Online Appendix to

“Fragmentation and Strategic Market-Making”

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The aim of this online Appendix is to present the proof of the main results, a number of additional results, a discussion on the relevance of our results for limit order book environments, the description of variables used in the regressions, and robustness tests.

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1 Proof

1.A Intermediaries' trading profits

Market-maker i's trading profit is given by:

$$V_i\left(p_1^D, p_2^D, p_1^S, p_2^S\right) = \begin{cases} \underbrace{p_i^D Q_D + p_i^S Q_S - TC_i(Q_D + Q_S)}_{\equiv v_i(Q_D+Q_S)} & \text{if } p_i^D Q_D < p_{-i}^D Q_D \text{ and } p_i^S Q_S < p_{-i}^S Q_S, \\ \underbrace{p_i^D Q_D - TC_i(Q_D)}_{\equiv v_i(Q_D)} & \text{if } p_i^D Q_D < p_{-i}^D Q_D \text{ and } p_i^S Q_S > p_{-i}^S Q_S, \\ \underbrace{p_i^S Q_S - TC_i(Q_S)}_{\equiv v_i(Q_S)} & \text{if } p_i^D Q_D > p_{-i}^D Q_D \text{ and } p_i^S Q_S < p_{-i}^S Q_S, \\ 0 & \text{if } p_i^D Q_D > p_{-i}^D Q_D \text{ and } p_i^S Q_S > p_{-i}^S Q_S. \end{cases}$$

where we denote by $TC_i(Q)(= r_i(Q) \times Q)$ the inventory costs of absorbing the shock Q for market-maker i ($Q = Q_S, Q_D$ or $Q_D + Q_S$), by p_i^D the price set by market-maker i in venue D, and by p_i^S the price posted by i in venue S, $i = 1, 2$. The price p_i^m is an ask price if $Q_m > 0$ and a bid price if $Q_m < 0$, $m = D, S$.[1]

1.B Proof of Lemma 1

We consider two cases separately.

Case 1 ("Consolidation"). We first look for the necessary conditions to be simultaneously filled to guarantee the existence of an equilibrium in which a single market-maker simultaneously absorbs the shock in the dominant venue and the shock in the satellite venue. Market-maker $i \in \{1, 2\}$ executes the global order flow in equilibrium if and only if she simultaneously posts the best price in the dominant venue and the satellite venue. The lowest ask price a_i^D prevailing in venue D, and the lowest ask price p_i^S (resp. highest bid price) prevailing in venue S when $Q_S > 0$ (resp. when $Q_S < 0$) are such that:

i: trading $Q_D + Q_S$ is profitable for market-maker i (i.e., $v_i(Q_D + Q_S) \geq 0$), and (i') not for market-maker $-i$ (i.e., $v_{-i}(Q_D + Q_S) < 0$);

ii: trading $Q_D + Q_S$ is more profitable for market-maker i than trading only Q_D (i.e., $v_i(Q_D + Q_S) \geq v_i(Q_D)$), or (ii') only Q_S (i.e., $v_i(Q_D + Q_S) \geq v_i(Q_S)$);

iii: undercutting market-maker i is not profitable for market-maker $-i$ neither in venue D (i.e., $v_{-i}(Q_D) < 0$), nor (iii') in venue S (i.e., $v_{-i}(Q_S) < 0$).

[1] Similar to Biais (1993), the utility function of intermediaries given in Equation (1) is linearized, under the assumption $Q_D < I_u - I_d$. Note that, in our transparent setting, the criticism on the linear approximation used by Biais (1993) for opaque markets raised by de Frutos and Manzano (2002) does not apply. The assumption $Q_D < I_u - I_d$ also guarantees that market-maker i has a probability of posting the best price in venue m which is strictly greater than 0 and strictly lower than 1, for $i = 1, 2$ and $m = D, S$.

Using the expression of market-makers' trading profits V, this set of conditions rewrites as follows:

$$
\begin{aligned}
\text{i} &: a_i^D Q_D + p_i^S Q_S \geq TC_i(Q_D + Q_S), \\
\text{i'} &: a_{-i}^D Q_D + p_{-i}^S Q_S < TC_{-i}(Q_D + Q_S); \\
\text{ii} &: a_i^D Q_D + p_i^S Q_S - TC_i(Q_D + Q_S) \geq a_i^D Q_D - TC_i(Q_D), \\
\text{ii'} &: a_i^D Q_D + p_i^S Q_S - TC_i(Q_D + Q_S) \geq a_i^S Q_S - TC_i(Q_S); \\
\text{iii} &: a_i^D Q_D < TC_{-i}(Q_D), \\
\text{iii'} &: p_i^S Q_S < TC_{-i}(Q_S).
\end{aligned}
$$

• *Conjecture 1: the best-quoting market-maker in all venues is market-maker 1.* Under Conjecture 1, conditions (ii) and (iii') write $a_1^D Q_D + p_1^S Q_S - TC_1(Q_D + Q_S) \geq a_1^D Q_D - TC_1(Q_D)$ and $p_1^S Q_S < TC_2(Q_S)$. Condition (ii) rewrites $p_1^S Q_S \geq TC_1(Q_D+Q_S) - TC_1(Q_D)$. Combining with (iii'), we deduce that:

$$
\text{(IA.1)} \qquad TC_1(Q_D + Q_S) < TC_1(Q_D) + TC_2(Q_S).
$$

Straightforward computations show further that if Eq. (IA.1) is verified, or equivalently $(I_1 - I_2 - Q_D) \times Q_S > 0$, then all conditions (i) to (iii') simultaneously hold, and Conjecture 1 is verified.

• *Conjecture 1a: the best-quoting market-maker in all venues is market-maker 2.* In that case, conditions (i) and (i') rewrite $a_2^D Q_D + p_2^S Q_S \geq TC_2(Q_D + Q_S)$ and $a_1^D Q_D + p_1^S Q_S < TC_1(Q_D+Q_S)$. Given that market-maker 2 is the best-quoter, we obtain $a_2^D Q_D < a_1^D Q_D$ and $p_2^S Q_S < p_1^S Q_S$.[2] However, recall that $I_1 > I_2$ or, equivalently, $TC_1(Q_D + Q_S) < TC_2(Q_D + Q_S)$. Therefore, condition (i) cannot hold in that case and Conjecture 1a is not verified.

Case 2 ("Specialization"). We now look for the necessary conditions to be simultaneously filled to guarantee the existence of an equilibrium in which each liquidity shock is absorbed by a different market-maker.

There exists an equilibrium such that market-maker i posts the lowest ask price a_i^D in venue D and the opponent $-i$ posts the lowest ask (resp. highest bid) price p_i^S in venue S when $Q_S > 0$ (resp. $Q_S < 0$) if and only if:

(I) trading Q_D is profitable for market-maker i (i.e., $v_i(Q_D) \geq 0$), and (I') trading Q_S is profitable for market-maker $-i$ (i.e., $v_{-i}(Q_S) \geq 0$).

(II) market-maker i is better off trading Q_D rather than Q_S (i.e., $v_i(Q_D) > v_i(Q_S)$) and (III') market-maker $-i$ is better off trading Q_S rather than Q_D (i.e., $v_{-i}(Q_S) > v_{-i}(Q_D)$);

(III) market-maker i is better off trading Q_D only rather than $Q_D + Q_S$ (i.e., $v_i(Q_D) > v_i(Q_D + Q_S)$) and (II') market-maker $-i$ is better off trading Q_S only rather than $Q_D + Q_S$ (i.e., $v_{-i}(Q_S) > v_{-i}(Q_D + Q_S)$) ;

[2] If $Q_S > 0$, $a_2^S < a_1^S$ and thus $a_2^S Q_S < a_1^S Q_S$. If $Q_S < 0$, $b_2^S > b_1^S$ and thus $b_2^S Q_S < b_1^S Q_S$. We thus can write $p_2^S Q_S < p_1^S Q_S$ for any sign of Q_S.

These conditions may be rewritten as follows:

$$\begin{aligned}
\text{I} &: a_i^D Q_D - TC_i(Q_D) \geq 0,\\
\text{I '} &: p_{-i}^S Q_S - TC_{-i}(Q_S) \geq 0,\\
\text{II} &: a_i^D Q_D - TC_i(Q_D) > p_i^S Q_S - TC_i(Q_S),\\
\text{II'} &: p_{-i}^S Q_S - TC_{-i}(Q_S) > a_{-i}^D Q_D - TC_{-i}(Q_D),\\
\text{III} &: a_i^D Q_D - TC_i(Q_D) > a_i^D Q_D + p_i^S Q_S - TC_i(Q_D + Q_S),\\
\text{III'} &: p_{-i}^S Q_S - TC_{-i}(Q_S) > a_{-i}^D Q_D + p_{-i}^S Q_S - TC_{-i}(Q_D + Q_S).
\end{aligned}$$

• *Conjecture 2: market-maker 1 trades Q_D and market-maker 2 trades Q_S.* Under Conjecture 2 and based on condition (III), we get $TC_1(Q_D + Q_S) - TC_1(Q_D) > p_1^S Q_S$. In case $Q_S > 0$, we know that $p_1^S > p_2^S$ and, using condition I', we get $TC_1(Q_D + Q_S) - TC_1(Q_D) > p_1^S Q_S > p_2^S Q_S > TC_2(Q_S)$. In case $Q_S < 0$, we get $-p_2^S Q_S \geq -p_1^S Q_S$, or using I' and III, we get $-TC_2(Q_S) > -p_2^S Q_S \geq -p_1^S Q_S > TC_1(Q_D) - TC_1(Q_D + Q_S)$. We thus obtain that:

$$TC_1(Q_D + Q_S) > TC_1(Q_D) + TC_2(Q_S) \tag{IA.2}$$

Straightforward computations show that if Eq. (IA.2) is verified then the set of conditions I to III' hold simultaneously and Conjecture 2 is verified.

• *Conjecture 2a: market-maker 1 trades Q_S and market-maker 2 trades Q_D.* Given that $I_1 < I_2$, straightforward computations lead to the following inequality:

$$TC_1(Q_D) + TC_2(Q_S) < TC_2(Q_D) + TC_1(Q_S). \tag{IA.3}$$

Under Conjecture 2a, we have $a_1^D Q_D > a_2^D Q_D$ and $p_2^S Q_S > p_1^S Q_S$. Combining Conjecture 2a with Inequality (IA.3), we obtain $a_1^D Q_D + p_2^S Q_S - TC_1(Q_D) - TC_2(Q_S) > a_2^D Q_D + p_1^S Q_S - TC_2(Q_D) - TC_1(Q_S)$, which contradicts conditions II and II' combined. Conjecture 2a is thus not verified. ■

1.C Proof of Proposition 1

From Lemma 1, we know that we must consider two cases according to the sign of $TC_1(Q_D + Q_S) - (TC_1(Q_D) + TC_2(Q_S))$, or, equivalently, of $(I_1 - I_2 - Q_D) \times Q_S$.

Case 1. Suppose that $(I_1 - I_2 - Q_D) \times Q_S > 0$ **("Virtual consolidation").** In that case, we know that market-maker 1 posts the best prices in all venues (Lemma 1). We now have to consider two sub-cases according to the sign of Q_S.

Case 1.1. Suppose that $Q_S > 0$**.** Following Condition (IA.1), we must have $I_1 - I_2 > Q_D$. Market-maker 1 posts the lowest selling price both in venue D and S. The ask prices a_1^D and a_1^S are the maximum prices that satisfy the set of conditions i to iii' (Lemma 1). Combining conditions (ii') and (iii) and conditions (ii) and (iii') and using reservation prices, we get:

$$\begin{aligned}
\text{ii' and iii} &: r_1(Q_D) + \rho\sigma^2 Q_S \leq a_1^D < r_2(Q_D),\\
\text{ii and iii'} &: r_1(Q_S) + \rho\sigma^2 Q_D \leq a_1^S < r_2(Q_S).
\end{aligned}$$

From the two first inequalities, natural candidates for the equilibrium are $(a_1^D)^* = r_2(Q_D) - \varepsilon$ and $(a_1^S)^* = r_2(Q_S) - \varepsilon$, as they are the maximum prices that satisfy conditions ii and iii, ii' and iii'. Straightforward computations show that they also satisfy conditions i and i' described above (details are omitted for brevity).

Case 1.2. Suppose that $Q_S < 0$. In that case, we must have $I_1 - I_2 < Q_D$ to satisfy Condition (IA.1). Market-maker 1 thus posts the lowest selling price in venue D and the highest bid price in venue S. The ask price a_1^D and the bid price b_1^S are such that they must satisfy the set of conditions (ii) to (iii') that we rewrite as follows:

$$\begin{aligned} &\text{ii' and iii} : r_1(Q_D) + \rho\sigma^2(Q_S) \leq a_1^D < r_2(Q_D), \\ &\text{ii and iii'} : r_2(Q_S) < b_1^S \leq r_1(Q_S) + \rho\sigma^2 Q_D. \end{aligned}$$

The natural candidates for the equilibrium are $a_1^D = r_2(Q_D) - \varepsilon$ and $b_1^S = r_2(Q_S) + \varepsilon$. These equilibrium prices must satisfy the following inequality $a_1^D Q_D + b_1^S Q_S < (a_2^D Q_D + b_2^S Q_S \leqslant)TC_2(Q_D + Q_S)$ (condition (i')). It is however not the case, implying that this constraint is binding and equilibrium prices must be such that:

$$(a_1^D)^* = r_2(Q_D + Q_S)\frac{(Q_D + Q_S)}{Q_D} + (b_1^S)^* \frac{(-Q_S)}{Q_D} - \varepsilon. \tag{IA.4}$$

First, using the expression of $(a_1^D)^*$ defined in Eq. (IA.4) in market-maker 1's trading profit, we obtain $v_1(Q_D + Q_S) = \rho\sigma^2(I_1 - I_2)(Q_D + Q_S)$. This expression does not depend on equilibrium prices. Consequently, there exists a continuum of prices that may sustain the equilibrium. Second, using $(a_1^D)^*$ defined in Eq. (IA.4) in conditions (ii') and (iii) combined, we get that $(b_1^S)^*$ must satisfy:

$$\text{ii' and iii} : r_2(Q_S) - \rho\sigma^2(I_1 - I_2)\frac{Q_D}{-Q_S} \leq (b_1^S)^* < r_2(Q_S) + \rho\sigma^2 Q_D.$$

We also know from conditions (ii) and (iii') combined that $(b_1^S)^*$ is such that:

$$\text{ii and iii'} : r_2(Q_S) < (b_1^S)^* \leq r_1(Q_S) + \rho\sigma^2 Q_D.$$

Since $I_1 > I_2$, we however have $r_2(Q_S) - \rho\sigma^2(I_1 - I_2)\frac{Q_D}{-Q_S} < r_2(Q_S)$ and $r_1(Q_S) + \rho\sigma^2 Q_D < r_2(Q_S) + \rho\sigma^2 Q_D$. The second inequality defined by (ii) and (iii') combined is constraining both the minimum and the maximum possible bid price in venue S. Within all equilibria defined by $(a_1^D)^*$ in Eq. (IA.4) and by $(b_1^S)^* \in (r_2(Q_S) + \epsilon, r_1(Q_S) + \rho\sigma^2 Q_D + \epsilon]$ we select the only equilibrium such that prices are continuous at $I_1 - I_2 = Q_D$, that is, $(a_1^D)^* = r_2(Q_D) + \rho\sigma^2(Q_S) - \epsilon \equiv \hat{r}_2(Q_D) - \epsilon$, from which we deduce that $(b_2^S)^* = r_2(Q_S) + \epsilon$.

Case 2. Suppose that $(I_1 - I_2 - Q_D) \times Q_S < 0$ ("Specialization"). From Lemma 1, we know that market-maker 1 posts the best price in venue D while market-maker 2 posts the best price in venue S. We now have to consider two sub-cases according to the sign of Q_S.

Case 2.1. Suppose that $Q_S > 0$. In that case, we must have $I_1 - I_2 < Q_D$ to satisfy Condition (IA.1). The ask price a_1^D posted by market-maker 1 and the ask price a_2^S posted by market-maker 2 are such that they must satisfy the set of conditions I to III', from which we deduce that:

$$\begin{aligned}
&\text{I and III'} : r_1(Q_D) \leq a_1^D < a_2^D < r_2(Q_D) + \rho\sigma^2 Q_S, \\
&\text{I' and III} : r_2(Q_S) \leq a_2^S < a_1^S < r_1(Q_S) + \rho\sigma^2 Q_D.
\end{aligned}$$

The candidates for the equilibrium are $a_1^D = r_2(Q_D)+\rho\sigma^2 Q_S-\varepsilon$ and $a_2^S = r_1(Q_S)+\rho\sigma^2 Q_D-\varepsilon$. These equilibrium prices must satisfy the following inequality $a_2^S Q_S - a_1^D Q_D (> a_2^S Q_S - a_2^D Q_D) > r_2(Q_S)Q_S - r_2(Q_D)Q_D$ (condition (II')). It is however not the case, implying that this constraint is binding and equilibrium prices must be such that:

$$\text{(IA.5)} \qquad (a_1^D)^* = r_2(Q_D) + ((a_2^S)^* - r_2(Q_S))\frac{Q_S}{Q_D} - \varepsilon.$$

First, if $(a_1^D)^*$ defined in Eq. (IA.5), then condition II always holds (given that $(I_1-I_2)(Q_D-Q_S) > 0$). Second, using $(a_1^D)^*$ defined in Eq. (IA.5) in conditions I and III' and I' and III combined, we get that $(a_2^S)^*$ must satisfy the following inequalities:

$$\begin{aligned}
&\text{I and III'} : r_2(Q_S) + (r_1(Q_D) - r_2(Q_D))\frac{Q_D}{Q_S} \leq (a_2^S)^* < r_2(Q_S) + \rho\sigma^2 Q_D, \\
&\text{I' and III} : r_2(Q_S) \leq (a_2^S)^* < r_1(Q_S) + \rho\sigma^2 Q_D.
\end{aligned}$$

Straightforward computations show that conditions I' and III combined is constraining the set of possible prices $(a_2^S)^*$. Third, we compute market-makers' equilibrium profits and show that, in that case, the trading profit of market-maker 2 writes: $v_2(Q_S) = ((a_2^S)^* - r_2(Q_S))Q_S$. Using the expression of $(a_2^D)^*$ defined in Eq. (IA.5), we then obtain that the trading profit of market-maker 1 writes:

$$v_1(Q_D) = \left(r_2(Q_D) + ((a_2^S)^* - r_2(Q_S))\frac{Q_S}{Q_D} - r_1(Q_D)\right) Q_D.$$

We observe that market-makers' profits are both strictly increasing in $(a_2^S)^*$. Consequently, market-makers' reaction functions are such that the best ask price in venue S is the highest possible one. From conditions I and III' combined, we deduce that $(a_S)^*$ is such that:

$$\text{(IA.6)} \qquad (a_2^S)^* = r_1(Q_S) + \rho\sigma^2 Q_D - \varepsilon, \text{ or } (a_2^S)^* = \hat{r}_1(Q_S) - \varepsilon,$$

from which we deduce that:

$$\text{(IA.7)} \qquad (a_1^D)^* = \hat{r}_2(Q_D) - \rho\sigma^2 Q_S \times \eta - \varepsilon,$$

where $\eta = \frac{(I_1-I_2)}{Q_D}$.

Consequently, there exists a unique equilibrium such that market-maker 1 posts $(a_1^D)^*$ (defined in Eq. (IA.7)) and trades Q_D while market-maker 2 posts the best ask price equal to $(a_2^S)^*$ (defined in Eq. (IA.6)) and trades Q_S.

Case 2.2. Suppose that $Q_S < 0$. In that case, we have $I_1 - I_2 > Q_D$ (Condition (IA.1)). Market-maker 1 posts the best ask price in D while market-maker 2 posts the best bid price in S. The ask price a_1^D in venue D and the bid price b_2^S in venue S are respectively the maximum and the minimum prices that satisfy the set of conditions I to III'. Combining Condition (II) and (III) and Condition (II') and (III'), we get:

$$\text{II and III} : r_1(Q_D) \le a_1^D < r_2(Q_D) + \rho\sigma^2 Q_S,$$
$$\text{II' and III'} : r_1(Q_S) + \rho\sigma^2 Q_D < b_2^S \le r_2(Q_S).$$

From the two first inequalities, $a_1^D = r_2(Q_D) - \rho\sigma^2(-Q_S) - \varepsilon$ and $b_2^S = r_1(Q_S) + \rho\sigma^2 Q_D + \varepsilon$ are natural candidates for the equilibrium. Straightforward computations show that they also satisfy conditions I and I'.■

1.D Proof of Proposition 2

We decompose the proof into two results, depending on the sign of Q_S.
Notations. For ease of computation in the proof, we use the following notations $q_m = Q_m$ for a net-buying order flow and $q_m = -Q_m$ for a net-selling order flow ($m = S, D$). Let us also define $v_d = \mu - \rho\sigma^2 I_d$, $v_u = \mu - \rho\sigma^2 I_u$, $x = \mu - \rho\sigma^2 I_1$ and $y = \mu - \rho\sigma^2 I_2$. The support of the uniform distribution function of x and y simplifies to $[v_u, v_d]$. We also note $d = \rho\sigma^2 q_D$ and $s = \rho\sigma^2 q_S$. Finally, let $a^{m,+}$ (resp. $a^{m,-}$) be the best ask price of venue m when liquidity demands have the same sign (resp. opposite sign) across venues.

Result 1 *Suppose that shocks have the same sign (with probability γ). Then, the expected ask prices quoted in the venues D and S are equal to:*
(IA.8)

$$E\left(\underline{a}^{m,+}\right) = \mu - \rho\sigma^2 \frac{2I_d + I_u}{3} + \frac{\rho\sigma^2 q_m}{2} + \rho\sigma^2 q_{-m}\left(\frac{q_D}{I_u - I_d} - \frac{1}{3}\left(\frac{q_D}{I_u - I_d}\right)^2\right), m = S, D.$$

Proof. We first compute the expected ask that prevails in venue D. By definition,

$$E\left(\underline{a}^{D,+}\right) = E\left(\min\left(a_1^D, a_2^D\right)\mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S>0}\right).$$

Given Proposition 1, the notations defined above, and the symmetry of our hypotheses, the latter equation writes:

$$\begin{aligned} E\left(\underline{a}^{D,+}\right) &= \frac{2}{(v_d - v_u)^2}\Bigg[\int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(y + \frac{d}{2})dydx + \int_{v_u}^{v_d}\int_{x}^{v_d}\left(y + \frac{d}{2} + s\left(\frac{d-(y-x)}{d}\right)\right)dydx \\ &\quad - \int_{v_u}^{v_d-d}\int_{x+d}^{v_d}\left(y + \frac{d}{2} + s\left(\frac{d-(y-x)}{d}\right)\right)dydx\Bigg]. \qquad \text{(IA.9)} \end{aligned}$$

We now turn to the expected ask prevailing in venue S using a similar reasoning. The expression writes:

$$
\begin{aligned}
E\left(\underline{a}^{S,+}\right) &= E\left(\min\left(a_1^S, a_2^S\right) \mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S>0}\right) \\
&= \frac{2}{\left(v_d - v_u\right)^2}\Bigg[\int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(y+\frac{s}{2})dydx + \int_{v_u}^{v_d}\int_{x}^{v_d}\left(x+\frac{s}{2}+d\right)dydx \\
&- \int_{v_u}^{v_d-d}\int_{x+d}^{v_d}\left(x+\frac{s}{2}+d\right)dydx\Bigg]. \qquad \text{(IA.10)}
\end{aligned}
$$

Computations based on Eq. (IA.9) and on Eq. (IA.10) yield the expressions given in Eq. (IA.8) for $m = D$ and $m = S$ respectively. Q.E.D.

Result 2 *Suppose that shocks have opposite signs (with probability $1-\gamma$), then the expected ask prices in venues D and S respectively write:*

$$
\begin{aligned}
E\left(\underline{a}^{D,-}\right) &= \mu - \rho\sigma^2\frac{2I_d + I_u}{3} + \frac{\rho\sigma^2 q_D}{2} - \rho\sigma^2 q_S, \qquad \text{(IA.11)} \\
E\left(\underline{a}^{S,-}\right) &= \mu - \rho\sigma^2\frac{2I_d + I_u}{3} + \frac{\rho\sigma^2 q_S}{2} + \rho\sigma^2\left[-q_D + \frac{\left(q_D\right)^2}{\left(I_u - I_d\right)} - \frac{\left(q_D\right)^3}{3\left(I_u - I_d\right)^2}\right]. \qquad \text{(IA.12)}
\end{aligned}
$$

Proof. We first compute the expected best ask prevailing in venue D (considering a sell shock in venue S):

$$E\left(\underline{a}^{D,-}\right) = E\left(\min\left(a_1^D, a_2^D\right)\mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S<0}\right),$$

which rewrites:

$$
\begin{aligned}
\text{(IA.13)} \quad E\left(\underline{a}^{D,-}\right) = \frac{2}{\left(v_d - v_u\right)^2}(&\int_{v_u}^{v_d-d}\int_{v_u}^{x+d}(y+\frac{d}{2}-s)dydx \\
&+ \int_{v_u}^{v_d}\int_{x}^{v_d}(y+\frac{d}{2}-s)dydx - \int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(y+\frac{d}{2}-s)dydx).
\end{aligned}
$$

Eq. (IA.11) immediately follows.
Symmetrically, the expected best ask prevailing in market S (considering now a sell shock in venue D) writes:

$$
\begin{aligned}
\text{(IA.14)} \quad E(\underline{a}^{S,-}) = \frac{2}{(v_d - v_u)^2}\Big(&\int_{v_u-d}^{v_d}\int_{v_u}^{x+d}(x+\frac{s}{2}+d)dydx + \int_{v_u}^{v_d}\int_{v_u}^{x}(y+\frac{s}{2})dydx \\
&- \int_{v_u-d}^{v_d}\int_{v_u}^{x+d}(y+\frac{s}{2})dydx\Big).
\end{aligned}
$$

Computations yield Eq. (IA.12). Q.E.D.
Let us define the half-spread as $s^m = a^m - \mu$ and $\phi_m = \frac{q_m}{I_u - I_d}$. Proposition 2 is then obtained from Results 1 and 2 considering the extensive form of the game represented in Figure IA.1.
■

1.E Proof of Corollary 1

Remind that $\underline{a}^c$ denotes the lowest ask price in a centralized market. From Ho and Stoll (1983), we know that:

$$E\left(\underline{a}^c\right) = \mu - \rho\sigma^2 \frac{2I_d + I_u}{3} + \frac{\rho\sigma^2(q_m + q_{-m})}{2}. \tag{IA.15}$$

Using Eq. (IA.8), (IA.11) and (IA.20) and the symmetry of the game, we deduce that the difference in expected transactions costs between a fragmented and a centralized market is:

$$\begin{aligned}\Delta E(TTrC) &= \gamma\left(E\left(\underline{a}^{D,+}\right)q_D + E\left(\underline{a}^{S,+}\right)q_S - E\left(\underline{a}^c\right)(q_D + q_S)\right)\\ &+ (1-\gamma)\left(E\left(\underline{a}^{D,-}\right)q_D - E\left(\overline{b}^{S,-}\right)q_S - E\left(\underline{a}^c\right)(q_D - q_S)\right).\end{aligned}$$

After straightforward computations the latter expression is equal to:

$$\Delta E(TTrC) = \rho\sigma^2 q_S\left(I_u - I_d\right)\left(-\frac{(\gamma+1)}{3}\right)P_\gamma(\phi_D), \tag{IA.16}$$

where $P_\gamma(x) = x^3 - 3x^2 + \frac{3}{(\gamma+1)}x + \frac{(\gamma-1)}{(\gamma+1)}$ for $x \in [0,1]$, and $\phi_D = \frac{q_D}{I_u - I_d}$.
To investigate whether expected transaction costs are larger or smaller in a centralized market, let us analyze the sign of the cubic polynomial P_γ. First, note that:

$$P'_\gamma(x) = 3x^2 - 6x + \frac{3}{(1+\gamma)} = 3\left(x - \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)\right)\left(x - \left(1 + \sqrt{\frac{\gamma}{1+\gamma}}\right)\right).$$

Given that $x \in [0,1]$, then $x - \left(1 + \sqrt{\frac{\gamma}{1+\gamma}}\right) < 0$, and the sign of $P'_\gamma(x)$ only depends on the sign of $\left(x - \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)\right)$. P_γ is increasing if $x < \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)$ and is decreasing if $x > \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)$. Thus, the local maximum is $P_\gamma(1 - \sqrt{\frac{\gamma}{1+\gamma}}) = \frac{\gamma\left(-1+2\sqrt{\frac{\gamma}{1+\gamma}}\right)}{1+\gamma}$.

• Consider the case where $\gamma \le \frac{1}{3}$. Straightforward computations show that $P_\gamma(1 - \sqrt{\frac{\gamma}{1+\gamma}}) \le 0$ (with $P_\gamma(1 - \sqrt{\frac{\gamma}{1+\gamma}}) = 0$ if $\gamma = \frac{1}{3}$). We therefore deduce that $P_\gamma \le 0$, i.e., $\Delta E(TTrC) \geqslant 0$ if $\gamma \le \frac{1}{3}$.

• Consider now the case where $\gamma > \frac{1}{3}$. We can show that $P_\gamma > 0$, or, equivalently, $\Delta E(TTrC) < 0$ iff $x \in [\Phi^1_\gamma, \Phi^2_\gamma]$ where $P_\gamma(\Phi^1_\gamma) = 0 = P_\gamma(\Phi^2_\gamma)$. Note that if $\gamma = 1$, then it is direct to show that $P_1 > 0$ if $x \in [0, \frac{(3-\sqrt{3})}{2}]$, or equivalently, $\Delta E(TTrC) < 0$ iff $\phi_D < \frac{(3-\sqrt{3})}{2}$.■

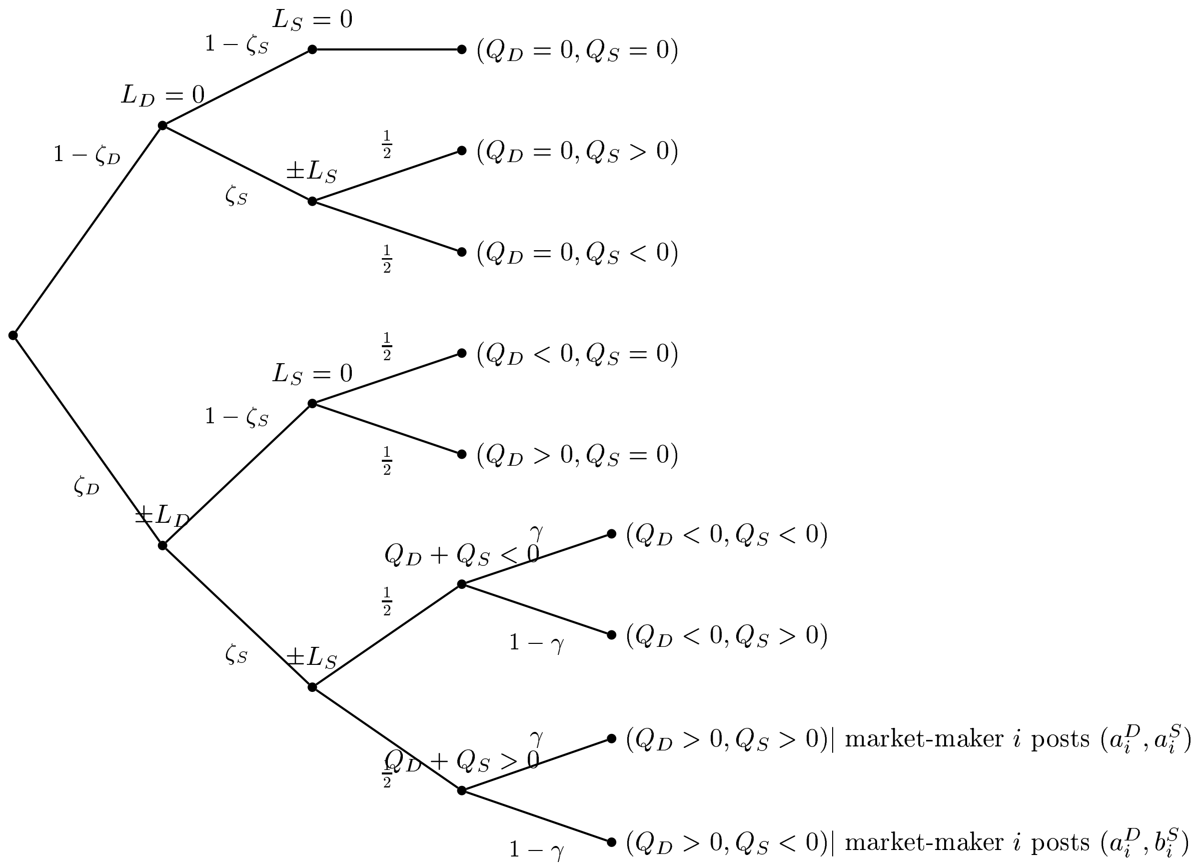

Figure IA.1: Tree of the quoting game across trading venues

Figure IA.1 represents the tree of the trading game. At date 1 (not represented on the Figure), market-maker i is endowed with an inventory position denoted I_i. At date 2, venue m is hit by a liquidity shock, denoted L_m, with probability ζ_m. L_m generates a liquidity demand Q_m, which is positive (resp. negative) with probability $\frac{1}{2}$ (resp. $\frac{1}{2}$). The probability that shocks simultaneously hit both venues is denoted ζ ($= \zeta_D \times \zeta_S$). The probability that shocks have the same sign is denoted γ. The paper analyzes price formation across venues when the global order flow is net-buying, i.e., $Q_D + Q_S > 0$. Symmetric results are obtained for a net-selling global order flow. At date 3, market-maker i posts simultaneously a price in venue D and a price in venue S. We denote a_i^m (resp. b_i^m) the ask price (resp. bid price) that i posts in venue m if $Q_m > 0$ (resp. $Q_m < 0$), $m = D, S$.

1.F Proof of Proposition 3

By definition, $Cov(s^D, s^S) = \gamma Cov(\underline{a}^{D,+} - \mu, \underline{a}^{S,+} - \mu) + (1-\gamma)Cov(\underline{a}^{D,-} - \mu, \mu - \overline{b}^{S,-}) = \gamma Cov(\underline{a}^{D,+}, \underline{a}^{S,+}) - (1-\gamma)Cov(\underline{a}^{D,-}, \overline{b}^{S,-})$. We decompose the proof into two results, depending on the sign of shocks across venues (similar or opposite).

Result 3 *Suppose that shocks have the same sign (with probability γ). The covariance between the ask price in venue D and the one in venue S is equal to:*

(IA.17)
$$\frac{Cov(\underline{a}^{D,+}, \underline{a}^{S,+})}{(\rho\sigma^2)^2(I_u - I_d)^2} = \frac{1}{18} - \phi_D\left(-\frac{\phi_D - \phi_S}{6} + \frac{2(\phi_S - \phi_D)}{9}\phi_D + \frac{15\phi_S - \phi_D}{12}\phi_D^2 + \frac{2\phi_S}{3}\phi_D^3 + \frac{\phi_S}{9}\phi_D^4\right),$$

where $\phi_m = \frac{q_m}{(I_u - I_d)}$, $m = D, S$.

Proof. By definition, $E\left(\underline{a}^{D,+}\underline{a}^{S,+}\right) = E\left(\min\left(a_1^D, a_2^D\right) \times \min\left(a_1^S, a_2^S\right) \mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S>0}\right)$. Using Proposition 1, and notations defined above, we get:

$$\text{(IA.18)} \quad E\left(\underline{a}^{D,+}\underline{a}^{S,+}\right) = \frac{2}{(v_d - v_u)^2}\Bigg[\int_{v_u}^{v_d - d}\int_{v_u}^{v_d - d}(y + \frac{s}{2})(y + \frac{d}{2})dydx$$
$$+ \int_{v_u}^{v_d}\int_{x}^{v_d}(x + \frac{s}{2} + d)(y + \frac{d}{2} + s\left(\frac{d - (y - x)}{d}\right))dydx$$
$$- \int_{v_u}^{v_d - d}\int_{x+d}^{v_d}(x + \frac{s}{2} + d)(y + \frac{d}{2} + s\left(\frac{d - (y - x)}{d}\right))\Bigg].$$

To compute $Cov(\underline{a}^{D,+}, \underline{a}^{S,+}) = E(\underline{a}^{D,+}\underline{a}^{S,+}) - E(\underline{a}^{D,+})E(\underline{a}^{S,+})$, we use the expression above and Result 1 for expressions of $E\left(\underline{a}^{D,+}\right)$ and $E\left(\underline{a}^{S,+}\right)$. Computations yield Eq. (IA.17). Q.E.D.

Result 4 *Suppose that shocks have opposite signs $(1-\gamma)$. The covariance between the best price in venue D and the one in venue S writes:*

$$\text{(IA.19)} \qquad \frac{Cov\left(\underline{a}^{D,-}, \overline{b}^{S,-}\right)}{(\rho\sigma^2)^2(I_u - I_d)^2} = \frac{1}{36} + \frac{(\phi_D)^2}{36}\left(3(\phi_D)^2 - 8\phi_D + 6\right).$$

Proof. If a sell shock hits venue S, the expected best bid in venue S is such that $E(\overline{b}^{S,-}) = E(max(b_1^S, b_2^S)\mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S<0})$, or:

$$\text{(IA.20)} \quad E(\overline{b}^{S,-}) = \frac{2}{(v_d - v_u)^2}(\int_{v_u}^{v_d - d}\int_{x+d}^{v_d}(x + \frac{s}{2} + d)dydx + \int_{v_u}^{v_d}\int_{x}^{v_d}(y + \frac{s}{2})dydx$$
$$- \int_{v_u}^{v_d - d}\int_{x+d}^{v_d}(y + \frac{s}{2})dydx)$$

When a buy shock hits venue D, the expected best ask price of venue D is thus described by Eq. (IA.11). Then $E(\underline{a}^{D,-}\overline{b}^{S,-})$ writes:

$$\text{(IA.21)}\quad E(\underline{a}^{D,-}\overline{b}^{S,-}) = \frac{2}{(v_d - v_u)^2}[\int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(y+\frac{d}{2}-s)(x+\frac{s}{2}+d)dydx$$
$$+\int_{v_u}^{v_d}\int_{x}^{v_d}(y+\frac{d}{2}-s)(y+\frac{s}{2})dydx - \int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(y+\frac{d}{2}-s)(y+\frac{s}{2})dydx].$$

Using Equations (IA.11), (IA.20) and (IA.21), we can deduce the expression of $Cov(\underline{a}^{D,-},\overline{b}^{S,-})$ described in Eq. (IA.19). Q.E.D.

From Results 3 and 4 and considering the extensive form of the game represented in Figure IA.1, we deduce that spreads co-vary jointly as follows:

$$\text{(IA.22)}\qquad cov(s^D, s^S) = \lambda\big(\rho\sigma^2(I_u - I_d)\big)^2\big(\gamma \times g_{\phi_D}(\phi_S) + a_{\phi_D}\big)$$

where a_{ϕ_D} and g_{ϕ_D} such that: $a_{\phi_D} = \frac{-1}{36} - (\phi_D)^2(\frac{1}{6} - \frac{2}{9}\phi_D)$ and

$$\text{(IA.23)}\qquad g_{\phi_D}(x) = \frac{3}{36} - \frac{(\phi_D)^4}{12} - x\phi_D\left(\frac{(\phi_D)^4}{9} - \frac{2(\phi_D)^3}{3} + \frac{5(\phi_D)^2}{4} - \frac{8\phi_D}{9} + \frac{1}{6}\right).$$

It is straightforward to show that g_{ϕ_D} is positive for any ϕ_D. We thus deduce that the covariance $cov(s^D, s^S)$ is an increasing function of γ.■

2 Model Extensions

2.A Non-strategic quotes in a fragmented market

Our paper assume that market-makers behave strategically within and across trading venues. Most of the literature analyzes, however, market fragmentation by assuming that liquidity suppliers behave competitively, setting prices such that a zero-profit condition holds. This appendix analyzes prices posted by competitive risk-averse market makers in our two-market setting. It allows us to better understand the impact of assuming *strategic* multi-venue market-makers. It also allows us to show that the "ultra-competitive effect" obtained in the model resulting from the strategic behavior of risk-averse market-makers is not obtained under the same conditions in a non-strategic inventory management model.

Proposition 2.A.1 *Assume that $I_1 > I_2$ and $Q_D + Q_S > 0$, and that market-makers behave competitively. Then they quote their true value for the asset, i.e., their own reservation price, that is:*
1. If $(I_1 - I_2 - Q_D)Q_S > 0$, then market-maker 1, with a longer position, posts the best prices across venues, that is:

(IA.24) $$(p_1^m)^{NS} = r_1(Q_D + Q_S) \quad \textit{for } m = D, S$$

2. If $(I_1 - I_2 - Q_D)Q_S \leq 0$, the longer market-maker posts the best price in the dominant market while the shorter market-maker posts the best price in the satellite market, that is:

(IA.25) $$((a_1^D)^{NS}, (p_2^S)^{NS}) = (r_1(Q_D), r_2(Q_S))$$

where p_i^m is a selling price when Q_m is a buy demand, and a bid price when Q_m is a sell demand ($i = 1, 2$, and $m = D, S$).

In a multi-venue environment, when intermediaries are competitive, best prices sometimes differ across venues for two reasons. First, when $(I_1 - I_2 - Q_D)Q_S \leq 0$, which is equivalent to $TC_1(Q_D+Q_S) \geq TC_1(Q_D)+TC_2(Q_S)$, market-maker 1 is capacity-constrained and cannot absorb both shocks. She is thus constrained to absorb the most efficient shock (in terms of risk sharing), which is the larger buy demand Q_D (see Lemma 1 in the baseline model). She thus posts her true value for executing Q_D, while market-maker 2 executes the shock in the satellite market at his reservation price for this liquidity demand Q_S. Second, because market-makers' reservation quotes reflect the price impact of trades of different size ($|Q_D| > |Q_S|$), when market-makers are not strategic, they post prices that reflect their true value for the magnitude of orders to execute, which sometimes differ (alternatively Q_D+Q_S or Q_D for market-maker 1, or Q_S for market-maker 2).

Let us now analyze the impact in terms of total trading costs. We assume also that market-makers behave non strategically in a centralized market, and thus post their true value for the asset. Market-maker 1 executes the net order flow at her reservation price $r_1(Q_D+Q_S)(< r_2(Q_D+Q_S))$. According to Lemma 1 in the baseline model, we have two cases to consider:

- If $(I_1 - I_2 - Q_D)Q_S \leq 0$, then total trading costs are the same. Fragmentation is thus innocuous in this case.

- If $(I_1 - I_2 - Q_D)Q_S \leq 0$, then the difference in total trading costs write:

  (IA.26)
  $$r_1(Q_D)Q_D + r_2(Q_S)Q_S - r_1(Q_D+Q_S) \times (Q_D+Q_S) = \rho \times \sigma^2 (I_1 - I_2 - Q_D) \times Q_S < 0.$$

  Market fragmentation is good for total trading costs in this case.

Total trading costs are lower in a two-venue setting with competitive market-makers, than in a centralized market. It is driven by a better risk sharing in the case market-maker 1 is capacity-constrained. This outcome is opposite to that obtained in our two-venue strategic duopoly model. Recall that when market-maker 1 is capacity-constrained, market-makers price high, by posting their "stay-at-home" price which takes into account their monopolist situation in their "home" venue. A better risk sharing leads to less competitive prices in our baseline model (see Appendix 2.C for a more formal proof on risk sharing).

In sum, the “intense competition” effect results from low divergence in inventories when market-makers behave non-strategically, whereas this effect is obtained when divergence in inventories is high if intermediaries behave strategically. Our main empirical result contained in Table 7 in the main model corroborates strategic inventory management.

2.B Endogenous fragmentation of the total order flow

This section extends the baseline model by assuming that a global liquidity demander has access to all venues simultaneously (through, for example, a smart order router). Let us assume that this liquidity demander must trade a given quantity denoted Q. He minimizes his total trading cost by optimally splitting orders across venues. Note that this section only extends the case in which shocks have exogenously the same sign in our baseline model. We also assume that market-maker 1 is longer than market-maker 2 and that Q is a buy order flow ($Q > 0$). Results for the case in which market-maker 2 is longer that market-maker 1 or for the case of a sell order are deduced by symmetry.

We consider that the global liquidity demander enjoys some private benefits denoted δ_m to trade in venue m. We assume that $\delta_D > \delta_S$, consistently with the dominant venue defined in the baseline model, and that $\delta_D - \delta_S < \rho\sigma^2 Q$.[3,4] The liquidity demander chooses the proportion α of the order flow routed to venue D (and $(1-\alpha)$ to venue S) so as to minimize his total trading cost.[5]

Based on assumptions in the baseline model, we suppose that the liquidity demander splits orders such that a larger demand is sent to the dominant market ($\alpha Q = Q_D \geq Q_S = (1-\alpha)Q$).[6] We thus investigates whether there exists an equilibrium when the liquidity trader optimally split orders across venues such that $\alpha \in [\frac{1}{2}; 1)$. In this interval, total trading costs write:

$$\text{(IA.27)} \quad TTrC(\alpha) = [((a_1^D)^*(\alpha Q) - \delta_D - \mu)\alpha + ((a_i^S)^*((1-\alpha)Q) - \delta_S - \mu)(1-\alpha)] \times Q.$$

[3]Numerous studies (see Froot and Dabora, 1999; Foerster and Karolyi, 1999; or Gagnon and Karolyi, 2010, among others) document the existence of a domestic bias, due to investment barriers, e.g., regulatory barriers, taxes, or information constraints. In Europe, brokerage fees charged in 2013 to trade in a foreign country or trading venue are 15 to 40% higher than those charged to trade in a national exchange, but the situation was even worse back in 2007. Differences in private benefits might also capture differences in terms of maker/taker spreads.

[4]When $\delta_D - \delta_S \geq \rho\sigma^2 Q$, the private benefits of trading in venue D are so large that it is never optimal for investors to split the quantity to be traded across trading platforms.

[5]Because markets are transparent in our set up, we assume that liquidity demanders perfectly anticipate what the best bid and ask prices will be.

[6]A complete proof of the existence and characterization of all the equilibria is available on request.

where $i = 1, 2$ depending on the divergence in inventories ($i = 1$ if $I_1 - I_2 > \alpha Q$, $i = 2$ otherwise, see Lemma 1 in the baseline model).

The following Proposition shows the existence and the characterization of an equilibrium α^*.

Proposition 2.B.1 *If $2\rho\sigma^2(I_1 - I_2) > (\delta_D - \delta_S)$, there exists an interior equilibrium α^*, such that it is optimal for the global liquidity demander to split orders across venues.*

Proof. We want to show that there exists an interior equilibrium, that is, an $\alpha^* \in [\frac{1}{2}, 1)$ that minimizes transaction costs $TTrC(.)$ described by Eq. (IA.27).
• We first conjecture that there exists an equilibrium characterized by a high divergence in intermediaries' inventories, i.e., $I_1 - I_2 - \alpha Q > 0$, or, $\frac{1}{2} \leq \alpha < \frac{I_1 - I_2}{Q}$. The first order condition (FOC) yields:

$$\alpha^H = \frac{1}{2} + \frac{\delta_D - \delta_S}{2\rho\sigma^2 Q}.$$

The two conditions for an interior equilibrium $\alpha \in [\frac{1}{2}, 1)$ to exist are thus: i. a condition ensuring that our conjecture holds, i.e., $\alpha^H < \frac{I_1 - I_2}{Q}$, and ii. a condition ensuring that the equilibrium is interior, i.e., $\alpha^H < 1$. The latter always holds under our assumption $\delta_D - \delta_S < \rho\sigma^2 Q$. Condition i. rewrites as follows:

$$I_1 - I_2 > \frac{1}{2}\left(Q + \frac{\delta_D - \delta_S}{\rho\sigma^2}\right). \tag{IA.28}$$

• We now conjecture that there exists an equilibrium characterized by a low divergence in intermediaries' inventories, i.e., $\alpha \geq \frac{I_1 - I_2}{Q}$. The FOC yields:

$$\alpha^L = \frac{1}{2} - \frac{\delta_D - \delta_S}{2\rho\sigma^2 Q} + \frac{(I_1 - I_2)}{Q}.$$

The three conditions for an interior equilibrium to exist are such that: (i) our conjecture must hold, i.e., $\alpha^L \geq \frac{I_1 - I_2}{Q}$; (ii) there exists an interior equilibrium, i.e., $\alpha^L < 1$; and (iii) $\alpha^L \geq \frac{1}{2}$. Condition (i) always holds under our assumption $\delta_D - \delta_S < \rho\sigma^2 Q$. Condition (ii) translates into $I_1 - I_2 < \frac{Q}{2} + \frac{\delta_D - \delta_S}{2\rho\sigma^2}$, which is the complement of the condition (IA.28) above. Notice that if $I_1 - I_2 = \frac{Q}{2} + \frac{\delta_D - \delta_S}{2\rho\sigma^2}$, then there exists an equilibrium such that $\alpha^* = 1$. Condition (iii) imposes $2\rho\sigma^2(I_1 - I_2) \geq \delta_D - \delta_S$.[7] ■

The liquidity demander trades off the benefits of price competition in a two-venue structure (related to the divergence of inventories, $I_1 - I_2$) to the private benefits of send-

[7] If $2\rho\sigma^2(I_1 - I_2) < \delta_D - \delta_S$, there is no solution to the FOC in$[\frac{1}{2}, 1)$. There is a corner equilibrium: $\alpha^* = 1$.

ing the entire demand to the dominant market ($\delta_D - \delta_S$). We conclude that, even when the demand splitting is endogenized, it is still the case that the market remains ex ante fragmented.

2.C Risk-sharing efficiency in a fragmented market

This section investigates the effect of having the possibility of absorbing the preferred shock (what we termed in the baseline model as the shock hitting their "home" venue) on risk sharing among market-makers. It is worth noticing that, when intermediaries specialize in their "home" venue, they obtain a better allocation of risk compared to the centralized market, as shown in the following corollary.

Corollary 2.C.1 *A fragmented market generates a more efficient outcome in risk sharing among market-makers than a centralized market in the sense that market-makers bear lower aggregate security risk.*

Proof. In our set up (equal risk aversion and identical pre-trade inventory distribution), we can measure intermediaries' aggregate post-trade risk by the sum of the variance of their post-trade wealths (Yin, 2005).

1. In a centralized market, the longer market-maker executes the net order flow. The aggregate post-trade risk, denoted by $(\sigma^2_{agg})^c$, is thus equal to:

(IA.29) $$(\sigma^2_{agg})^c = Var((I_1 - Q_D - Q_S)\tilde{v}) + Var(I_2 \times \tilde{v}).$$

2. In a fragmented market, post-trade allocations depend on the sign of $(I_1 - I_2 - Q_D)\, Q_S$ (See Lemma 1 in the baseline model).

   • If $(I_1 - I_2 - Q_D)\, Q_S > 0$, the aggregate post-trade risk is equal to that in a centralized market, since the longer market-maker consolidates the global order flow:

$$(\sigma^2_{agg})^{cons} = Var((I_1 - Q_D - Q_S)\tilde{v}) + Var(I_2 \times \tilde{v}) = (\sigma^2_{agg})^c.$$

   • If $(I_1 - I_2 - Q_D)\, Q_S \leq 0$, each shock is absorbed by a different market-maker and the aggregate post-trade risk is equal to:

(IA.30) $$(\sigma^2_{agg})^{frag} = Var((I_1 - Q_D)\tilde{v}) + Var((I_2 - Q_S)\tilde{v}).$$

Then, subtracting Eq. (IA.30) from Eq. (IA.29) is equal to
$(\sigma^2_{agg})^{frag} - (\sigma^2_{agg})^c = 2Q_S(I_1 - I_2 - Q_D) < 0$, which is negative in the case considered here.■

The intuition is as follows: in a centralized market, orders are crossed when they are of opposite direction. This implies that, if $Q_S < 0$, market-makers only execute the remaining order imbalance of $Q_D + Q_S < Q_D$. In a multiple-venue setting, orders cannot be crossed, market-makers are however able to choose to execute only trades with a desirable impact on their inventory position. In the case $Q_S < 0$, market-maker 1 chooses to execute only Q_D when she is very long ($I_1 - I_1 - Q_D > 0$), while the shorter market-maker absorb the shock in S, which results in a better risk sharing than in the centralized market. The better allocation of risk does not however necessarily lead to more competitive prices as detailed in Proposition 1 in the baseline model. This result is in the spirit of the one obtained in Biais et al. (1998).

2.D Introduction of a pre-stage inter-dealer market

This section analyzes whether our results are sensitive to the introduction of a pre-stage inter-dealer market. We assume that, at stage 0, intermediaries are able to optimally share inventory risks before setting quotes in the customer-dealer market. It could be the case that they prefer sharing risks in an inter-dealer market to avoid multi-venue competition in the customer-dealer market starting at stage 1.

In a conservative approach, we assume that intermediaries independently and non-strategically maximize their expected profit in the inter-dealer market, then maximize their expected profit in the customer-dealer market (the model is solved sequentially).[8]

Even in the presence of a pre-stage risk-sharing round, intermediaries may find more profitable ex ante not to trade in the inter-dealer market as shown by the following Corollary:

Corollary 2.D.1 *The set of parameters for which intermediaries choose not to trade in the inter-dealer market is non-empty.*

Proof. We consider two stages.

First stage: the inter-dealer market (ID). If market-maker 1 sells a quantity q at price p to market-maker 2 in the inter-dealer market, the profits of market-maker 1 and 2 respectively write:

$$\left(v_1^{ID} = \left[p - \mu - \frac{\rho\sigma^2}{2}(q - 2I_1)\right] q ; v_2^{ID} = \left[\mu - \frac{\rho\sigma^2}{2}(q + 2I_2) - p\right] q\right).$$

We maximize market-makers' profits with respect to q to find market-maker 1's supply function, and market-maker 2's demand function. The crossing of the supply and demand curves yields the following symmetric equilibrium in the inter-dealer market:

$$\left(q_{ID}^* = \frac{I_1 - I_2}{2} ; p_{ID}^* = \mu - \rho\sigma^2 \frac{I_1 + I_2}{2}\right).$$

At equilibrium in the inter-dealer market, market-makers' profits write $\left(v_1^{ID}\right)^* = \left(v_2^{ID}\right)^* = \frac{\rho\sigma^2}{8}(I_1 - I_2)^2$. Notice that market-makers find it optimal to perfectly share risk:

[8] In the case in which intermediaries strategically trade in the inter-dealer market *after* observing the realization of the order flows in venue D and S, we find that they may find optimal to reinforce the divergence in inventories in order to maximize their trading profit in the customer-dealer market. The inter-dealer market is not a way to optimize risk-sharing, but to enhance divergence in inventories. Multi-venue competition in the customer-dealer is thus emphasized in this case.

after trading in the inter-dealer market, market-makers 1 and 2 end up with the same inventory position, $I_1' = I_2'$.

Second stage: the customer-dealer market (CD). Given market-makers' inventory positions after their trades in the inter-dealer market, their equilibrium profits in the customer-dealer market can be computed at the limit when $I_1' \to I_2'$ using the formula derived in the proof of Proposition 1. We find: $\left(v_1^{CD|ID}\right)^* = \left(v_2^{CD|ID}\right)^* = \rho\sigma^2 q_D q_S$.

Comparison. We finally compute market-makers' expected profits in the presence of an inter-dealer market, namely $V^{CD+ID} = E\left(\left(v_i^{CD|ID}\right)^* + \left(v_i^{ID}\right)^*\right)$, and compare them with the expected profits they obtain in the absence of an inter-dealer market, namely $\left(V^{CD}\right)^* = E\left(\left(v_i^{CD}\right)^*\right)$. Computations yield:

$$V^{CD+ID} = \frac{\rho\sigma^2}{48}(I_u - I_d)^2 + \gamma\rho\sigma^2 q_D q_S, \tag{IA.31}$$

and

$$\begin{aligned} V^{CD} &= \frac{\rho\sigma^2}{6}(I_u - I_d)\left(q_D + (2\gamma - 1)q_S\right) \\ &+ \frac{\rho\sigma^2 q_S}{(I_u - I_d)^2} \times \left[\begin{array}{l} (1-\gamma)(I_u - I_d)^3 - \left(3(1-\gamma)q_D + \frac{1}{2}\gamma q_S\right)(I_u - I_d)^2 \\ + \left\{(1-\gamma)q_D + \frac{1}{2}\gamma q_S\right\} q_D \left(3(I_u - I_d) - q_D\right) \end{array}\right]. \end{aligned} \tag{IA.32}$$

To assess the impact of the existence of an inter-dealer market on intermediaries' expected profits, one needs to compare the expressions given in Eq. (IA.31) and (IA.32). Closed-form solutions are difficult to interpret. However there exist parameters' values such that intermediaries would prefer not to share risk in an inter-dealer market, that is, $V^{CD} > V^{CD+ID}$. Figure IA.2 shows that intermediaries are better off trading ex ante in an inter-dealer market only when (i) the probability that shocks have the same sign, γ, is high, and (ii) the size of the liquidity demand sent to the satellite venue, q_S, is small.

As illustrated by Figure IA.2, there exist cases (white squared surface) in which intermediaries find more profitable ex ante not to trade in the inter-dealer market (for different values of γ and q_S) and trade directly in the customer-dealer market.■

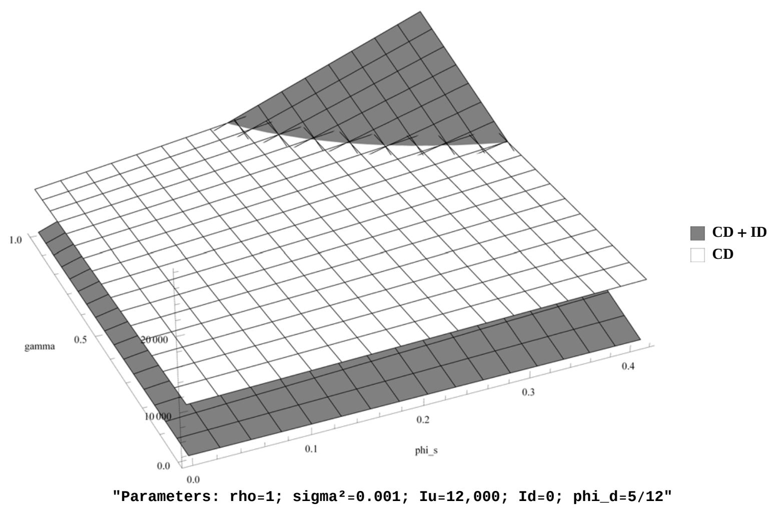

Figure IA.2: Impact of the inter-dealer market on dealers' expected profits.

Figure IA.2 represents intermediaries' expected profits with or without an initial trading round in an inter-dealer market, as a function of γ (the probability that shocks have the same sign) and ϕ_S, for $\phi_S \leq \phi_D$ and $\phi_D \leq I_u - I_d$. The white squared surface plots the expected trading profit in the customer-dealer market (CD) only, the grey squared surface plots the total expected trading profit if intermediaries engage in an inter-dealer round before trading in the customer-dealer market (CD+ID).

2.E No trade-through

This section explores whether a market order can execute at a price worse than the best quoted price, termed as trade-through. Note that a trade-through can only occur if orders sent to S and D have the same sign. The question is thus: do we observe different prices across trading venues when orders Q_D and Q_S are of same direction and same size?

Corollary 2.E.1 *There is no trade-through possible in our model.*

Proof: We use Proposition 1 when Q_D and Q_S have the same sign, and consider that $Q_S = Q_D = Q$.

- If $(I_1 - I_2 - Q_D)Q_S > 0$ and $Q_D = Q_S = Q > 0$, then it is straightforward to show that $(a_1^D)^* = (a_1^S)^* = r_2(Q)$.
- When $(I_1 - I_2 - Q_D)Q_S \leq 0$ and $Q_D = Q_S = Q > 0$, easy computations (below) show that $(a_1^D)^* = (a_2^S)^*$.

Observe that $\hat{r}_2(Q) - \rho\sigma^2 Q \times \eta = r_2(Q) + \rho\sigma^2 Q - \rho\sigma^2(I_1 - I_2) = r_1(Q) + \rho\sigma^2 Q = \hat{r}_1(Q)$. We deduce that $(a_1^D)^* = (a_2^S)^*$. ■

3 Relevance for limit order book environments

3.A Transaction costs, risk premia and rents

In this section, we analyze the effects of fragmentation on transaction costs and the economic forces driving differences in transaction costs between a fragmented and a centralized markets. To do so, we compare the level of total transaction costs (denoted $TTrC$) incurred by liquidity traders in the two market structures, i.e., $TTrC - TTrC^c = (a_1^D)^* Q_D + (p_i^S)^* Q_S - (a_1^c)^*(Q_D + Q_S)$.

In our model, fragmentation enables market-makers to compete for only a part of the total order flow, which is not possible in a centralized market in which order flow is batched. This possibility has an impact on both the inventory risk exposure of market-makers and the intensity of price competition. In order to better understand this dual impact, we decompose total transaction costs into an aggregate risk premium component and an aggregate rent component. Let $\mathbf{1}_{i,S}$ be a dummy that takes value 1 if market-maker i is the best quoting market-maker in market S. Total transaction costs re-write:

$$\begin{aligned} TTrC = & \underbrace{\left(r_1\left(Q_D + Q_S \mathbf{1}_{1\_S}\right) - \mu\right)\left(Q_D + Q_S \mathbf{1}_{1,S}\right) + \left(r_2\left(Q_S\right) - \mu\right) Q_S \mathbf{1}_{2,S}}_{\text{aggregate risk premium}} \\ & + \underbrace{\left(a_1^D - r_1\left(Q_D + Q_S \mathbf{1}_{1,S}\right)\right) Q_D + \left(p_1^S - r_1\left(Q_D + Q_S\right)\right) Q_S \mathbf{1}_{1,S} + \left(p_2^S - r_2\left(Q_S\right)\right) Q_S \mathbf{1}_{2,S}}_{\text{aggregate rent}}. \end{aligned} \tag{IA.3}$$

The difference in total transaction costs between a fragmented and a centralized market is driven by changes in risk premium and in rent:

$$TTrC - TTrC^c = \Delta\text{agg. risk premium} + \Delta\text{agg. rent}.$$

Figure IA.3 illustrates our decomposition and differences in total transaction costs between the two market structures. The parameters' space is split along two dimensions, namely (i) whether competition is high ($(I_1 - I_2 - Q_D) \times Q_S > 0$) or low ($(I_1 - I_2 - Q_D) \times Q_S < 0$) and (ii) whether Q_S is positive (on the top) or negative (on the bottom). The domain is divided into regions A1, A2, B, C and D, exactly corresponding to those in Figure 1.[9] Total transaction costs in a centralized market are in white (with the legend "Benchmark"). The risk premium component is in dashed blue, while the rent component in plain blue. Total transaction costs in a fragmented market are in green (when smaller) or orange (when larger).

[INSERT FIGURE IA.3]

In accordance with previous sections, we divide the analysis into the intense competition case and the low competition case.

The intense competition case (regions B and C in Figure IA.3). In this case, remind that inventory costs of market-maker 1 are small enough to let her absorb the total order flow. Market-maker 1 consolidates the fragmented order flow as she would do in a centralized market. The inventory risk exposure of both market-makers 1 and 2 is unchanged compared to the centralized market (Δagg. risk premium = 0). Variation in the rent depends on the intensity of price competition, as explained below:

- When $Q_S > 0$ (region B), competition is very high due to the threat exerted by the possibility of the shorter market-maker to post aggressive quotes in only one venue that forces the longer market-maker to respond by posting "ultra-competitive" quotes, which are even more competitive than in a centralized market. The rent extracted by market-makers is thus smaller in a fragmented market, leading to lower total transaction costs:

$$TTrC - TTrC^c = \underbrace{0}_{\Delta\text{agg. risk premium}} \underbrace{-\rho\sigma^2 Q_S Q_D}_{\Delta\text{agg. rent}<0} < 0.$$

  It is worth noticing that the reduction in transaction costs in a fragmented market is purely driven by competitive pressure, since the aggregate risk premium remains unchanged.

[9]Note that the x-axis designates $(I_1 - I_2 - Q_D)$ in Figure 1 while the x-axis designates $((I_1 - I_2 - Q_D) \times Q_S)$ in Figure IA.3. For the case in which $Q_S < 0$, regions C and D are reversed.

- In case $Q_S < 0$, price competition in region C is less intense than in region B and comparable to a centralized market. Consequently,

$$TTrC - TTrC^c \;=\; \underbrace{0}_{\Delta\text{agg. risk premium}} + \underbrace{0}_{\Delta\text{agg. rents}} = 0.$$

The low competition case (regions A1, A2, and D in Figure IA.3). In this case, the cost of producing liquidity for market-maker 1 is too high to profitably absorb all shocks across venues. The total order flow is thus split among the two market-makers. The possibility of executing only a part of the total order flow reduces inventory risk exposure in a fragmented market in comparison with a centralized market (i.e., Δagg. risk premium < 0).[10] A smaller risk premium in aggregate could result in more competitive prices in a fragmented market. The lower ability of market-maker 1 to post aggressive quotes due to her capacity constraint is however anticipated by her opponent, and has an impact on the intensity of competition, determining the magnitude of rents.

- When $Q_S > 0$ (region A1 and A2), the rent component varies highly, depending on the divergence of market-makers' inventories which tunes the intensity of competition. In region A2, competition is still intense and the rent extracted from imperfect competition is not large enough to offset the benefits of a smaller risk premium component. Total transaction costs are thus still smaller in a fragmented market. In region A1, the competition pressure is low and market-makers extract large rents (and in particular, larger than those in a centralized market), leading to larger total transaction costs:

$$TTrC - TTrC^c = \underbrace{-\rho\sigma^2 Q_S(Q_D - I_1 + I_2)}_{\Delta\text{agg. risk premium}<0} + \underbrace{\rho\sigma^2 Q_S(2Q_D - 3(I_1 - I_2))}_{\Delta\text{agg. rent}\gtrless 0} \begin{cases} > 0 \text{ if } I_1 - I_2 < Q_D/2 \\ < 0 \text{ if } I_1 - I_2 > Q_D/2 \end{cases}$$

[10] Market-makers bear actually lower aggregate security risk in a fragmented market, which results in a more efficient outcome in risk sharing among market-makers than a centralized market (see Corollary 2.C.1 of this Online Appendix).

- When $Q_S < 0$ (region D), market-maker 1 (very long in this case) is not able to compete for sell orders submitted to the satellite venue, that would further increase inventory risk. This is anticipated by market-maker 2. Both market-makers behave as local monopolists. The rent extracted from imperfect competition is unambiguously higher in a fragmented market. Its magnitude more than offsets the benefits of the improvement in risk sharing (lower aggregate risk premium), leading to worse transaction costs :

$$TTrC - TTrC^c = \underbrace{-\rho\sigma^2(-Q_S)(I_1 - I_2 - Q_D)}_{\Delta\text{agg. risk premium}<0} + \underbrace{2\rho\sigma^2(-Q_S)(I_1 - I_2 - Q_D)}_{\Delta\text{agg. rent}>0} > 0$$

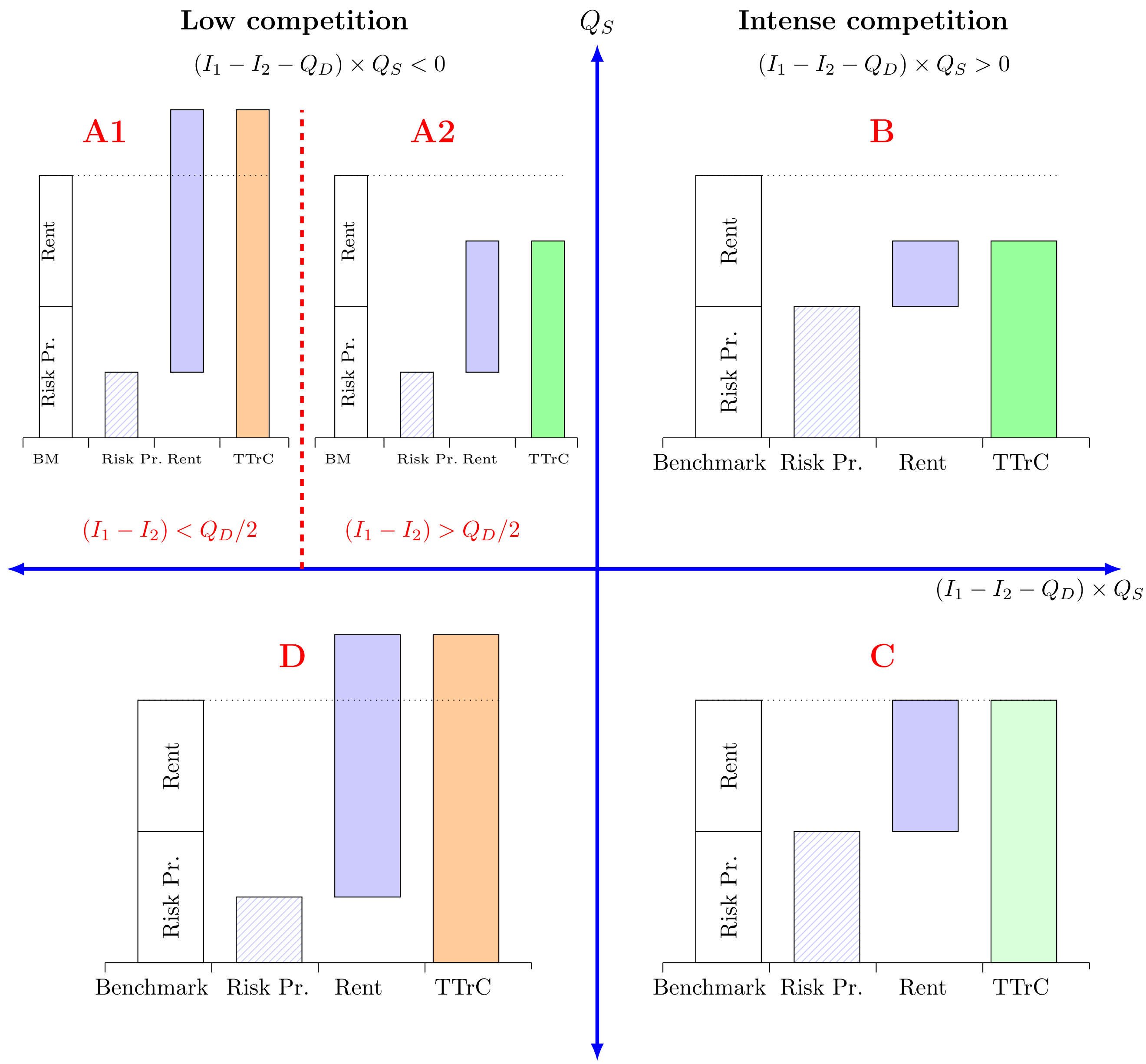

Figure IA.3: Is market fragmentation good for transactions costs?

Figure IA.3 partitions the set of parameters into 5 different regions. The y-axis represents the shock hitting S, denoted Q_S, which might be positive or negative. The x-axis represents $I_1 - I_2 - Q_D$ varying from $-Q_D$ to $I_u - I_d - Q_D$. The product $(I_1 - I_2 - Q_D) \times Q_S$ determines whether market-maker 1 is capacity-constrained (see Lemma 1). Total transaction costs in a centralized market are in transparent white (with the legend "Benchmark"). The risk premium component is in dashed blue, while the rent component in plain blue. Total transaction costs in a fragmented market are in green (when smaller) or orange (when larger). Regions A1, A2, B, C and D correspond to regions on Figure 1.

3.B Model implications in a limit order book environment

In this section, we discuss the robustness of our empirical predictions to our modelling choices. We test implications 1, 2 and 3 of our model using a limit order book environment, but model imperfect competition in quote-driven markets. Order-driven markets might differ along three dimensions: i. capacity to share inventory exposure risks, ii. time priority, iii. timing. Besides, some modern financial markets are very fragmented (e.g., US cash equity markets), and the entry of multi-venue market-makers could be endogenous to the number of trading platforms.

Risk sharing. Risk sharing may be more easily obtained in a consolidated limit order book than in a centralized dealer market (see, for instance, Biais et al., 1998). One key prediction of our model is the "ultra-competitive" effect. Crucially though, this result is not driven by favorable effects on the risk premium (and better risk sharing) but by the strategic quoting behavior of market-makers. As illustrated in region B of Figure IA.3, the risk premium is invariant between a fragmented and a centralized market in the ultra-competitive case. Only a smaller rent component due to more intense competition explains the smaller transaction costs observed in a fragmented market. We thus deduce that the ultra-competitive effect, independent of the impact of risk sharing, would also be a driving force of the impact of fragmentation in imperfectly competitive limit order markets.[11]

Time priority. The timing of quote/limit order submission may be an important driver of competition in fragmented limit order markets when price then time priority is enforced within a venue but not across venues, as documented by Foucault and Menkveld (2008). In our model, market-makers post quotes simultaneously. This assumption is however not crucial in our context because market-makers compete in prices and undercut their competitor by ε, a small positive number. Our results are robust to a sequential entry of market-makers

[11] Note that theoretical models of frictionless limit order books such as Biais et al. (1998) involve simultaneously efficient risk sharing and competitive pricing. In practice, frictions such as the choice of an allocation rule may prevent perfect competition. Biais et al. (2010) for instance find evidence of imperfect competition in Island limit order book before Nasdaq decimalization.

and to the introduction of a time priority rule, as long as market-makers are able to post quotes before the arrival of order flows.

Timing. In our model, market-makers first observe shocks, and then compete to absorb one or both shocks. The chronology of events would be reversed in limit order markets in which market-makers post limit orders before the arrival of a liquidity shock, which changes market-makers' information set. This difference may be crucial in the presence of asymmetric information (see Glosten, 1994). In contrast, in the absence of asymmetric information on the fundamental value of the asset as in our model, it is often equivalent to consider a setting in which market-makers post a price schedule first, or a setting in which investors first submit demand schedules to market-makers. This equivalence does not hold in fragmented markets, because market-makers can only condition their price in one market upon the execution of orders sent to this market, and not on orders sent to other markets. This prevents them from setting prices equal to actual marginal costs.

Changing the assumption on the timing of the model so that market-makers would post limit orders or quotes before observing the realization of order flows would require to replace the actual cost of supplying liquidity in venue m by an *expected* cost of supplying liquidity that would depend on the probability to observe a given realization of the order flow in venue $-m$, conditional on the order flow observed in venue m. This would impact: (i) the condition that drives the market-makers' strategic decision to absorb the order flow in part or in totality (that is, $(I_1 - I_2 - Q_D) \times Q_S$ positive or negative), and, potentially, (ii) equilibrium prices that depend on "stay-at-home" prices. However, our "ultra-competitive" effect obtained when divergence in inventories is large and the probability to observe same-sign shocks (γ) is sufficiently high, would be preserved. Equilibrium prices in this case are driven by competition. They are set such that the opponent cannot undercut in any venue, and are independent of the cross-market cost linkage that would be affected by a different timing.

Endogenous entry of market-makers. The number of venues is fixed in our model, as well as the number of market-makers. The question could arise whether having more venues could lead to the entry of more market-makers, if entry decisions are made endogenous. An increase in the number of market-makers would have two opposite effects in our model. On the one hand, it would decrease the rent extracted each by market-makers. On the other hand, it would possibly decrease the probability to be in the ultra-competitive case as observing a high inventory divergence could be less likely. This is due to our assumption that the support of the distribution of inventories ($[I_d, I_u]$) is equal for every market-maker. In a more general model, this support could be heterogenous. Which effect dominates would ultimately depend on the parameters of the model: the size of liquidity shocks, the distribution of inventories, and the distribution of entry costs across market-makers.

4 Robustness checks

4.A Quoting aggressiveness

This section shows why our measure of quoting aggressiveness, illustrated in Figure IA.4, isolates inventory effects and controls for asymmetric information effects. The best price of the market, say the best ask, at time t of the transaction, can be written as:

$$A_t^* = E[\tilde{v}_t|\Omega_{t-1}, \text{buy-initiated trade at } t] + \gamma - \beta z_t,$$

where Ω_{t-1} is the information known right up to the trade, γ is the order-processing cost per share, β reflects inventory costs and z_t is the inventory of the trader quoting the best ask before the trade at t. We can rewrite traders' estimate of the asset payoff as:

$$E[\tilde{v}_t|\Omega_{t-1}, \text{buy-initiated trade at } t] = E[\tilde{v}_t|\Omega_{t-1}] + \lambda(q_t - E(q_t|\Omega_{t-1})) + \varepsilon_t,$$

where q_t is the trade at t, $\lambda(q_t - E(q_t|\Omega_{t-1}))$ is the trade innovation and λ measures the degree of information asymmetry. The variable ε_t captures the effects of public information other than information incorporated in trades. Suppose that order-processing costs per share are symmetric among traders, and that traders make the same Bayesian update. The best ask quote placed by intermediary i writes:

$$A_t^i = E[\tilde{v}_t|\Omega_{t-1}] + \lambda(q_t - E(q_t|\Omega_{t-1})) + \varepsilon_t + \gamma - \beta^i z_t^i,$$

where β^i represents inventory costs of i and z_t^i the aggregate inventory of i before the trade at t. It can be shown that changes in quoting aggressiveness over the interval $[t, t+\tau]$ write:

$$\Delta|A^* - A^i|_{[t,t+\tau]} = -\beta^i(z_{t+\tau}^i - z_t^i) + \beta(z_{t+\tau} - z_t).$$

Because the transaction at t has been executed by intermediary i, $(z^i_{t+\tau} - z^i_t) = q_t$ and $(z_{t+\tau} - z_t) = 0$ (by hypothesis), changes in aggressiveness are function of inventory only:

$$\Delta|A^* - A^i|_{[t,t+\tau]} = -\beta^i q_t.$$

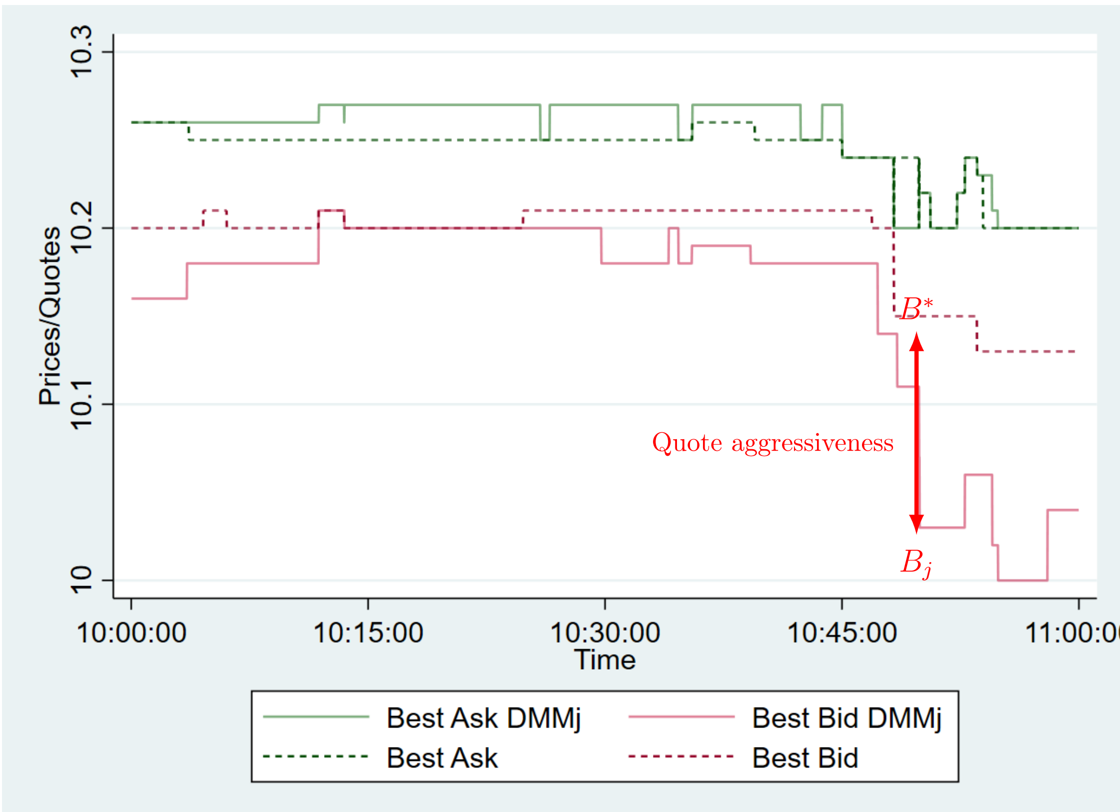

Figure IA.4: An example of quoting aggressiveness (Galapagos, 3rd January 2007)

Figure IA.4 plots the best bid and best ask quotes posted by an intermediary and the best bid and ask prices of the (satellite) market (the "Best limits") on January 3, 2007 between 10:00 am and 11:00 am for the stock Galapagos. The figure illustrates our measure of quoting aggressiveness, which is the distance between the best individual quote of intermediary j (denoted Q_j) and the best market quotes at time t, Q^*: $|Q^* - Q_j|_t$. Here $Q = B$ (Bid).

4.B Variables description

Table IA.1: **Variables description**

Table IA.1 defines the variables used in the paper.

Variable	Definition
Intermediary Variables	
DMM	Dummy variable that takes the value 1 if the intermediary is an exchange-regulated market-maker
$\lvert \Delta I \rvert$	Magnitude of the inventory change expressed in euros caused by a trade
TR_PRICE	(Log) price of the trade
TR_TYPE	Dummy variable that takes the value 1 if the intermediary has triggered the execution (active trade that "takes" liquidity) and 0 if intermediary's quote was hit (passive transaction that "makes" liquidity)
XM_INV_MSG	Dummy variable that takes the value 1 if intermediary i submits at least 1 inventory-driven message in venue m during the 60 seconds immediately following a trade she has absorbed in $-m$
PROP. XM_INV_MSG	Ratio between cross-market inventory-driven messages over all cross-market messages submitted by intermediary i in venue m during the 60 seconds immediately following a trade she has absorbed in $-m$
TOT_XM_INV_MSG	Total number of cross-market messages submitted by an intermediary in venue m in the 60 seconds following a trade in venue $-m$
DIST_Q_t^i	Quoting aggressiveness measured as the distance between the quote (limit order price) posted by intermediary i and the best market quote at time t. This measure is standardized by the midpoint of the prevailing inside spread at t (Q =A (Ask),B (Bid)).
$\Delta\text{DIST\_Q}_{t,t+20}^i$	Change in quoting aggressiveness by intermediary i on venue m during the 20 seconds following the trade execution at time t on venue $-m$.
$\Delta\text{DIST\_Q}^{-i}\rvert_{t,[t+21,t+60]}$	Change in quoting aggressiveness of competitors $-i$ on venue m between t and the last 40 seconds of the 60-second window following the trade of intermediary i at t in venue $-m$.
$\overline{\text{DIST\_Q}}_{i,s,t}$	Time-weighted quoting aggressiveness over each 20-min interval, based on the most aggressive side at time t.
RI^i	Divergence of intermediary i's inventory built as the distance at any time t between the signed aggregated inventory of i (in euros) to the median inventory across all peers excluding i. Inventories must be comparable and are standardized using the methodology described in Hansch et al. (1998).
$\overline{RI}$	Divergence in intermediaries' inventories defined as the average over all intermediaries of the distance between their respective inventory to the median inventory. Inventories are standardized following the Hansch et al. (1998) methodology.
Stock Variables	
VOLUME_{t-300}	Volume traded in the stock during the 300 seconds prior to the trade at t expressed in euros
$\lvert\text{RET}_{t-300}\rvert$	Absolute stock return calculated from midquote over the 300 seconds prior to the trade at t
SAME	Dummy variable that takes the value 1 if order flows have the same direction across venues on a given 20-min interval ($TrIMB\_D \times TrIMB\_S > 0$) , and zero if order flows have opposite signs. Tr_IMB_m is defined as the number of buyer initiated trades minus the number of seller-initiated trades during the last 20-min interval.
$\Delta RBAS$	Changes in the bid-ask spread between 2 consecutive 20-min intervals

4.C Analysis of cross-market messages after a trade in venue $-m$

In this section, we analyze inventory-driven messages submitted by an intermediary in venue m in reaction to a change in inventory due to a trade in venue $-m$ ($m = D, S$). Our model implies the following:

Implication 4.C.1 *(Cross-venue message activity) Multi-venue market-makers update existing limit orders or submit new orders in venue m after a trade in venue $-m$, in a direction that is associated with the inventory change induced by the trade. The intensity of the cross-market message activity is related to the magnitude of the inventory change.*

Formulating our hypothesis in the context of the limit-order-book environment of Euronext, we test whether, for instance, after buying (i.e., executing a sell order) in venue $-m$, a multi-venue market-maker is more likely to cancel an existing buy order in venue m, or modify it for a *less aggressive* price (negative revision), or post a new sell limit order m or modify an existing sell order for *a more aggressive price* (positive revision).

We construct two measures of cross-market messages activity. The first measure (XM_INV_MSG) is a dummy variable that takes the value one if intermediary i sends at least one inventory-driven message in venue m during the 60 seconds following a trade in $-m$. The second measure (PROP. XM_INV_MSG) is the proportion of cross-market inventory-driven messages submitted by intermediary i, namely, the number of inventory-driven messages submitted by i in venue m during the 60 seconds following a trade in $-m$, standardised by the total number of messages submitted by i in m during these 60 seconds. We run the following regression model:

$$\text{(IA.34)} \quad Y^i_{s,[t,t+60]} = \alpha + \beta_1 DMM^i_s + \beta_2 |\Delta I^i_{s,t}| + \beta_3 DMM^i_s \times |\Delta I^i_{s,t}| + \lambda_d + \mu_s + \gamma W^i_{s,t} + \varepsilon^i_{s,t},$$

where the dependent variable Y is one of our two measures of cross-market messages activity described above (XM_INV_MSG and PROP. XM_INV_MSG). Given the nature of the dependent variables, we use a logistic regression model for the first measure and a censored tobit model for the second measure. The explanatory variables are a dummy variable,

DMM_s^i that takes the value 1 if i is registered as a DMM at least on one venue on which stock s is traded, the (log) absolute change in inventory of i in euros due to a trade at time t in stock s ($|\Delta I_{s,t}^i|$), and the interaction between both. λ_d is a day fixed-effect, μ_s is a stock fixed-effect, and $W_{s,t}^i$ is a vector of control variables which includes the (log) transaction price (TR_PRICE), and the type of trade triggering (passive vs. active triggering), TR_TYPE. We also include the lagged trading volume over the past 300 seconds (VOLUME_{t-300}) and the return volatility during the 300 seconds prior to the transaction ($|\text{RET}_{t-300}|$), which control for market conditions in stock s at time t of the transaction (Hasbrouck and Saar, 2009). Our variable of interest is the coefficient β_3 which captures the impact of an inventory change on cross-market messages activity for a given category of intermediaries (DMMs vs. non-DMMs).

Table IA.2 reports two specifications according to the measure of cross-message activity used. Results of the estimated logit model are reported in columns 1 and 3. Columns 2 and 4 present the estimation results for the tobit model. The most important result from Table IA.2 is that, following a transaction in venue $-m$, DMMs submit significantly more inventory-driven messages in venue m, the larger the change in inventory. Across all specifications, we observe a systematic and statistically significant coefficient on the interaction variable between the group of DMMs and the (absolute) magnitude of inventory change, with the expected sign. A one-standard deviation increase in the change of DMMs' inventory increases the likelihood of a cross-venue inventory-driven message by 4.8% in satellite venues, and by 5.6% in dominant venues. Using the estimates of the tobit regression, a one-standard deviation increase in the change of DMMs' inventory increases the proportion of cross-venue inventory-driven message by 6.95% in dominant venues and by 5.55% in satellite venues. These results support implication 4.C.1 presented in this section.[12]

[12]Results are unchanged if we consider a 20-second window instead of a 60-second window; the first inventory-driven message takes place, on average, around 4 seconds after a trade in the other venue.

Table IA.2: **Cross-market messages activity after trading in the other venue**

This table presents the determinants of inventory-driven messages submitted by a multi-venue intermediary in a venue after trading in another venue. Columns 1 and 3 report Logit regression results for using a dummy variable, XM_INV_MSG, that takes 1 if the trader posts at least one inventory-driven message in venue $-m$ in the 60 seconds following a trade in venue m. Column 2 and 4 report Tobit regression results for using the variable PROP_XM_INV_MSG, which is the proportion of inventory-driven messages over all messages posted by the trader in venue m in the 60 seconds following a trade in venue $-m$. We include the following determinants (trader/transaction/stock): DMM is an indicator variable taking the value one if the trader is a Designated Market-Maker for the stock and zero otherwise; DMM $\times$ $|\Delta I|$ is the DMM indicator interacted with the (log) change in inventory due to the transaction expressed in euros. $|\Delta I|$ is the (log) change in inventory due to the transaction expressed in euros. TR_PRICE is the (log) price of the transaction. TR_TYPE is related to the type of trade: 1 being used if intermediary i actively takes liquidity from the limit order book; 0 for a transaction passively triggered, i.e., making liquidity. VOLUME$_{t-300}$ is the lagged dollar volume traded in the stock during the 300 seconds prior to the transaction. $|\text{RET}|_{t-300}$ is the absolute 300-seconds stock return prior to the transaction. TOT_XM_MSG is the total number of messages submitted by i to m after a trade in $-m$ ($m = D, S$). This variable is used in columns 2 and 4 to control for a possible denominator effect in the dependant variable. Stock fixed-effects and day-fixed effects are included. Standard errors are double-clustered by trader and stock. The symbols ***, **, * denote significance levels of 1%, 5% and 10%, respectively, for the two-tailed hypothesis test that the coefficient equals zero.

	IN S AFTER TRADING IN D				IN D AFTER TRADING IN S			
	XM_INV_MSG		PROP_XM_INV_MSG		XM_INV_MSG		PROP_XM_INV_MSG	
Determinants	1		2		3		4	
DMM $\times$ $\lvert\Delta I\rvert$	0.254	***	4.556	***	0.172	**	4.756	**
	(3.67)		(2.69)		(2.09)		(2.53)	
$\lvert\Delta I\rvert$	-0.027		-0.965		0.01		0.08	
	(-1.27)		(-1.50)		(0.56)		(0.16)	
DMM	-0.674		2.907		-1.064		-37.944	*
	(-0.83)		(0.13)		(-1.49)		(-1.82)	
TR_PRICE	0.055		1.162		0.914		21.913	
	(0.05)		(0.04)		(0.92)		(0.77)	
TR_TYPE	0.105		3.711		0.016		2.356	
	(1.37)		(1.62)		(0.14)		(0.74)	
VOLUME$_{t-300}$	0.016	***	0.007		0.025	***	-0.402	**
	(3.50)		(0.05)		(3.41)		(-2.05)	
$\lvert\text{RET}\rvert_{t-300}$	0.21	*	-0.311		0.064		-3.665	
	(1.93)		(-0.12)		(0.55)		(-1.17)	
TOT_XM_MSG			4.556	***			1.448	***
			(2.69)				(3.55)	
Intercept	-0.89		-22.575		-3.752		-92.193	
	(-0.25)		(-0.25)		(-1.23)		(-1.04)	
Stock/Day FE	Yes		Yes		Yes		Yes	
N	506,508		506,508		338,082		338,082	
Pseudo-R-squared	0.23		0.08		0.05		0.02	

4.D Cross-market quoting aggressiveness after making or taking liquidity

In this section, we split the sample by category (DMM or non-DMM) and by type of transaction (actively or passively triggered). An intermediary takes liquidity from the limit order book when she triggers an execution (TR_TYPE = 1), while she makes liquidity when her best quote is passively hit by an incoming aggressive order (TR_TYPE = 0). Table IA.3 reports coefficient estimates from a panel regression of changes in quoting aggressiveness on changes in inventory.

Table IA.3 shows that changes in quoting aggressiveness for DMMs are positive and significant when they provide liquidity in D, consistent with quotes updates due to non-constant cross-inventory cost (coefficients of columns 1 and 3 of Panel A are significantly positive with t-stat of 2.25 and 2.69). In contrast, columns 2 and 4 show that DMMs do not update significantly their quotes in S immediately after they take liquidity from D (ruling out any alternative explanation based on arbitrage strategies). On the opposite side, DMMs seem to be more aggressive after they make liquidity (expected negative sign) but the coefficients are not statistically significant. In contrast, non-DMMs significantly decrease their aggressiveness on the same side of the transaction (significant negative coefficient in columns 6 and 8 of Panel A) and significantly increase their aggressiveness on the opposite side (significant negative sign in columns 6 and 8 of Panel B) when they *take* liquidity from the market (TR_TYPE = 1), which is consistent with arbitrage strategies rather than market-making strategies. These results show therefore that the quoting aggressiveness changes are supportive of market-making and inventory-related strategies for DMMs, while they are consistent with arbitrage strategies for non-DMMs.

Table IA.3: **Cross-market quotes updates by intermediaries' category and by transaction type**

This table documents the determinants of cross-market quote updates in S after a trade in D by category of intermediaries (DMM vs non-DMM) and by type of transaction (TR_TYPE). Panel A reports how much intermediaries revise quotes in S after a transaction in D *on the same side* of the trade. Panel B refers to cross-market quote updates in S after a trade that was executed in D *on the opposite* side. The dependent variable $\Delta\text{DIST\_Q}^i_{t,t+20}$ ($Q = A, B$) is a measure of changes in quoting aggressiveness defined in Table IA.1. The main independent variable $|\Delta I|$ is the magnitude of the change in euro inventory due to the transaction, expressed in log. All control variables (TR_PRICE, VOLUME_{t-300}, and $|\text{RET}|_{t-300}$) are detailed in the caption of Table IA.2. Estimates are from panel regressions with stock and day fixed effects. T-statistics are calculated using clustered (by stock-intermediary) standard errors. The symbols ***, **, * denote significance levels of 1%, 5% and 10%, respectively, for the two-tailed hypothesis test that the coefficient equals zero.

PANEL A: SAME SIDE	DMM				non-DMM			
	$\Delta\text{DIST\_B}^i_{t,t+20}$		$\Delta\text{DIST\_A}^i_{t,t+20}$		$\Delta\text{DIST\_B}^i_{t,t+20}$		$\Delta\text{DIST\_A}^i_{t,t+20}$	
	TR_TYPE=0	TR_TYPE=1	TR_TYPE=0	TR_TYPE=1	TR_TYPE=0	TR_TYPE=1	TR_TYPE=0	TR_TYPE=1
Determinants	1	2	3	4	5	6	7	8
$\|\Delta\text{I}\|$	0.200**	0.023	0.171**	0.045	-0.008	0.048**	0.012	0.055**
	(2.25)	(0.76)	(2.69)	(0.81)	(-0.57)	(2.32)	(0.49)	(2.53)
Intercept	1.259	4.16**	5.928	3.148	-1.024	2.527	-3.867*	0.964
	(0.35)	(2.13)	(1.47)	(1.02)	(-0.42)	(0.77)	(-1.80)	(0.32)
Control Variables	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock/Day FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	7,843	8,844	7,963	9,041	28,426	15,405	28,368	16,067
Adj. R^2	0.04	0.01	0.03	0.01	0.03	0.01	0.03	0.01

PANEL B: OPPOSITE SIDE	DMM				non-DMM			
	$\Delta\text{DIST\_B}^i_{t,t+20}$		$\Delta\text{DIST\_A}^i_{t,t+20}$		$\Delta\text{DIST\_B}^i_{t,t+20}$		$\Delta\text{DIST\_A}^i_{t,t+20}$	
	TR_TYPE=0	TR_TYPE=1	TR_TYPE=0	TR_TYPE=1	TR_TYPE=0	TR_TYPE=1	TR_TYPE=0	TR_TYPE=1
Determinants	1	2	3	4	5	6	7	8
$\|\Delta\text{I}\|$	-0.028	0.001	-0.005	0.013	-0.024	-0.068***	0.008	-0.112**
	(-0.52)	(0.01)	(-0.19)	(0.13)	(-2.65)	(2.32)	(0.29)	(-2.94)
Intercept	-0.345	5.578	3.669	5.155	-5.492*	2.527	-3.872*	-5.222
	(-0.10)	(1.36)	(1.44)	(1.45)	(-1.78)	(0.77)	(-1.55)	(-1.13)
Control Variables	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock/Day FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	8,580	8,844	8,491	8,371	13,952	15,405	30,884	13,087
Adj. R^2	0.03	0.01	0.04	0.01	0.03	0.01	0.01	0.01

4.E Cross-venue aggressiveness and divergence in intermediaries' inventory

This section complements the analysis of Section III.D by focusing on how changes in relative inventory affect quoting aggressiveness. We use a setting similar to that of Table 4. The generating event is a transaction executed by Designated Market-Makers (DMMs) in the dominant venue. We focus on DMMs as it is the only group of multi-venue intermediaries with a quoting behavior consistent with our multi-venue competition model (see Tables 4 and IA.3). We denote RI_i the inventory divergence of intermediary i, also referred to as "relative inventory" in the literature. We run the following regression:

$$\Delta\text{DIST\_Q}^i_{s,[t,t+20]} = \alpha + \beta_1 \Delta RI^i_t + \lambda_d + \mu_s + \gamma W^i_{s,t} + \epsilon^i_{s,t}, \tag{IA.35}$$

where the variable of interest is ΔRI, the change in relative inventory. RI^i is the distance between the signed aggregated inventory of i (in euros) relative to the median inventory across all peers excluding i. Note that RI requires to standardize inventories to be able to compare them. We adopt the methodology proposed by Hansch et al. (1998). It follows that $RI^i = \text{std}(I^i) - \text{std}(I^M)$. RI^i is calculated using a panel "trader x stock x second". We then take the difference between $t-1$ and $t+1$ (t being the time of the transaction expressed in second) to calculate ΔRI^i.

Note that the interpretation of the results must be made with care because we need to take into account the signed changes in relative inventory. Table IA.4 summarizes the potential changes in cross-quoting aggressiveness along with the associated changes in relative inventory. For instance, after the execution of an incoming buy order triggering a sell for the DMM, the change in relative inventory might imply that i is too short relative to its peers to stay on the same side ($\Delta RI^i < 0$), and she updates her quotes to move from the best ask in all venues ($\Delta\text{DIST\_A}^i_{t,t+20} > 0$) to, potentially, be closer to the opposite side, i.e., to the best bid ($\Delta\text{DIST\_B}^i_{t,t+20} < 0$). We thus expect $\beta_1 < 0$ after a sell for the DMM, as reported in Table IA.4. The expected sign is the reverse after the execution of an incoming sell order triggering a buy for the DMM.

Table IA.4: **Quoting aggressiveness and associated inventory changes**

Table IA.4 summarizes the expected sign between the changes in quote aggressiveness (on the ask and bid side) and the associated changes in relative inventory immediately following a buy or sell transaction for the intermediary.

Expected sign	AFTER BUYING $\Delta RI^i > 0$		AFTER SELLING $\Delta RI^i < 0$	
SAME SIDE Less aggressive: $\Delta\text{DIST\_Q}^i_{t,t+20} \geq 0$	(Bid)	+	(Ask)	-
OPPOSITE SIDE More aggressive: $\Delta\text{DIST\_Q}^i_{t,t+20} \leq 0$	(Ask)	-	(Bid)	+

Table IA.5 reports results. Panel A presents results following a buy for the intermediary, while Panel B reports results following a sell. Results show that, after buying, a higher inventory divergence is associated with a less aggressive quoting behavior of DMMs on the bid side and more aggressive on the sell side. Opposite results hold after selling, i.e. when inventory divergence is lower. Only results on the bid side after buying are statistically significant.

Table IA.5: **Quoting aggressiveness and relative inventory changes**

Table IA.5 documents the determinants of cross-market quote updates in S after a trade in D by a DMM. Panel A reports how much DMMs revise quotes in S after *buying* in D. Panel B refers to cross-market quote updates in S after *selling* in D. The dependent variable $\Delta\text{DIST\_Q}^i_{t,t+20}$ ($Q = A, B$) is a measure of changes in quoting aggressiveness defined in the caption of Table 4. The main independent variable ΔRI^i is the change in relative inventory due to the transaction. All control variables (TR_PRICE, VOLUME_{t-300}, and $|\text{RET}|_{t-300}$) are detailed in the caption of Table IA.2. Estimates are from panel regressions with stock and day fixed effects. T-statistics are calculated using clustered (by stock-intermediary) standard errors. The symbols ***, **, * denote significance levels of 1%, 5% and 10%, respectively, for the two-tailed hypothesis test that the coefficient equals zero.

PANEL A: AFTER BUYING	$\Delta\text{DIST\_Q}^i_{t,t+20}$	
	SAME SIDE	OPPOSITE SIDE
Determinants	1	2
ΔRI^i	1.858**	-1.341
	(2.12)	(-1.17)
Intercept	3.554	-0.089
	(0.32)	(-0.01)
Control Variables	Yes	Yes
Stock/Day FE	Yes	Yes
N	7,990	8,725
Adj. R^2	0.03	0.05

PANEL B: AFTER SELLING	$\Delta\text{DIST\_Q}^i_{t,t+20}$	
	SAME SIDE	OPPOSITE SIDE
Determinants	1	2
ΔRI^i	-0.888	1.887
	(-1.03)	(0.93)
Intercept	6.709	-14.18
	(1.56)	(-1.57)
Control Variables	Yes	Yes
Stock/Day FE	Yes	Yes
N	7,525	6,944
Adj. R^2	0.04	0.05

4.F Inventory measures and cross-market aggressiveness

In this section, we pool inventory measures together in the same regression model and we run the following regression:

$$\Delta \text{DIST\_Q}^i_{s,[t,t+20]} = \alpha + \beta_1 |\Delta \text{I}^i_{s,t}| + \beta_2 \Delta RI^i_{s,t} + \beta_3 \, \text{std}(Inv^i_{s,t-1}) + \gamma W^i_{s,t} + \varepsilon^i_{s,t} \tag{IA.36}$$

where the variable $|\Delta \text{I}_i|$ is the magnitude of the change in intermediary i's inventory, RI^i is the change in the relative inventory RI^i (RI^i is the distance of i's inventory to the median inventory across all peers excluding i), and $\text{std}(Inv^i_{t-1})$ is the level of i's aggregate and standardized inventory (in euros) the second before the transaction. W is a vector of control variables described in the caption of Table IA.2.

Table IA.6 reports the results. The results show that a larger change both in inventory or in the relative inventory is associated with a less aggressive quoting behavior of DMMS on the same side of the trade. Across all specifications, both the magnitude of the change in inventory and the change in relative inventory have the expected signs. Both inventory measures are correlated (around 20%) as they depend both on the trade size executed in t. The magnitude of the change in inventory is however the only measure for which we observe a statistically significant coefficient across all specifications (t-statistics vary from 1.80 to 2.31). The weaker relationship of the variable ΔRI^i could be due to the standardization, making it potentially noisier than the magnitude of the change in inventory.

Table IA.6: **Quoting aggressiveness, inventory, and relative inventory**

Table IA.6 documents the relationship between measures of inventory and cross-market quote updates in S after a trade in D by a DMM. Panel A reports how much DMMs revise quotes in S after *buying* in D. Panel B refers to DMM's cross-market quote updates in S after *selling* in D. The dependent variable $\Delta\text{DIST\_Q}^i_{s,[t,t+20]}$ is a measure of changes in quoting aggressiveness defined in the caption of Table 4 ($Q = A, B$). The main independent variables are ΔRI^i which is the change in relative inventory due to the transaction, $|\Delta \text{I}_i|$ which is the magnitude of the change in inventory (expressed in euros), and $\text{std}(I^i_{t-1})$ which is the level of the (lagged) standardized inventory the second prior to the transaction. All control variables (TR_PRICE, VOLUME_{t-300}, and $|\text{RET}|_{t-300}$) are detailed in the caption of Table IA.2. Estimates are from panel regressions with stock and day fixed effects. T-statistics are calculated using clustered (by stock-intermediary) standard errors. The symbols ***, **, * denote significance levels of 1%, 5% and 10%, respectively, for the two-tailed hypothesis test that the coefficient equals zero.

PANEL A: AFTER BUYING		SAME SIDE $\Delta\text{DIST\_B}^i_{s,[t,t+20]}$					
Determinants	Expected sign	1		2		3	
ΔRI^i	+	1.858	**	1.442	*	1.381	
		(2.12)		(1.72)		(1.64)	
$\|\Delta \text{I}^i\|$	+			0.203	*	0.207	*
				(1.80)		(1.84)	
$\text{std}(I^i_{t-1})$	+					0.289	
						(1.59)	
Intercept		3.554		1.234		2.923	
		(0.32)		(0.12)		(0.27)	
Control Variables		Yes		Yes		Yes	
Stock/Day FE		Yes		Yes		Yes	
N		7,990		7,990		7,990	
Adj. R^2		0.03		0.03		0.03	

PANEL B: AFTER SELLING		SAME SIDE $\Delta\text{DIST\_A}^i_{s,[t,t+20]}$					
Determinants	Expected sign	1		2		3	
ΔRI^i	-	-0.89		-0.71		-0.71	
		(-1.03)		(-0.84)		(-0.83)	
$\|\Delta \text{I}^i\|$	+			0.166	**	0.166	**
				(2.31)		(2.30)	
$\text{std}(I^i_{t-1})$	+					-0.01	
						(-0.12)	
Intercept		6.709		4.634		4.483	
		(1.56)		(1.06)		(0.87)	
Control Variables		Yes		Yes		Yes	
Stock/Day FE		Yes		Yes		Yes	
N		7,525		7,525		7,523	
Adj. R^2		0.04		0.04		0.04	

4.G Timeline of tests

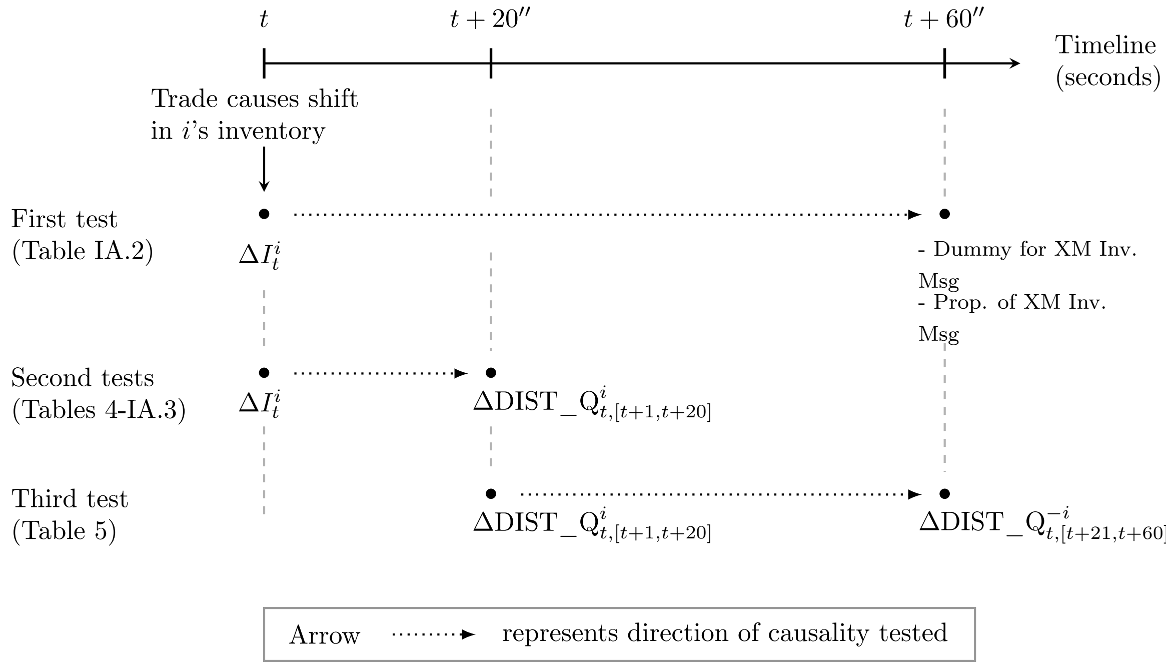

Figure IA.5: Timeline of tests related to implications 1a, 1b and 4.C.1

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