-skewness and market factors. We plot the time-series of the twelve-month moving average of the month-by-month cross-sectional regression estimates of the price of co-skewness risk. The estimates for the price of co-skewness risk in the bivariate models in Table 2 are thus the averages of the time series in Figure A1. Figure A1 suggests that the insignificant estimates for the price of co-skewness risk in Table 2 are due to the fact that the regression estimates are noisy.

The essence of the Fama-MacBeth procedure is to estimate the price of risk by averaging the time series of cross-sectional estimates. The fact that the estimates in Figure A1 are positive for some months therefore does not constitute a problem in itself. But it is clear that the cross-sectional estimates vary a lot over time, and that they are often positive, even when the averages reported in Table 2 are negative. Figure A1 therefore suggests that noise in the month-by-month estimates is an important problem with regression-based estimation of the price of co-skewness risk.

We now present additional results on the comparison of the predictive performance of the option-implied and regression-based estimates of the co-skewness price of risk. The results in Table 3 use a twelve-month moving average for the price of risk and a 120 month-window for the factor loadings. Table A1 reports on alternative rolling windows, both for the price of risk and the factor loadings. We report on the ΔR^2 for the 100 portfolios, but conclusions based on ΔMSE are similar. Panel A reports the ΔR^2 as a function of the rolling window and Panel B reports the corresponding bootstrapped p-values. As expected, the evidence in favor of the option-based price of risk is strongest when the estimate of the price of risk is based on a single month. This is not surprising because as the length of the moving average used for the prices of risk increases, both prices of risk converge toward their unconditional means, and the conditional nature of the test disappears. Perhaps most importantly, the ΔR^2 are always positive regardless of the estimation windows and are often highly statistically significant.

Table A2 presents cross-sectional prediction results using the different co-skew estimates in Figure 3. Panel A of Table A2 first reports estimates of the average price of risk obtained using the different approaches which can be compared with the benchmark case in Table 1. Note that the Heston model tends to deliver relatively low average physical variance estimates leading to an average price of co-skewness risk that is larger in magnitude. However, the differences are small and of course what ultimately matters is the dynamics of the estimated price of risk.

Panel B of Table A2 contains the ΔMSE and ΔR^2 metrics for comparing the option-implied and regression-based estimates of the price of co-skew risk. A comparison with Panels A and B of Table 3 shows that the results in Table 3 are very robust. The p-values are small in Table A2 for three of the four sets of test assets, indicating that the option-implied estimates of co-skew risk are significantly better than the regression-based estimates in a ΔMSE-sense. The one-sided p-values for the significance of ΔR^2 are very small everywhere in Table A2 leading to the same conclusion: The option-implied co-skew risk estimates deliver a superior fit to the cross-section of equity portfolio ex-post co-skewness returns regardless of which risk-neutral and physical variance estimate is used.

In Table A3 we consider the 1986-2007 sample period. This provides insights into the impact of the turbulent 2008-2009 financial crisis period on the performance of the option-implied price of risk. Panel A of Table A3 shows the ΔMSE results and should be compared with Panel A in Table 3. Note that our results are robust and if anything even stronger in favor of the option-implied estimates than the benchmark results in Table 3. Using all 100 test assets, the ΔMSE is 0.241 which is higher than the average of 0.193 obtained during the

1986-2012 sample period. We conclude that the superior performance of the new estimate of the price of co-skewness risk is not driven by the recent financial crisis.

Panel B of Table A3 shows that the ΔR^2 are again large. Using all test assets, the ΔR^2 is equal to 5.09%, compared to 3.70% in Table 3. We conclude that the improvements offered by the option-implied estimates of co-skewness risk over the regression-based benchmark are pervasive across different time periods and are not driven by the financial crisis.

A.4 The Price of Co-Kurtosis Risk

In this section we first discuss the estimation of the conditional third moment, which can be used to estimate the price of co-kurtosis risk. Subsequently we present and discuss estimates of the conditional price of co-kurtosis risk.

A.4.1 Estimating the Physical Third Moment

It is well known that modeling the third moment dynamics under the physical measure is challenging, partly because it is much less persistent than the second moment–particularly so at the monthly frequency. Our own empirical implementation confirmed these challenges. The unconditional third moment estimate for monthly S&P500 returns during 1986-2012 is not statistically different from zero at conventional confidence levels, and moreover it is very small compared to the estimates of risk-neutral moments in our sample.

We therefore present and discuss different estimates of the price of co-kurtosis risk. In our benchmark implementation, we set the physical third moment equal to zero. Setting the physical third moment to zero is preferable to using noisy estimates. We also report on the price of co-kurtosis risk using two alternative estimates for the physical third moment: a constant third moment computed using daily data; and a fully dynamic physical third moment, estimated using a version of the dynamic moment model in Jondeau and Rockinger (2003).

Our implementation of the Jondeau and Rockinger (2003) model is close to the model they refer to as Model 2. We implement this model using monthly data. The model is given by

$$R_{m,t} = h_t z_t \qquad\qquad z_t \sim GT(z_t|\eta_t, \lambda_t),$$

where $R_{m,t}$ is the return on the market in month t, GT denotes the generalized student-t distribution, and where the higher-moment dynamics are modeled via

$$\begin{aligned}
h_t^2 &= a_0 + b_0^+ \left(R_{m,t-1}^+\right)^2 + b_0^- \left(R_{m,t-1}^-\right)^2 + c_0 h_{t-1}^2, \\
\widetilde{\eta}_t &= a_1 + b_1^+ R_{m,t-1}^+ + b_1^- R_{m,t-1}^-, \\
\widetilde{\lambda}_t &= a_2 + b_2^+ R_{m,t-1}^2, \\
\eta_t &= g_{]2,+30]}\left(\widetilde{\eta}_t\right), \text{ and } \lambda_t = g_{]-1,1]}\left(\widetilde{\lambda}_t\right)
\end{aligned}$$

where $R_m^+ = \max(R_m, 0)$ and $R_m^- = \max(-R_m, 0)$. The logistic map

$$g_{]x_L,x_U]}(x) = x_L + \frac{x_U - x_L}{1 + \exp(-x)}$$

ensures that $2 < \eta_t < \infty$ and $-1 < \lambda_t < 1$, which are necessary conditions for the existence of the *GT* distribution. Note that we have set the conditional mean return to zero here because it is difficult to model and unlikely to matter much for the dynamics of higher moments.

The density of Hansen's (1994) *GT* distribution is defined by

$$GT(z_t|\eta_t, \lambda_t) = \begin{cases} b_t c_t \left(1 + \frac{1}{\eta_t - 2}\left(\frac{b_t z_t + a_t}{1 - \lambda_t}\right)^2\right)^{-(\eta_t+1)/2} & \text{if } z_t < -a_t/b_t, \\ b_t c_t \left(1 + \frac{1}{\eta_t - 2}\left(\frac{b_t z_t + a_t}{1 + \lambda_t}\right)^2\right)^{-(\eta_t+1)/2} & \text{if } z_t \geq -a_t/b_t, \end{cases}$$

where

$$a_t \equiv 4\lambda_t c_t \frac{\eta_t - 2}{\eta_t - 1}, \quad b_t \equiv 1 + 3\lambda_t^2 - a_t^2, \quad c_t \equiv \frac{\Gamma\left(\left(\eta_t + 1\right)/2\right)}{\sqrt{\pi\left(\eta_t - 2\right)}\Gamma\left(\eta_t/2\right)}.$$

We need the non-centered second and third conditional moments, which can be computed as follows

$$E_t^P\left[R_{m,t+1}^2\right] = h_{t+1}^2,$$

and

$$E_t^P\left[R_{m,t+1}^3\right] = h_{t+1}^3\left[m_{3,t+1} - 3a_{t+1}m_{2,t+1} + 2a_{t+1}^3\right]/b_{t+1}^3.$$

where

$$m_{2,t} = 1 + 3\lambda_t^2, \quad m_{3,t} = 16c_t\lambda_t\left(1 + \lambda_t^2\right)\frac{\left(\eta_t - 2\right)^2}{\left(\eta_t - 1\right)\left(\eta_t - 3\right)},$$

Note that the third moment exists in the model so long as $\eta_t > 3$. We estimate the model monthly by maximum likelihood using 10-year rolling windows of returns.

A.4.2 The Price of Co-Kurtosis Risk: Empirical Results

Panel A of Table A4 reports on the price of co-kurtosis risk using the benchmark method, which sets the physical third moment equal to zero, and two alternative estimates for the physical third moment: first, a constant third moment computed using daily data; second, a fully dynamic physical third moment, estimated using a version of the dynamic moment model in Jondeau and Rockinger (2003) discussed above. Our implementation is close to the model Jondeau and Rockinger (2003) refer to as Model 2, which is among the more parsimonious models they consider and which is sufficiently richly parameterized for our purposes. Panel A of Table A4 indicates that one of the resulting estimates of the price of co-kurtosis risk is very similar to the estimate of 0.022 obtained using a zero physical third moment; the third estimate is also positive but smaller.

Figure A2 reports on estimates of co-kurtosis risk obtained using Fama-MacBeth regressions from a bivariate model that includes the co-kurtosis and market factors. Figure A2 indicates that the twelve-month average of the month-by-month estimates of the price of co-kurtosis risk vary considerably over time, and that they are often negative. However, the average of the time-series estimates are all positive for the four sets of test assets and range from 0.0069 for Size/Short-term reversal to 0.0210 for Size/Book-to-market. In line with the conclusions drawn for the regression-based estimates of co-skewness risk, the statistical significance of these estimates is weak and strongly depends on the set of test assets considered.

Tables A4 and A5 report on the point estimates as well as the statistical significance for the out-of-sample ΔMSE and ΔR^2 metrics applied to the co-kurtosis price of risk, constructed as in Section V.A. Panel B of Table A4 reports on a bivariate model with the market price of risk and the co-kurtosis price of risk. Table A5 reports on the model that also includes the co-skew price of risk. In Table A4 we report on all three methods used to compute the physical third moment. In Table A5 we only report on two implementations with zero and constant physical skew, but we get similar conclusions when using the other estimate of the physical third moment.

All of the ΔMSE and ΔR^2 in Table A4 are positive. Regardless of the model used for the physical third moment, the improvements are smallest for the 25 size/LTR portfolios, and the statistical evidence in favor of the co-skew price of risk is weakest for these test portfolios. For the other three sets of test portfolios, the p-values are very small both for the ΔMSE and ΔR^2 measures. This is also the case when using all 100 test assets in the last column.

When using both the co-skew and the co-kurtosis price of risk in Table A5, the statistical evidence in favor of the option-implied estimates is even stronger, in the sense that the p-values are now also small for the 25 size/LTR portfolios. Note that these are out-of-sample tests, and that the performance of the model therefore cannot simply be attributed to the fact that more pricing factors are included.

We conclude that the model with co-kurtosis risk performs well in predicting the cross-section of portfolio returns when using price of risk estimated from option-based risk premia.

Figure A1: Regression-Based Estimates of the Price of Co-Skewness Risk

Notes to Figure: We plot time series of the cross-sectional prices of co-skewness risk, multiplied by 100 for expositional convenience. Each month, we estimate factor loadings using a 120-month rolling window of monthly returns from a time series regression of excess portfolio returns on the factors. We then run a cross-sectional regression of returns on estimated loadings to obtain the risk premiums and take a twelve-month moving average. We consider four sets of test portfolios: 25 Size/Book-to-Market, 25 Size/Momentum, 25 Size/Short-Term Reversal and 25 Size/Long-Term Reversal. The sample period is from January 1986 through December 2012.

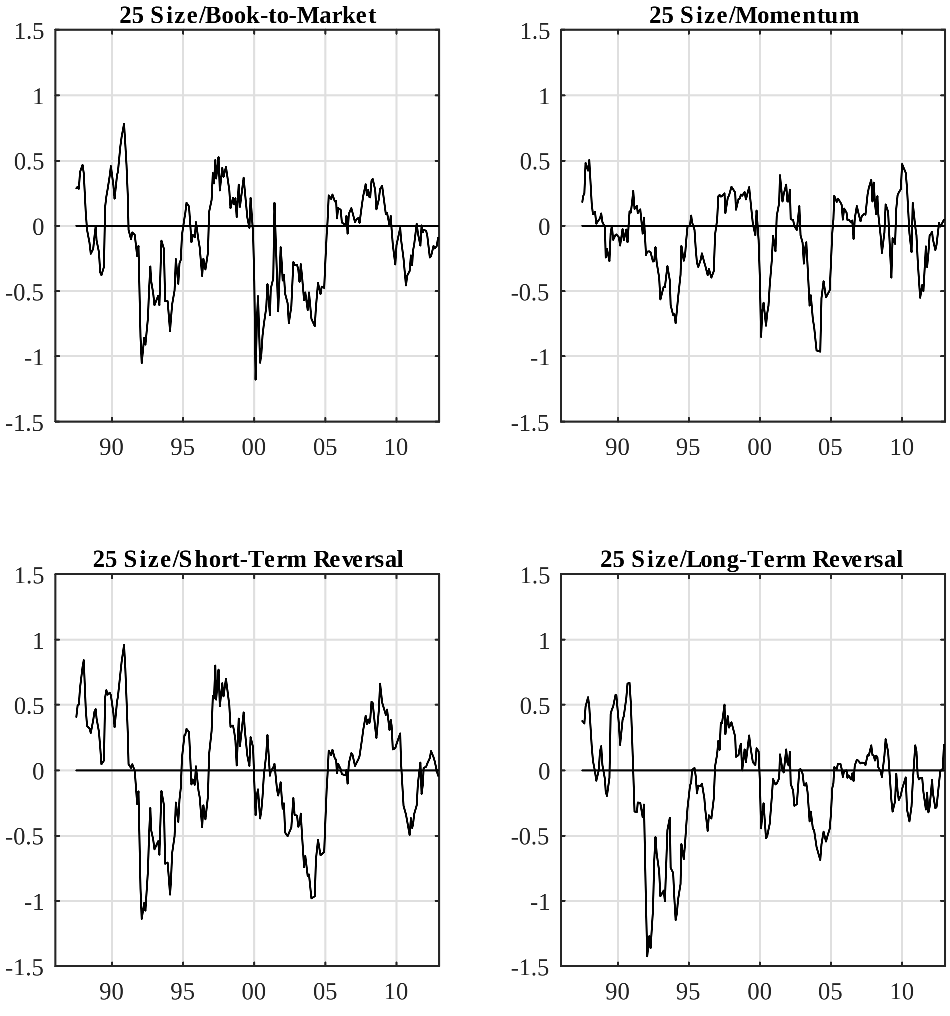

Figure A2: Regression-Based Estimates of the Price of Co-Kurtosis Risk

Notes to Figure: We plot time series of the cross-sectional prices of co-kurtosis risk, multiplied by 100 for expositional convenience. Each month, we estimate factor loadings using a 120-month rolling window of monthly returns from a time series regression of excess portfolio returns on the factors. We then run a cross-sectional regression of returns on estimated loadings to obtain the risk premiums and take a twelve-month moving average. We consider four sets of test portfolios: 25 Size/Book-to-Market, 25 Size/Momentum, 25 Size/Short-Term Reversal and 25 Size/Long-Term Reversal. The sample period is from January 1996 through December 2012.

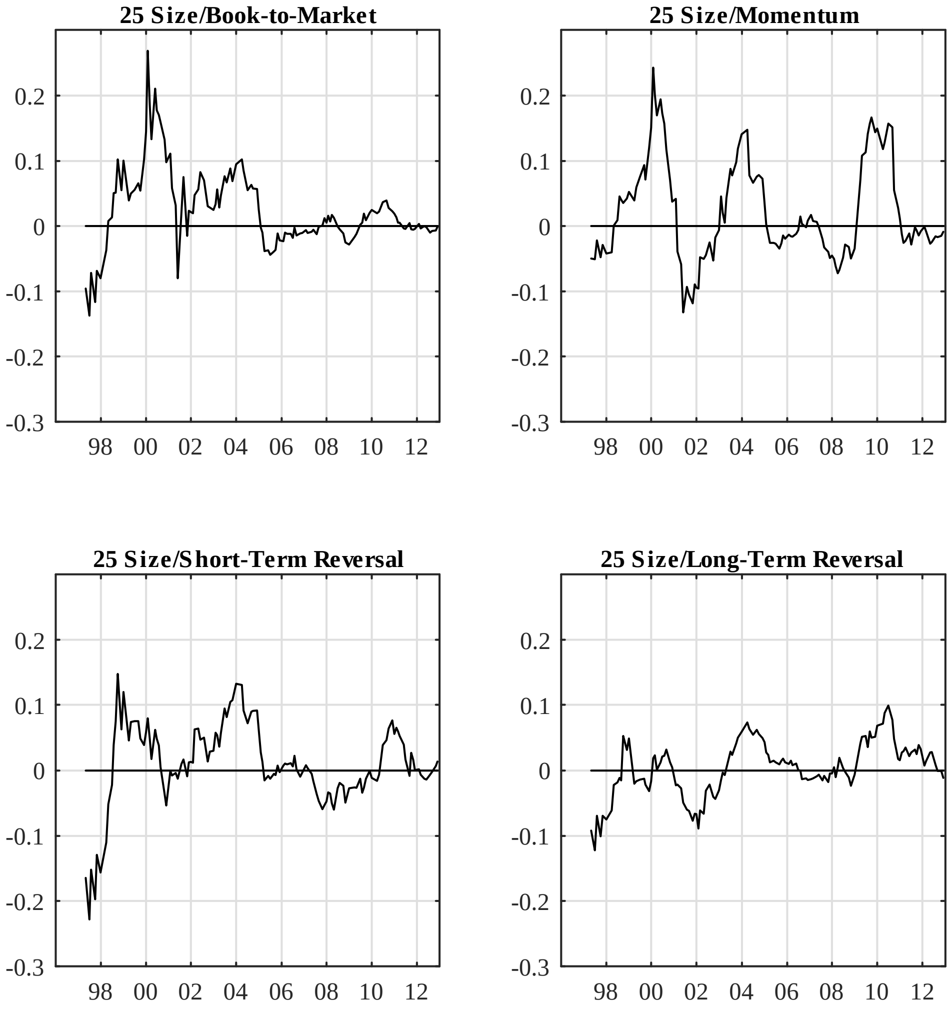

Table A1: Robustness Analysis: Estimation Window for Loadings and the Price of Risk

Notes to Table: Panel A reports the relative R^2 for the 100 test portfolios. Panel B reports the corresponding p-values. The estimation of loadings is based on rolling windows ranging from 36 to 120 months. The estimation of the price of risk is based on rolling windows ranging from 1 to 120 months. We refer to Table 3 and Appendices A.1 and A.2 for details on the computation of the relative R^2 and the bootstrapped p-values. The sample periods vary depending on the estimation windows used for the price of risk. For example, the sample period is 1986-2012 when using unsmoothed monthly estimates (the first row in both panels). It is 1991-2012 when using ten-year averages of the monthly estimates (the next-to-last row in both panels).

Panel A: Relative R-Squared

# of Months used to Compute Price of Risk.	# of Months Used to Estimate Loading: 36	48	60	72	84	96	108	120	Average
1	10.19	10.17	11.87	12.49	14.77	16.94	17.50	16.45	13.80
12	2.83	2.30	2.17	2.81	2.49	3.99	3.89	3.70	3.02
24	2.10	1.37	0.53	1.17	0.85	1.46	1.13	1.24	1.23
36	1.88	1.15	0.56	0.98	0.90	1.42	1.23	1.46	1.20
48	1.29	0.99	0.41	0.88	0.72	0.93	0.72	0.85	0.85
60	0.76	0.72	0.35	0.65	0.65	0.63	0.41	0.71	0.61
72	0.33	0.46	0.24	0.59	0.67	0.57	0.47	0.67	0.50
84	0.04	0.44	0.26	0.53	0.74	0.66	0.60	0.82	0.51
96	0.05	0.48	0.24	0.44	0.68	0.61	0.57	0.68	0.47
108	0.23	0.63	0.64	0.78	0.96	0.82	0.73	0.78	0.70
120	0.20	0.61	0.52	0.58	0.67	0.61	0.56	0.62	0.55
Average	1.81	1.76	1.62	1.99	2.19	2.60	2.53	2.54	2.13

Panel B: One-Sided Bootstrapped P-Values

# of Months used to Compute Price of Risk.	# of Months Used to Estimate Loading: 36	48	60	72	84	96	108	120	Average
1	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
12	0.2%	0.6%	1.2%	1.0%	1.5%	0.0%	0.1%	0.1%	0.6%
24	0.1%	0.5%	20.6%	6.6%	15.7%	4.5%	11.5%	9.6%	8.6%
36	0.0%	0.4%	11.1%	5.0%	8.1%	1.0%	3.7%	2.6%	4.0%
48	0.0%	0.1%	14.0%	3.6%	8.7%	2.6%	9.4%	9.1%	5.9%
60	1.5%	0.8%	12.5%	4.3%	8.5%	7.3%	20.5%	12.3%	8.5%
72	14.7%	5.2%	15.2%	2.7%	3.8%	4.8%	13.1%	10.0%	8.7%
84	43.5%	5.6%	9.6%	2.6%	1.3%	1.2%	4.9%	3.9%	9.1%
96	41.9%	4.2%	10.1%	4.0%	1.2%	0.6%	3.3%	4.9%	8.8%
108	17.2%	1.2%	0.1%	0.1%	0.1%	0.1%	1.0%	2.3%	2.8%
120	18.6%	0.5%	0.6%	1.3%	1.8%	1.0%	3.0%	3.8%	3.8%
Average	12.5%	1.7%	8.6%	2.8%	4.6%	2.1%	6.4%	5.3%	5.5%

Table A2: Robustness Analysis for the Price of Co-Skewness Risk

Notes to Table: In Panel A, we provide estimates of the average price of co-skewness risk using alternative estimators of the monthly physical and risk-neutral moments. The moments and prices of risk are multiplied by 100 for expositional convenience. In Panel B, we document the ΔMSE and R^2 for each set of test assets and for the 100 portfolios (last column), using different moment estimators for the option-implied price of co-skewness risk. We refer to Table 3 and Appendices A.1 and A.2 for details on the computation of these statistics and their bootstrapped (B.S.) p-values. The sample periods are 1986-2012 and 1996-2012.

Panel A: Average Price of Co-Skewness Risk

Sample Period	Physical 2nd Moment	Risk-Neutral 2nd Moment	Physical Estimate	Risk-Neutral Estimate	Average Price of Co-Skewness Risk
1986-2012	Heston	VIX^2	0.2519	0.4499	-0.1979
1986-2012	AR	VIX^2	0.2921	0.4499	-0.1578
1996-2012	HAR	BKM	0.3405	0.5153	-0.1748
1996-2012	Heston	BKM	0.2944	0.5153	-0.2209
1996-2012	AR	BKM	0.3367	0.5153	-0.1786

Panel B: Difference in Mean Squared Error and Relative R^2

	25 Size/BM	25 Size/Mom	25 Size/STR	25 Size/LTR	All
		Heston & VIX^2 (1986-2012)			
ΔMSE	0.330	0.128	0.226	0.074	0.190
B.S. p-value	*0.11%*	*4.23%*	*0.18%*	*15.98%*	*0.19%*
ΔR^2	5.87	2.48	4.55	1.53	3.61
B.S. p-value	*0.02%*	*3.44%*	*0.07%*	*20.45%*	*0.21%*
		AR & VIX^2 (1986-2012)			
ΔMSE	0.325	0.129	0.231	0.080	0.191
B.S. p-value	*0.16%*	*3.89%*	*0.14%*	*14.00%*	*0.18%*
ΔR^2	5.79	2.54	4.67	1.69	3.67
B.S. p-value	*0.03%*	*2.90%*	*0.06%*	*17.81%*	*0.14%*
		HAR & BKM (1996-2012)			
ΔMSE	0.492	0.223	0.207	0.107	0.257
B.S. p-value	*0.14%*	*1.92%*	*1.45%*	*8.91%*	*0.26%*
ΔR^2	7.42	3.74	3.50	1.97	4.16
B.S. p-value	*0.01%*	*1.36%*	*1.27%*	*12.60%*	*0.23%*
		Heston & BKM (1996-2012)			
ΔMSE	0.489	0.215	0.191	0.097	0.248
B.S. p-value	*0.14%*	*2.73%*	*2.51%*	*11.87%*	*0.48%*
ΔR^2	7.33	3.61	3.21	1.72	3.97
B.S. p-value	*0.02%*	*2.07%*	*2.34%*	*16.57%*	*0.54%*
		AR & BKM (1996-2012)			
ΔMSE	0.493	0.229	0.218	0.109	0.262
B.S. p-value	*0.17%*	*1.62%*	*1.12%*	*8.15%*	*0.24%*
ΔR^2	7.44	3.88	3.70	2.03	4.26
B.S. p-value	*0.01%*	*0.99%*	*0.93%*	*11.65%*	*0.19%*

Table A3: Robustness Analysis: 1986-2007 Sample

Notes to Table: Panel A reports the ΔMSE for each set of test assets. The option-implied price of co-skewness risk is measured using HAR and VIX^2. Below the ΔMSE estimates, we report the bootstrapped one-sided p-values, allowing for one autocorrelation lag of the monthly differences in mean squared errors, and the bootstrapped 90% confidence interval bounds. We refer to Table 3 and Appendices A.1 and A.2 for details on the computation of ΔMSE and the bootstrapped (B.S.) confidence bounds. Panel B reports for each set of test assets the relative R^2, the bootstrapped p-values of the ΔR^2 and the lower and upper 90% confidence interval bounds. In the last column of both panels, we report on the pricing performance when all 100 portfolios are considered. We refer to Appendices A.1 and A.2 for further details about the methodology used to estimate the p-values and confidence bounds of the R^2. All bootstrap results are based on 100,000 sample draws. The data are monthly and the sample period is from January 1986 to December 2007.

Panel A: Difference in Mean Squared Error

	25 Size/BM	25 Size/Mom	25 Size/STR	25 Size/LTR	All
ΔMSE	0.379	0.188	0.267	0.129	0.241
B.S. p-value	*0.20%*	*1.27%*	*0.19%*	*6.12%*	*0.11%*
B.S. 5 Percentile Bound	0.191	0.051	0.125	-0.006	0.118
B.S. 95 Percentile Bound	0.582	0.326	0.417	0.267	0.367

Panel B: Relative R^2

	25 Size/BM	25 Size/Mom	25 Size/STR	25 Size/LTR	All
ΔR^2	7.01	3.93	5.99	3.43	5.09
B.S. p-value	*0.04%*	*1.00%*	*0.04%*	*7.78%*	*0.09%*
B.S. 5 Percentile Bound	3.69	1.00	2.88	-0.54	2.37
B.S. 95 Percentile Bound	10.29	6.76	8.89	7.53	7.78

Table A4: Co-Kurtosis Out-of-Sample Tests, 1996-2012

Notes to Table: For each portfolio p in a given set of test assets, we estimate model k's monthly forecast error as

$$\epsilon_{t+1}^{p,k} = R_{p,t+1} - \lambda_{RB,t}^{MKT} \beta_{p,t}^{CAPM} - \lambda_{k,t}^{COKU} \beta_{p,t}^{COKU},$$

where $\lambda_{RB,t}^{MKT}$ is the 12-month moving average of the regression based price of market risk, and $\beta_{p,t}^{F}$ is portfolio p's exposure to factor F estimated by OLS regression based on the most recent 120 months, including month t. $\lambda_{RB,t}^{COKU}$ is the 12-month moving average of the regression based price of co-kurtosis risk and $\lambda_{OI,t}^{COKU}$ is the 12-month moving average of the option-implied price of co-kurtosis risk. Each month, we calculate the cross-sectional difference in mean squared error of the models' forecasts using all portfolios p in a given set of test assets according to

$$\Delta MSE_t = \left(\frac{1}{25} \sum_{p=1}^{2} 5 (\epsilon_t^{p,RB})^2 - (\epsilon_t^{p,OI})^2 \right) \times 12 \times 100,$$

where we multiply by 12 to annualize and by 100 to express the difference in percentages. We then compute the sample average of this measure for each test asset, $\Delta MSE = \frac{\sum_{t=1}^{T} \Delta MSE_t}{T}$. We compute the percentage relative R^2 across portfolios according to

$$\Delta R^2 = \frac{1}{25} \sum_{p=1}^{2} 5 \left(1 - \frac{\sum_{t=1}^{T} (\epsilon_t^{p,OI})^2}{\sum_{t=1}^{T} (\epsilon_t^{p,RB})^2} \right) \times 100.$$

In Panel A, we provide estimates of the average price of co-kurtosis risk using alternative estimators of the monthly physical and risk-neutral moments. Panel B reports the ΔMSE for each set of test assets, the bootstrapped (B.S.) one-sided p-value of ΔMSE, as well as the 90% confidence interval bounds. We also report the ΔR^2 for each set of test assets, its bootstrapped p-value and 90% confidence bounds. In the last column of Panel B, we report on results when considering all 100 portfolios. We refer to Appendices A.1 and A.2 for further details about the methodology developed to estimate p-values and confidence bounds. All bootstrap results are based on 100,000 draws. The data are monthly and the sample period is from January 1996 to December 2012.

Panel A: The Average Price of Co-Kurtosis Risk

Physical Third Moment Specification	Average Physical Third Moment	Average Risk-Neutral Third Moment	Average Price of Co-Kurtosis Risk
Zero Skewness	0	-0.0220	0.0220
Constant Skewness from Daily Returns	-0.0006	-0.0220	0.0214
Jondeau and Rockinger (2003)	-0.0104	-0.0220	0.0116

Panel B: Difference in Mean Squared Error and Relative R^2

	25 Size/BM	25 Size/Mom	25 Size/STR	25 Size/LTR	All
		Zero Skewness & BKM			
ΔMSE	0.199	0.488	0.296	0.016	0.250
B.S. p-value	*0.93%*	*0.04%*	*0.25%*	*37.83%*	*0.01%*
ΔR^2	3.45	8.66	5.63	0.44	4.54
B.S. p-value	*0.39%*	*0.00%*	*0.05%*	*35.37%*	*0.00%*
		Constant Skewness & BKM			
ΔMSE	0.199	0.487	0.294	0.017	0.249
B.S. p-value	*0.95%*	*0.04%*	*0.26%*	*36.95%*	*0.01%*
ΔR^2	3.44	8.64	5.61	0.46	4.54
B.S. p-value	*0.41%*	*0.00%*	*0.06%*	*34.51%*	*0.00%*
		Jondeau and Rockinger (2003) & BKM			
ΔMSE	0.182	0.464	0.278	0.001	0.231
B.S. p-value	*2.04%*	*0.07%*	*0.40%*	*50.55%*	*0.03%*
ΔR^2	3.16	8.29	5.33	0.12	4.22
B.S. p-value	*1.06%*	*0.00%*	*0.10%*	*46.63%*	*0.00%*

Table A5: Co-Skewness and Co-Kurtosis Out-of-Sample Tests, 1996-2012

Notes to Table: For each portfolio p in a given set of test assets, we estimate model k's monthly forecast error as

$$\epsilon_{t+1}^{p,k} = R_{p,t+1} - \lambda_{RB,t}^{MKT}\beta_{p,t}^{CAPM} - \lambda_{k,t}^{COSK}\beta_{p,t}^{COSK} - \lambda_{k,t}^{COKU}\beta_{p,t}^{COKU},$$

where $\lambda_{RB,t}^{MKT}$ is the 12-month moving average of the regression based price of market risk, and $\beta_{p,t}^{F}$ is portfolio p's exposure to factor F estimated by OLS regression based on the most recent 120 months, including month t. $\lambda_{RB,t}^{COSK}$ and $\lambda_{RB,t}^{COKU}$ are the 12-month moving average of the regression based prices of risk. $\lambda_{OI,t}^{COSK}$ and $\lambda_{OI,t}^{COKU}$ are the 12-month moving average of the option-implied prices of risk. Each month, we calculate the cross-sectional difference in mean squared error of the models' forecast using all portfolios p in a given set of test assets according to

$$\Delta MSE_t = \left(\frac{1}{25}\sum_{p=1}^{2} 5(\epsilon_t^{p,RB})^2 - (\epsilon_t^{p,OI})^2\right) \times 12 \times 100,$$

where we multiply by 12 to annualize and by 100 to express the difference in percentages. We then compute the sample average of this measure for each test asset, $\Delta MSE = \frac{\sum_{t=1}^{T}\Delta MSE_t}{T}$. We compute the percentage relative R^2 across portfolios according to

$$\Delta R^2 = \frac{1}{25}\sum_{p=1}^{2} 5\left(1 - \frac{\sum_{t=1}^{T}(\epsilon_t^{p,OI})^2}{\sum_{t=1}^{T}(\epsilon_t^{p,RB})^2}\right) \times 100.$$

Panel A reports the ΔMSE for each set of test assets, the bootstrapped (B.S.) one-sided p-value of ΔMSE and its 90% confidence interval bounds. Panel B reports the average R^2 for each set of test assets, its bootstrapped p-value and 90% confidence bounds. In the last column of both panels, we report results when considering all 100 portfolios. We refer to Appendix A.1 and A.2 for further details about the methodology developed to estimate p-values and confidence bounds. All bootstrap results are based on 100,000 draws. The data are monthly and the sample period is from January 1996 to December 2012.

Panel A: Difference in Mean Squared Error

	25 Size/BM	25 Size/Mom	25 Size/STR	25 Size/LTR	All
	COSK: HAR& BKM, COKU: Zero Skew & BKM				
ΔMSE	0.483	0.542	0.360	0.258	0.411
B.S. p-value	*0.00%*	*0.00%*	*0.00%*	*1.30%*	*0.00%*
B.S. 5 Percentile Bound	0.265	0.307	0.169	0.049	0.235
B.S. 95 Percentile Bound	0.710	0.775	0.560	0.464	0.585
	COSK: HAR& BKM, COKU: Constant Skew & BKM				
ΔMSE	0.486	0.544	0.360	0.260	0.413
B.S. p-value	*0.03%*	*0.01%*	*0.22%*	*2.00%*	*0.01%*
B.S. 5 Percentile Bound	0.267	0.313	0.167	0.048	0.236
B.S. 95 Percentile Bound	0.710	0.781	0.562	0.469	0.591

Panel B: Average Relative R2

	25 Size/BM	25 Size/Mom	25 Size/STR	25 Size/LTR	All
	COSK: HAR& BKM, COKU: Zero Skew & BKM				
ΔR^2	7.80	9.40	6.37	5.49	7.27
B.S. p-value	*0.00%*	*0.00%*	*0.00%*	*1.30%*	*0.00%*
B.S. 5 Percentile Bound	4.46	6.05	3.13	1.10	4.24
B.S. 95 Percentile Bound	10.92	12.81	9.72	10.10	10.03
	COSK: HAR& BKM, COKU: Constant Skew & BKM				
ΔR^2	7.84	9.41	6.38	5.54	7.29
B.S. p-value	*0.00%*	*0.00%*	*0.09%*	*1.73%*	*0.00%*
B.S. 5 Percentile Bound	4.61	5.97	3.10	1.08	4.29
B.S. 95 Percentile Bound	10.94	12.68	9.69	10.00	10.20