Internet appendix to “Liquidation, bailout, and bail-in: Insolvency resolution mechanisms and bank lending”

Bart M. Lambrecht
Cambridge Judge Business School and CEPR

Alex S.L. Tse
University of Exeter Business School

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Abstract

This internet appendix consists of four major parts. The first part provides supplementary details about the model and the results. The second part contains technical materials complementing the proofs in the main paper. The third part covers miscellaneous extensions such as the inclusion of a leverage constraint, randomized insolvency resolution mechanisms, and the incorporation of transaction costs and decreasing returns to scale. Finally, the last part offers further numerical results comparing equity-conversion bail-in and write-down bail-in.

1 Further discussion of the model

1.1 Definition of managers' claim under bailout regime

We provide further justification behind the definition of managers' claim under bailout regime as introduced in Section 2.2. Recall that managers survive a bailout with some probability $p_o \in [0,1]$ and their stake in the inside equity is diluted by a factor $\xi_o (\leq 1)$. The effect of dismissal and stake dilution upon the arrival of a shock can be captured by a random variable X which takes on value ξ_o with probability p_o or value 0 otherwise. In the former case insiders still have a claim after the shock, albeit a reduced one as reflected by the factor $\xi_o \leq 1$. In the latter case insiders lose their claim entirely.

Let $\Lambda_k \equiv \prod_{n=1}^{k} X_n$ be the cumulative dismissal-adjusted dilution factor after k shocks have arrived, where $X_n \sim X$ are i.i.d random variables independent of the net worth dynamics. Denote by T_k is the random arrival time of the k^{th} crash. Managers' optimization problem under the bailout regime can be stated as:

$$M_o(N) = \max_{q_t, l_t, f} E\left(\int_0^{T_1} e^{-\delta t} U(q_t N_t) dt + \sum_{k=1}^{\infty} \int_{T_k}^{T_{k+1}} e^{-\delta t} U(\Lambda_k q_t N_t) dt \Big| N_0 = N \right) \quad \text{(A.1)}$$

It is straightforward to verify that:

$$E\left(\Lambda_k^{1-\eta}\right) = \left[E\left(X^{1-\eta}\right)\right]^k = [p_o \xi_o^{1-\eta}]^k$$

using the i.i.d. property of $X_n \sim X$. Due to the power form of the utility function, and X_n and N_t being independent, (A.1) can be expressed as:

$$M_o(N) = \max_{q_t, l_t, f} E\left(\int_0^{T_1} e^{-\delta t} U(q_t N_t) dt + \sum_{k=1}^{\infty} \int_{T_k}^{T_{k+1}} e^{-\delta t} [p_o \xi_o^{1-\eta}]^k U(q_t N_t) dt \Big| N_0 = N \right)$$

1.2 Convexity, corner solution and the role of "skin in the game"

Proposition 4 in the main paper suggests that a corner solution $f_b = 1$ is observed in the liquidation regime while interior solutions are observed in the bailout and bail-in regime provided that $v_o > 0, v_i > 0$. Such phenomena could be understood via the convexity behaviors of the managers' objective function. For simplicity of exposition, suppose l is fixed and we just focus on the optimal choice of f. For small value of f such that $f \leq 1/l$, the firm remains solvent when a crash arrives and the insiders' net worth recovery rate is $\phi_s(f) = 1 - fl$ (the dependence on l is suppressed as we consider l fixed here). For large value of f where $f > 1/l$, the bank becomes insolvent during a crash and the insiders' net worth recovery rate is $\phi_j(f)$ under IRM j. The recovery rate as a function of all values of f can be compactly written as:

$$\phi(f) = \begin{cases} 1 - fl, & f \leq 1/l \\ \xi_j(1-f), & 1/l < f \leq 1 \end{cases}$$

As explained in Section 2.2, the managers' claim value is the sum of expected utility of the payout extracted up to the random arrival time of the macroshock and the residual claim value. The former is indeed linear in f while the latter is proportional to $p_j[\phi(f)]^{1-\eta}$,[1] and hence the convexity of the managers' objective function in f solely depends on that of the residual component. It is thus sufficient to analyze the convexity

[1] See for example equation (34) in the main paper where the last term is indeed corresponding to such residual claim value.

of the following function:

$$V(f) = \begin{cases} (1 - fl)^{1-\eta}, & f \leq 1/l \\ v_j(1 - f)^{1-\eta}, & 1/l < f \leq 1 \end{cases}$$

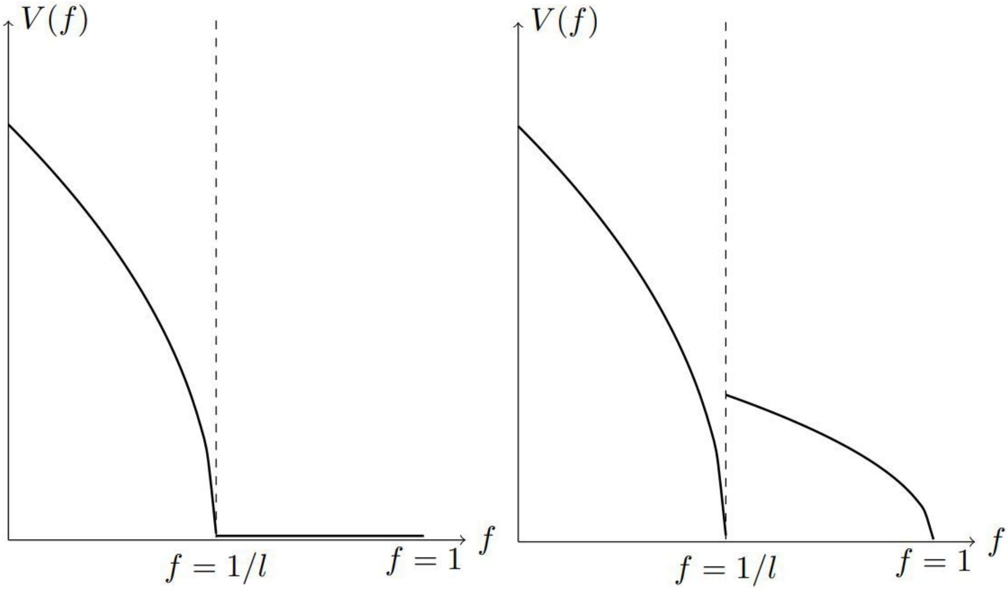
with $v_b = 0$, $v_o > 0$, $v_i > 0$. The stylized plots of this function under $j = b$ and $j = o, i$ are shown in Figure 1.

When the IRM is liquidation, the managers are fired during the crash and they receive nothing thereafter. The continuation value is thus zero on $f > 1/l$ which is a convex function. The creates the possibility of a corner solution at $f = 1$ which we have verified its optimality.

When the IRM is bailout or bail-in, the managers can freeride on the government subsidy or the severance claim payment. This is reflected by the discontinuity of the continuation value function at $f = 1/l$. In particular, the managers will strictly prefer a marginally insolvent firm to a marginally solvent one. However, the free subsidy also creates local concavity near $f = 1$. This risk aversion introduced deters the managers from putting the entire bank at risk.

Complement to Proof of Proposition 2 and 4: bailout regime. We provide further technical details to identify the maximizer of $H_o(f)$ in the bailout regime. Consider first the range $f \leq \hat{f}_o$. We have:

$$H_o'(f) = \frac{d}{df} G_o(\hat{l}(f); f) = -\frac{\mu - \rho}{f^2} + \frac{\sigma^2 \eta}{f^3} - \frac{v_o \lambda}{(1 - f)^\eta} \equiv \Gamma_o(f)$$



(a) The IRM is liquidation

(b) The IRM is bailout/bail-in with $v_o, v_i > 0$.

Figure 1: The illustration of the convexity of managers' objective function.

Observe that $\Gamma_o(0) = \infty$ and $\Gamma_o(1) = -\infty$. Furthermore:

$$
\begin{aligned}
\Gamma_o'(f) &= \frac{2(\mu-\rho)}{f^3} - \frac{3\sigma^2\eta}{f^4} - v_o\lambda\eta(1-f)^{-\eta-1} \\
&= \frac{1}{f^3}\left(2(\mu-\rho) - \frac{3\sigma^2\eta}{f}\right) - v_o\lambda\eta(1-f)^{-\eta-1} \\
&\leq \frac{1}{f^3}\left(2\left(\frac{\sigma^2\eta}{f} - \kappa\lambda f\right) - \frac{3\sigma^2\eta}{f}\right) - v_o\lambda\eta(1-f)^{-\eta-1} \\
&= -\frac{1}{f^3}\left(\frac{\sigma^2\eta}{f} + 2\kappa\lambda f\right) - v_o\lambda\eta(1-f)^{-\eta-1} < 0
\end{aligned}
$$

where we have used the fact that $\frac{\mu+\kappa\lambda f-\rho}{\sigma^2\eta} \leq \frac{1}{f}$ over $f \leq \hat{f}_o$. Then we conclude $\Gamma_o(f) = 0$ must have exactly one root $\tilde{f}_o \in (0,1)$. We are going to show that the condition of $\frac{\mu-\rho}{\sigma^2\eta} > 1 + \frac{v_o}{\kappa}$ will imply $\hat{f}_o < \tilde{f}_o$ such that $\Gamma_o(f) > 0$ for all $f < \hat{f}_o$. Hence $H_o(f)$ is strictly increasing over $f \leq \hat{f}_o$. As a result, any maximum must be attained at some $f \geq \hat{f}_o$.

Now consider the range of $\hat{f}_o < f \leq 1$. We have:

$$H_o'(f) = \frac{d}{df} G_o \left(\frac{\mu + \kappa\lambda f - \rho}{\sigma^2 \eta}; f \right) = \frac{\kappa\lambda(\mu + \kappa\lambda f - \rho)}{\sigma^2 \eta} - \frac{v_o \lambda}{(1-f)^\eta} \equiv \lambda\kappa\Theta_o(f)$$

If $v_o = 0$, we have $H_o'(f) > 0$ and the maximum must be attained at $f = 1$. Otherwise, check that $\Theta_o(1) = -\infty$, $\Theta_o(0) = \frac{\mu-\rho}{\sigma^2\eta} - \frac{v_o}{\kappa} > 1 > 0$, and we can compute:

$$\Theta_o''(f) = -v_o \lambda \eta (1+\eta)(1-f)^{-\eta-2} < 0$$

Thus Θ_o is a strictly concave function starting with a positive value and ending with a negative value, which must change sign from positive to negative exactly once at some $f = f_o$ given by the solution to $\Theta_o(f) = 0$. We are going to show that the condition $\frac{\mu-\rho}{\sigma^2\eta} > 1 + \frac{v_o}{\kappa}$ will result in $\hat{f}_o < f_o$. Hence $H_o(f)$ must attain its maximum at some interior point $f_o \in (\hat{f}_o, 1)$.

Recall that $\hat{f}_o$, f_o and $\tilde{f}_o$ are defined as the solutions to:

$$\begin{aligned}
\zeta_o(f) &\equiv \frac{\mu + \kappa\lambda f - \rho}{\sigma^2\eta} - \frac{1}{f} = 0 \\
\Theta_o(f) &\equiv \frac{\mu + \kappa\lambda f - \rho}{\sigma^2\eta} - \frac{v_o \lambda}{\kappa(1-f)^\eta} = 0 \\
\Gamma_o(f) &\equiv -\frac{\mu - \rho}{f^2} + \frac{\sigma^2\eta}{f^3} - \frac{v_o\lambda}{(1-f)^\eta} = 0
\end{aligned}$$

respectively. The final step of the proof is to verify that if $\frac{\mu-\rho}{\sigma^2\eta} > 1 + \frac{v_o}{\kappa}$, then $\hat{f}_o < \tilde{f}_o$ and $\hat{f}_o < f_o$.

We first show that $\hat{f}_o < \tilde{f}_o$. The equations $\zeta_o(f) = 0$ and $\Gamma_o(f) = 0$ can be restated as:

$$\begin{aligned}
L_1(f) \equiv \kappa f &= \frac{\sigma^2\eta}{\lambda}\left(\frac{1}{f} - \frac{\mu-\rho}{\sigma^2\eta}\right) \equiv \chi_1(f) \\
L_2(f) \equiv \frac{v_o f^2}{(1-f)^\eta} &= \frac{\sigma^2\eta}{\lambda}\left(\frac{1}{f} - \frac{\mu-\rho}{\sigma^2\eta}\right) = \chi_1(f)
\end{aligned}$$

Then:

$$
\begin{aligned}
L_2\left(\frac{\sigma^2\eta}{\mu-\rho}\right) &= v_o\left(\frac{\sigma^2\eta}{\mu-\rho}\right)^2\left(1-\frac{\sigma^2\eta}{\mu-\rho}\right)^{-\eta} < v_o\left(\frac{\sigma^2\eta}{\mu-\rho}\right)^2\left(1-\frac{\sigma^2\eta}{\mu-\rho}\right)^{-1} \\
&= v_o\left(\frac{\sigma^2\eta}{\mu-\rho}\right)\frac{\frac{\sigma^2\eta}{\mu-\rho}}{\left(1-\frac{\sigma^2\eta}{\mu-\rho}\right)} < v_o\left(\frac{\sigma^2\eta}{\mu-\rho}\right)\frac{(1+v_o/\kappa)^{-1}}{(1-(1+v_o/\kappa)^{-1})} \\
&= \kappa\left(\frac{\sigma^2\eta}{\mu-\rho}\right) = L_1\left(\frac{\sigma^2\eta}{\mu-\rho}\right)
\end{aligned}
$$

where we have used the condition $\frac{\mu-\rho}{\sigma^2\eta} > 1+\frac{v_o}{\kappa}$ and in turn $\frac{\sigma^2\eta}{\mu-\rho} < (1+v_o/\kappa)^{-1}$.

It can be easily checked that L_2 is an increasing convex function on $f \in [0,1)$. Since $L_1(f)$ is linear, $L_1(0) = 0 = L_2(0)$ and $L_1\left(\frac{\sigma^2\eta}{\mu-\rho}\right) > L_2\left(\frac{\sigma^2\eta}{\mu-\rho}\right)$, we must have $L_1(f) > L_2(f)$ for $0 < f < \frac{\sigma^2\eta}{\mu-\rho}$. Moreover, since $\chi_1\left(\frac{\sigma^2\eta}{\mu-\rho}\right) = 0$, we must have $\tilde{f}_o < \frac{\sigma^2\eta}{\mu-\rho}$ and $\hat{f}_o < \frac{\sigma^2\eta}{\mu-\rho}$.

We are going to establish $\hat{f}_o < \tilde{f}_o$ by argument of contradiction. Suppose instead we have $\hat{f}_o \geq \tilde{f}_o$. Then:

$$\chi_1(\hat{f}_o) = L_1(\hat{f}_o) > L_2(\hat{f}_o) \geq L_2(\tilde{f}_o) = \chi_1(\tilde{f}_o)$$

but this contradicts the fact that χ_1 is a decreasing function.

To show that $\hat{f}_o < f_o$, we restate the equations $\zeta_o(f) = 0$ and $\Theta_o(f) = 0$ as:

$$
\begin{aligned}
L_3(f) \equiv \frac{1}{f} &= \frac{\mu+\kappa\lambda f-\rho}{\sigma^2\eta} \equiv \chi_2(f) \\
L_4(f) \equiv \frac{v_o}{\kappa(1-f)^\eta} &= \frac{\mu+\kappa\lambda f-\rho}{\sigma^2\eta} = \chi_2(f)
\end{aligned}
$$

Since $L_4(f)$ is an increasing convex function with $L_4(0) = \frac{v_o}{\kappa} \leq 1 < \frac{\mu-\rho}{\sigma^2\eta} = \chi_2(0)$ and $\chi_2(f)$ is linear, $L_4(f)$ and $\chi_2(f)$ can cross only once where the crossing point gives the solution f_o. Moreover:

$$L_4\left(\frac{1}{1+v_o/\kappa}\right) = \frac{v_o}{\kappa}(1-(1+v_o/\kappa)^{-1})^{-\eta} < \frac{v_o}{\kappa}(1-(1+v_o/\kappa)^{-1})^{-1}$$
$$= 1+v_o/\kappa < \frac{\mu-\rho}{\sigma^2\eta} < \frac{\mu+\kappa\lambda(1+v_o/\kappa)^{-1}-\rho}{\sigma^2\eta} = \chi_2\left(\frac{1}{1+v_o/\kappa}\right)$$

Hence $\frac{1}{1+v_o/\kappa} < f_o$ and thus $\frac{1}{f_o} < 1 + \frac{v_o}{\kappa} < \frac{\mu-\rho}{\sigma^2\eta}$. On the other hand:

$$\frac{\mu-\rho}{\sigma^2\eta} < \frac{\mu+\kappa\lambda\hat{f}_o-\rho}{\sigma^2\eta} = \chi_2(\hat{f}_o) = L_3(\hat{f}_o) = \frac{1}{\hat{f}_o}$$

We obtain $\frac{1}{f_o} < \frac{\mu-\rho}{\sigma^2\eta} < \frac{1}{\hat{f}_o}$ and in turn $\hat{f}_o < f_o$. ■

Complement to Proof of Proposition 2 and 4: bail-in regime.

The goal here is almost identical to that for the bailout regime. Write $\hat{f}_i$, f_i and $\tilde{f}_i$ as the solutions to:

$$\zeta_i(f) \equiv \frac{\mu+[\kappa-(1-\tau)(1+h)]\lambda f+\lambda h(1-\tau)-\rho}{\sigma^2\eta} - \frac{1}{f} = 0$$
$$\Theta_i(f) \equiv \frac{\mu+[\kappa-(1-\tau)(1+h)]\lambda f+\lambda h(1-\tau)-\rho}{\sigma^2\eta} - \frac{v_i\lambda}{[\kappa-(1-\tau)(1+h)](1-f)^\eta} = 0$$
$$\Gamma_i(f) \equiv -\frac{\mu-\rho+\lambda h(1-\tau)}{f^2} + \frac{\sigma^2\eta}{f^3} - \frac{v_i\lambda}{(1-f)^\eta} = 0$$

respectively. We want to verify that if $\frac{\mu-\rho+\lambda h(1-\tau)}{\sigma^2\eta} > 1 + \frac{v_i}{\kappa-(1+h)(1-\tau)}$ and $\kappa > (1+h)(1-\tau)$, then $\hat{f}_i < \tilde{f}_i$ and $\hat{f}_i < f_i$.

We first show that $\hat{f}_i < \tilde{f}_i$. The equations $\zeta_i(f) = 0$ and $\Gamma_i(f) = 0$ can be restated as:

$$L_1(f) \equiv [\kappa-(1-\tau)(1+h)]f = \frac{\sigma^2\eta}{\lambda}\left(\frac{1}{f} - \frac{\mu-\rho+\lambda h(1-\tau)}{\sigma^2\eta}\right) \equiv \chi_1(f)$$
$$L_2(f) \equiv \frac{v_i f^2}{(1-f)^\eta} = \frac{\sigma^2\eta}{\lambda}\left(\frac{1}{f} - \frac{\mu-\rho+\lambda h(1-\tau)}{\sigma^2\eta}\right) = \chi_1(f)$$

Then:

$$
\begin{aligned}
L_2\left(\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right) &= v_i\left(\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right)^2\left(1-\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right)^{-\eta} \\
&< v_i\left(\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right)^2\left(1-\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right)^{-1} \\
&= v_i\left(\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right)\frac{\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}}{\left(1-\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right)} \\
&< v_i\left(\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right)\frac{[1+v_i/(\kappa-(1+h)(1-\tau))]^{-1}}{(1-[1+v_i/(\kappa-(1+h)(1-\tau))]^{-1})} \\
&= [\kappa-(1-\tau)(1+h)]\left(\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right) = L_1\left(\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right)
\end{aligned}
$$

where we have used the condition $\frac{\mu-\rho+\lambda h(1-\tau)}{\sigma^2\eta} > 1+\frac{v_i}{\kappa-(1+h)(1-\tau)}$ and in turn $\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)} < [1+v_i/(\kappa-(1+h)(1-\tau))]^{-1}$.

One can verify that L_2 is an increasing convex function on $f \in [0,1)$. Since $L_1(f)$ is linear, $L_1(0) = 0 = L_2(0)$ and $L_1\left(\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right) > L_2\left(\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right)$, we must have $L_1(f) > L_2(f)$ for $0 < f < \frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}$. Moreover, since $\chi_1\left(\frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}\right) = 0$, we must have $\tilde{f}_i < \frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}$ and $\hat{f}_i < \frac{\sigma^2\eta}{\mu-\rho+\lambda h(1-\tau)}$.

We can establish $\hat{f}_i < \tilde{f}_i$ by argument of contradiction. Suppose instead we have $\hat{f}_i \geq \tilde{f}_i$. Then:

$$\chi_1(\hat{f}_i) = L_1(\hat{f}_i) > L_2(\hat{f}_i) \geq L_2(\tilde{f}_i) = \chi_1(\tilde{f}_i)$$

but this contradicts the fact that χ_1 is a decreasing function.

To show that $\hat{f}_i < f_i$, we restate the equations $\zeta_i(f) = 0$ and $\Theta_i(f) = 0$ as:

$$
\begin{aligned}
L_3(f) \equiv \frac{1}{f} &= \frac{\mu+[\kappa-(1-\tau)(1+h)]\lambda f-\rho+\lambda h(1-\tau)}{\sigma^2\eta} \equiv \chi_2(f) \\
L_4(f) \equiv \frac{v_i}{[\kappa-(1-\tau)(1+h)](1-f)^\eta} &= \frac{\mu+[\kappa-(1-\tau)(1+h)]\lambda f-\rho+\lambda h(1-\tau)}{\sigma^2\eta} = \chi_2(f)
\end{aligned}
$$

Write $\omega \equiv 1 - \frac{v_i}{\kappa-(1-\tau)(1+h)+v_i} \in (0,1)$. Note that χ_2 is a linear increasing function. Then:

$$
\begin{aligned}
L_3(\hat{f}_i) = \chi_2(\hat{f}_i) \geq \chi_2(0) = \frac{\mu-\rho+\lambda h(1-\tau)}{\sigma^2\eta} > 1 + \frac{v_i}{\kappa-(1+h)(1-\tau)} \\
= \frac{1}{\omega} = L_3(\omega)
\end{aligned}
$$

and hence we have $\omega > \hat{f}_i$ as L_3 is a decreasing function. On the other hand:

$$
\begin{aligned}
L_4(f_i) = \chi_2(f_i) \geq \chi_2(0) = \frac{\mu-\rho+\lambda h(1-\tau)}{\sigma^2\eta} > 1 + \frac{v_i}{\kappa-(1+h)(1-\tau)} \\
= \frac{v_i}{[\kappa-(1-\tau)(1+h)](1-\omega)} > \frac{v_i}{[\kappa-(1-\tau)(1+h)](1-\omega)^\eta} = L_4(\omega)
\end{aligned}
$$

and thus $f_i > \omega$ since L_4 is increasing. Then the result follows as $f_i > \omega > \hat{f}_i$. ■

Complement to Proof of Proposition 5. We complete the proof by establishing a useful identity. Suppose T_1 is the arrival time of the first Poisson shock (such that T_1 has an $Exp(\lambda)$ distribution) and recall that the net worth process under the optimally chosen l_j satisfies:

$$
\begin{aligned}
\frac{dN_t}{N_t} &= \{[\mu+\kappa\lambda f_j - \rho_j(l_j)]l_j + \rho_j(l_j) - mq_j\}\,dt + \sigma l_j dB_t + (\Phi_j - 1)dy_t \\
&\equiv g_j dt + \sigma l_j dB_t + (\Phi_j - 1)dy_t
\end{aligned}
$$

We are going to show that if $\delta + \lambda - g_j > 0$, then:

$$
E\left(e^{-\delta T_1} N_{T_1}\right) = \frac{\Phi_j \lambda}{\delta+\lambda-g_j} N_0 \tag{A.2}
$$

To begin with, note that N_t yields a closed-form expression:

$$
N_t = N_0 \Phi_j^{Y_t} \exp\left(\left(g_j - \frac{\sigma^2 l_j^2}{2}\right)t + \sigma l_j B_t\right)
$$

where B_t is a Brownian motion and Y_t is a Poisson process. By construction, $Y_{T_1} = 1$ with probability one. Then:

$$
\begin{aligned}
E\left(e^{-\delta T_1} N_{T_1}\right) &= N_0 \Phi_j E\left[\exp\left(\left(g_j - \delta - \frac{\sigma^2 l_j^2}{2}\right) T_1 + \sigma l_j B_{T_1}\right)\right] \\
&= N_0 \Phi_j E\left\{E\left[\exp\left(\left(g_j - \delta - \frac{\sigma^2 l_j^2}{2}\right) T_1 + \sigma l_j B_{T_1}\right) |T_1\right]\right\} \\
&= N_0 \Phi_j E[e^{(g_j - \delta)T_1}] = N_0 \Phi_j \int_0^{\infty} \lambda e^{-\lambda t} e^{(g_j - \delta)t} dt = \frac{\Phi_j \lambda}{\delta + \lambda - g_j} N_0
\end{aligned}
$$

For the sake of completeness, we also derive the expression of W_b the net value created under the liquidation regime:

$$
\begin{aligned}
W_b &= E\left[\int_0^{T_1} e^{-\delta t}(r_t + d_t) dt\right] - N_0 = m q_b E\left[\int_0^{T_1} e^{-\delta t} N_t dt\right] - N_0 \\
&= m q_b N_0 E\left[\int_0^{\infty} e^{-\delta t} I(T_1 > t) \exp\left(\left(g_b - \frac{\sigma^2 l_b^2}{2}\right) t + \sigma l_b B_t\right) dt\right] - N_0 \\
&= m q_b N_0 \int_0^{\infty} \left\{e^{-\delta t} E\left[I(T_1 > t)\right] \times E\left[\exp\left(\left(g_b - \frac{\sigma^2 l_b^2}{2}\right) t + \sigma l_b B_t\right)\right]\right\} dt - N_0
\end{aligned}
$$

where we have used the independence of B_t and Y_t. The second expectation term is equal to $\exp(g_b t)$, while the first term is:

$$
E\left[I(T_1 > t)\right] = E\left[I(Y_t = 0)\right] = P(Y_t = 0) = \exp(-\lambda t)
$$

Hence:

$$
W_b = m q_b N_0 \int_0^{\infty} \exp[-(\lambda + \delta - g_b)t] dt - N_0 = \left(\frac{m q_b}{\lambda + \delta - g_b} - 1\right) N_0
$$

Finally, the corresponding expression of W_i (the net value created under the bail-in regime) can be obtained in an identical fashion together with the help of (A.2). ■

2 Extensions and further analysis

This section presents extensions of the model.

2.1 Leverage constraint

We incorporate a capital requirement by imposing that the gearing ratio l does not exceed an exogenously given constant l^{max}. Proposition 4 in the main paper can be easily modified to reflect the new solution structure.

Proposition 1 *For the insolvency regime (i.e. $\kappa \geq \underline{\kappa}_j$), the optimal investment policy (l_j^c) for the liquidation, bailout and bail-in regime in the presence of a leverage constraint is the lower of the unconstrained gearing ratio and the maximum allowable regulatory gearing, i.e. $l_j^c = \min\left(l_j(f_j), l^{max}\right)$, where $l_j(f_j)$ (for $j = b, o, i$) is the unconstrained gearing level defined in Proposition 4 in the main paper. The optimal crash exposure level is as described in Proposition 4 of the main paper for the unconstrained bank but by replacing in equations (17) and (18) in the main paper the unconstrained gearing level $l_j(f_j)$ by the constrained level $l_j^c(f_j)$. Junior debt is always risky under bail-ins.*

Introducing a capital requirement reduces the optimal investment and risk exposure in a fairly trivial fashion. The ranking of the optimal policies is not affected by a minimum capital requirement.

Proof. Recall that the underlying optimization problem is:

$$\max_{(l,f):l\leq l^{\max}} G_j(l,f) \equiv \max_{(l,f):l\leq l^{\max}} \left\{ [\mu + \kappa\lambda f - \rho_j(l,f)]l - \frac{\sigma^2\eta}{2}l^2 + \rho_j(l,f) + \frac{\lambda p_j}{1-\eta}[\phi_j(l,f)]^{1-\eta} \right\}$$

over each regime $j \in \{s, b, o, i\}$. An additional constraint $fl \leq 1$ ($fl > 1$) is imposed when $j = s$ ($j = b, o, i$).

Asset sales regime

If $l^{\max} \geq \frac{\mu-\rho}{\eta\sigma^2} = l_s$ then the unconstrained solution is feasible. Otherwise if $l^{\max} < \frac{\mu-\rho}{\eta\sigma^2}$, the original optimal leverage level $l_s = \frac{\mu-\rho}{\eta\sigma^2}$ is no longer feasible. Recall that the first order condition with respect to f is given by:

$$\frac{\partial G_s}{\partial f} = \lambda l \left(\kappa - \frac{1}{(1-fl)^\eta} \right) = 0$$

and it is easy to verify that $\frac{\partial^2 G_s}{\partial f^2} < 0$ for any $l > 0$. Hence if the optimization problem is unconstrained then the optimal solution (l, f) is expected to lie on the curve $\frac{1}{(1-fl)^\eta} = \kappa$. Write $f(l) = \frac{1-\kappa^{-1/\eta}}{l}$. To find the optimal l, we maximize $G_s(l, f(l))$. Differentiation gives:

$$\frac{d}{dl} G_s(l, f(l)) = \mu + \kappa\lambda f(l) - \rho - \sigma^2 \eta l - \kappa\lambda f(l) = \mu - \rho - \sigma^2 \eta l$$

If $l^{\max} < \frac{\mu-\rho}{\eta\sigma^2}$ then the candidate solution $l = \frac{\mu-\rho}{\eta\sigma^2}$ is not feasible. Since $\frac{d}{dl} G_s(l, f(l)) > 0$ for $l < \frac{\mu-\rho}{\eta\sigma^2}$, we should pick l to be as large as possible such that the optimal l must be $l = l^{\max}$. To summarize, the optimal (l, f) in presence of leverage constraint is given by:

$$l_s^c = \min\left(\frac{\mu-\rho}{\eta\sigma^2}, l^{\max} \right) = \min(l_s, l^{max}), \qquad f_s^c = \frac{1-\kappa^{-1/\eta}}{l_s^c}$$

Note that $f_s^c l_s^c = 1 - \kappa^{-1/\eta} < 1$.

In the rest of this section, we will assume $l^{\max} > \frac{\mu-\rho}{\eta\sigma^2}$ such that the solution structure under the asset sales regime will not be affected by the leverage constraint. If the value of $l^{\max}$ is too low, insolvency may not arise at all ex-ante because managers are not allowed to reasonably leverage the bank. Under this assumption, we have shown in the main paper that the managerial claim value under the asset sale regime is related to the constant:

$$H_s \equiv G_s(l_s^c, f_s^c) = \frac{(\mu-\rho)^2}{2\sigma^2\eta} + \kappa\lambda + \rho + \frac{\lambda\eta}{1-\eta}\kappa^{-\frac{1-\eta}{\eta}}$$

We are going to view H_s as a function of κ. As in the main paper, the existence of a critical κ above which the managers will optimally switch from the asset sales regime to the insolvency regime can be established by verifying that $J_j(\kappa) := H_j(\kappa) - H_s(\kappa)$ is increasing in κ. Here $H_j \equiv G_j(l_j^c, f_j^c)$ is viewed as a function of κ for $j \in \{b, o, i\}$.

Liquidation regime

The first order condition of G_b with respect to f is:

$$\frac{\partial G_b}{\partial f} = [\kappa - (1-\tau)(1-c_b)]\lambda l > 0$$

and thus the optimal value of f is always $f_b^c = 1$ no matter there is leverage constraint or not. To find the optimal l, we just need to solve:

$$\max_{l: 1 < l \le l^{\max}} G_b(l, f=1) = \max_{l: 1 < l \le l^{\max}} [\mu + (\kappa - 1 + \tau)\lambda - \rho] l - \frac{\sigma^2 \eta}{2} l^2 + \rho + \lambda(1-\tau)$$

where the optimal value of l is trivially given by:

$$l_b^c = \min\left(\frac{\mu + (\kappa - (1-\tau))\lambda - \rho}{\eta\sigma^2}, l^{\max}\right) = \min(l_b, l^{max})$$

The managerial claim value under this regime is linked to the constant:

$$H_b \equiv G_b(l_b^c, f_b^c) = \begin{cases} \frac{(\mu + (\kappa - 1 + \tau)\lambda - \rho)^2}{2\sigma^2\eta} + \rho + \lambda(1-\tau), & l_b \le l^{max} \\ (\mu + (\kappa - 1 + \tau)\lambda - \rho) l^{max} - \frac{\sigma^2\eta}{2}(l^{max})^2 + \rho + \lambda(1-\tau), & l_b > l^{max} \end{cases}$$

But it is clear that $l_b \le l^{max} \iff \kappa \le K_b^*$ where K_b^* is some constant. Then:

$$J_b'(\kappa) = \begin{cases} \lambda\kappa^{-\frac{1}{\eta}} + \lambda(l_b - 1) > 0, & \kappa \le K_b^* \\ \lambda\kappa^{-\frac{1}{\eta}} + \lambda(l^{\max} - 1) > 0, & \kappa > K_b^* \end{cases}$$

Hence the conclusion that $J_b(\kappa)$ being strictly increasing remains unchanged.

Bailout regime

If $v_o = 0$, then the problem is similar to that of the liquidation regime where the optimal jump exposure is $f_o^c = 1$ and the optimal leverage level is simply:

$$l_o^c = \min\left(\frac{\mu + \kappa\lambda - \rho}{\eta\sigma^2}, l^{\max}\right) = \min(l_o, l^{max})$$

We now consider the case of $v_o > 0$. Suppose the optimal leverage and jump exposure in the unconstrained case are given by l_o and f_o. Then it is clear that if $l^{\max} \geq l_o$ the original solution (l_o, f_o) will remain feasible. We are interested in the case where $l^{\max} < l_o$. The first two derivatives of G_o with respect to l and f are:

$$\frac{\partial G_o}{\partial l} = \mu + \kappa\lambda f - \rho - \sigma^2 \eta l, \qquad \frac{\partial^2 G_o}{\partial l^2} = -\sigma^2\eta < 0$$
$$\frac{\partial G_o}{\partial f} = \kappa\lambda l - \lambda v_o(1-f)^{-\eta}, \qquad \frac{\partial^2 G_o}{\partial f^2} = -\lambda v_0 \eta (1-f)^{-\eta-1} < 0$$

Hence for any fixed f, G_0 is increasing (resp. decreasing) in l on the region $l \leq l_o(f) \equiv \frac{\mu+\kappa\lambda f-\rho}{\sigma^2\eta}$ (resp. $l \geq l_o(f)$). Likewise, for any fixed l, G_o is increasing (resp. decreasing) in f on the region $f \leq f_o(l) \equiv 1 - \left(\frac{v_o}{\kappa l}\right)^{1/\eta} \iff l \geq \frac{v_o}{\kappa(1-f)^\eta} \equiv \mathcal{D}_o(f)$ (resp. $f \geq f_o(l) \iff l \leq \mathcal{D}_o(f)$). Note that the unconstrained optimal policy (l_o, f_o) is given by the intersection point of the functions $l_o(f)$ and $\mathcal{D}_o(f)$ on the (f, l) plane. A simple graphical consideration can reveal that $l_o(f) > \mathcal{D}_o(f)$ for $f < f_o$. Thus for any $l \leq l^{max} < l_o = l_o(f_o)$, we have:

$$G_o(l, f) \leq G_o(l, f_o(l)) \leq G_o(l^{max}, f_o(l)) \leq G_o(l^{max}, f_o(l^{max}))$$

i.e. $l_o^c = l^{max}$ and $f_o^c = 1 - \left(\frac{v_o}{\kappa l^{max}}\right)^{1/\eta}$. Finally, using the same arguments as in Section 2 of this internet appendix, the condition $l^{max} > \frac{\mu-\rho}{\sigma^2\eta} > 1 + \frac{v_o}{\kappa}$ will ensure $f_o^c l_o^c > 1$.

More generally, the optimal leverage ratio is given by $l_o^c = \min(l_o, l^{max})$. Recall that the unconstrained leverage ratio is given by $l_o = \frac{\mu+\kappa\lambda f_o-\rho}{\eta\sigma^2}$ where f_o solves the equation:

$$\frac{\mu + \kappa\lambda f - \rho}{\eta\sigma^2} - \frac{v_o}{\kappa(1-f)^\eta} = 0$$

It is easy to see that the solution f_o is increasing in κ. Hence l_o must be increasing in κ. We can conclude there must exist K_o^* such that $l_o^c = l_o$ ($l_o^c = l^{max}$) whenever $\kappa \leq K_o^*$

$(\kappa \geq K_o^*)$. Let $H_o \equiv G_o(l_o^c, f_o^c)$. Then:

$$H_o = \begin{cases} G_o(l_o(f_o(\kappa);\kappa), f_o(\kappa);\kappa), & \kappa \leq K_o^* \\ G_o(l^{max}, f_o^c(\kappa), \kappa), & \kappa \geq K_o^* \end{cases}$$

We have verified in the main paper that:

$$\frac{d}{d\kappa} G_o(l_o(f_o(\kappa);\kappa), f_o(\kappa);\kappa) = \lambda f_o l_o$$

Similarly, using the fact that f_o^c satisfies the first order condition, we have:

$$\frac{d}{d\kappa} G_o(l^{max}, f_o^c(\kappa), \kappa) = \frac{\partial G_o}{\partial f}\Big|_{l=l^{max}, f=f_o^c} \times \frac{\partial f_o^c}{\partial \kappa} + \frac{\partial G_o}{\partial \kappa}\Big|_{l=l_o, f=f_o} = 0 + \lambda f_o^c l^{max}$$

Hence for $J_o \equiv H_o - H_s$ the conclusion

$$J_o'(\kappa) = \lambda \kappa^{-\frac{1}{\eta}} + \lambda(f_o^c l_o^c - 1) > 0$$

still holds.

Bail-in regime

The analysis is identical to that of the bailout case and is thus omitted. The only additional consideration is that we need to verify the junior debt is indeed risky under the condition $l^{max} > \frac{\mu-\rho}{\sigma^2\eta} > \frac{\mu-\rho+\lambda h(1-\tau)}{\sigma^2\eta} > 1 + \frac{v_i}{\kappa-(1+h)(1-\tau)}$, but this again could be achieved by following the same arguments in Section 2 of this internet appendix. ■

2.2 Government commitment and randomized IRMs

In reality, the government has no obligation to commit to a particular IRM. Therefore, the bank's insiders and debtholders may not know ex ante which IRM will be adopted. In what follows we assume they have a common prior belief over the probability π_j that a particular IRM j will be applied (with $\pi_b + \pi_o + \pi_i = 1$ and $\pi_j \in [0, 1]$ for $j = b, o, i$).

We first derive the cost of debt as a function of (l, f). Recall the notation introduced in Table 1 of the main paper. What matters from the risk-neutral bondholders' perspective is the expected recovery rate which is given by:

$$\bar{\Omega} = \pi_b \Omega_b + \pi_o \Omega_o + \pi_i \Omega_i = (1 - \pi_o - \pi_i)\Omega_b + \pi_o \Omega_o + \pi_i \Omega_i.$$

To simplify the exposition, we assume zero bankruptcy cost for the liquidation regime such that $c_b = 0$. The expected loss in default and the fair after-tax cost of debt are, respectively:

$$\begin{aligned} 1 - \bar{\Omega} &= (1 - \pi_o - \pi_i)(1 - \Omega_b) + \pi_o(1 - \Omega_o) + \pi_i(1 - \Omega_i) \\ &= (1 - \pi_o - \pi_i)\frac{fl - 1}{l - 1} + \pi_i \frac{(f + h - fh)l - 1}{l - 1} \quad \text{and} \\ \bar{\rho}(l, f) &= \rho + \lambda(1 - \tau)\left[(1 - \pi_o - \pi_i)\frac{fl - 1}{l - 1} + \pi_i \frac{(f + h - fh)l - 1}{l - 1}\right]. \end{aligned}$$

From the perspective of the risk averse inside equityholders, the uncertainty regarding the IRM affects the net worth adjustment following a shock. Suppose the bank is risky (i.e. $fl > 1$). Then, immediately after a shock, there is a probability π_b that the managers get nothing (when the bank is liquidated), a probability π_o that the managers receive a continuation value $p_o M(\phi_o N)$ (when the bank is bailed out) and a probability π_i that the managers receive a severance claim value $p_i M(\phi_i N)$ (when the bank is bailed-in). The HJB equation associated with the managerial claim value can be suitably modified. The modified version of Proposition 4 in the main text is as follows.

Proposition 2 *In the insolvency regime, the optimal investment policy ($\bar{l}$) under a random IRM is:*

$$\bar{l} \;=\; \bar{l}(\bar{f}) \;=\; \frac{\mu - \rho}{\eta\sigma^2} \;+\; \frac{\lambda\left\{\kappa\bar{f} - (1 - \pi_o - \pi_i)(1 - \tau)\bar{f} - \pi_i(1 - \tau)[\bar{f}(1 + h) - h]\right\}}{\eta\sigma^2}$$

If $\pi_o v_o = \pi_i v_i = 0$*, then managers adopt maximum crash risk exposure* ($\bar{f} = 1$)*. Otherwise, the optimal exposure level is given by some* $\bar{f} \in (0,1)$ *which is the unique solution to the equation:*

$$[\kappa - (1-\tau)(1-\pi_o-\pi_i) - \pi_i(1-\tau)(1+h)]\,\bar{l}(f) \; - \; \frac{\pi_o v_o}{(1-f)^\eta} - \frac{\pi_i v_i}{(1-f)^\eta} = 0$$

The bank's optimal investment and risk exposure policies are a weighted average of the policies we previously derived for the case where the IRM is known ex ante. One can recover our earlier solutions by setting π_j equal to 1 for one of the probabilities.

Proof. Under randomized IRMs, the Hamilton-Jacobi-Bellman (HJB) equation of the optimization problem becomes:

$$\begin{aligned}\delta M(N_t) = \max_{q_t, l_t, f}\Bigg\{ & u(q_t N_t) \; - m q_t N_t \frac{\partial M(N_t)}{\partial N_t} + [\mu + \kappa\lambda f - \bar{\rho}(l_t, f)] l_t N_t \frac{\partial M(N_t)}{\partial N_t} \\ & + \frac{1}{2}\sigma^2 l_t^2 N_t^2 \frac{\partial^2 M(N_t)}{\partial N_t^2} + \bar{\rho}(l_t, f) N_t \frac{\partial M(N_t)}{\partial N_t} \\ & + \lambda \sum_{j\in\{b,o,i\}} [\pi_j p_j M(\phi_j(l_t, f) N_t) - M(N_t)] \Bigg\}\end{aligned} \tag{A.3}$$

The form of the value function remains the same as $M(N) = \frac{CN^{1-\eta}}{1-\eta}$ for some constant C. Then after substitution and slight rearrangement, the HJB equation becomes:

$$\begin{aligned}\frac{\lambda+\delta}{1-\eta} = \max_{q>0,l,f}\Bigg\{ & \frac{q^{1-\eta}}{C(1-\eta)} - mq + [\mu + \kappa\lambda f - \bar{\rho}(l,f)]l - \frac{\sigma^2\eta}{2} l^2 \\ & + \bar{\rho}(l,f) + \frac{\lambda \pi_o p_o}{1-\eta}[\phi_o(l,f)]^{1-\eta} + \frac{\lambda\pi_i p_i}{1-\eta}[\phi_i(l,f)]^{1-\eta} \Bigg\}\end{aligned} \tag{A.4}$$

The right-hand-side of (A.4) decouples into:

$$\max_{q>0}\left\{ \frac{q^{1-\eta}}{C_j(1-\eta)} - mq \right\} + \max_{l,f} \bar{G}(l,f)$$

where:

$$\bar{G}(l,f) \equiv \left\{ [\mu + \kappa\lambda f - \bar{\rho}(l,f)]l - \frac{\sigma^2\eta}{2} l^2 + \bar{\rho}(l,f) + \frac{\lambda\pi_o p_o}{1-\eta}[\phi_o(l,f)]^{1-\eta} + \frac{\lambda\pi_i p_i}{1-\eta}[\phi_i(l,f)]^{1-\eta} \right\}$$

Recall the notation $v_j \equiv p_j \xi_j^{1-\eta}$. Using the expressions of $\bar{\rho}$ and ϕ_j, the objective function can be further written as:

$$\bar{G}(l,f) = (\mu + \kappa\lambda f - \rho)l + \rho - \lambda(1-\tau)\left[(1-\pi_o-\pi_i)(fl-1) + \pi_i((f+fh-h)l-1)\right] - \frac{\sigma^2\eta}{2}l^2 + \frac{\lambda\pi_o v_o}{1-\eta}(1-f)^{1-\eta} + \frac{\lambda\pi_i v_i}{1-\eta}(1-f)^{1-\eta}$$

It is then straightforward to write down the first order conditions for l and f as:

$$\mu - \rho + \lambda\left\{\kappa f - (1-\pi_o-\pi_i)(1-\tau)f - \pi_i(1-\tau)[f(1+h)-h]\right\} - \sigma^2\eta l = 0$$

and:

$$l\left[\kappa - (1-\tau)(1-\pi_o-\pi_i) - \pi_i(1-\tau)(1+h)\right] - \frac{\pi_o v_o}{(1-f)^\eta} - \frac{\pi_i v_i}{(1-f)^\eta} = 0$$

Then the optimal (l,f) can be characterized by the solutions to the above simultaneous equations. The remaining technical gaps lie with checking: 1) the optimal solutions indeed exist over the risky regime $fl > 1$, and 2) the first order conditions indeed yield a global maximum. These can be achieved using the similar techniques considered in the main paper. Note that if $\pi_o v_o = \pi_i v_i = 0$, then the left-hand-side of the first order condition of f will be strictly positive. This implies the objective function is increasing in f and thus the optimal jump exposure is automatically given by $f = \bar{f} = 1$. ■

2.3 Extension to asset sales with transaction costs

Our benchmark model assumes that asset sales can be performed in a frictionless manner. In this section, we briefly discuss how the model can potentially be generalized to incorporate transaction costs associated with asset rebalancing.

In presence of transaction costs, it is well known that a constant investment level is not optimal but instead one should trade minimally to keep the allocation in the risky asset within a certain interval. This intuition is due to Magill and Constantinides (1976), which later is verified rigorously by Davis and Norman (1990) and Shreve and Soner (1994). No closed-form solution exists. The optimal policies have to be identified as a part of the solution to a non-linear free boundary value problem. To simplify analysis and facilitate comparison with our benchmark setup, we make the following two assumptions.

First, transaction costs apply to asset balancing only during the arrival of an economic downturn. This is a realistic assumption if we interpret transaction costs as a liquidity premium. While trading loans is relatively inexpensive during good times when rebalancing only involves small changes in the firm's assets, risk appetite of market participants goes down when the economy is experiencing distress (i.e. arrival of a large negative shock). After a large negative macro-shock, financial institutions must sell a significant fraction of their assets to rebalance and they may find it difficult to find a counterparty unless the assets are sold at a discount. The discrepancy between the asset book value and the actual executable price could be viewed as the transaction cost.

Second, we impose that the bank must adopt a constant investment level $l \equiv A_t/N_t$. A constant asset to net worth ratio is optimal in the absence of transaction costs, but may be suboptimal from a theoretical point of view when there are transaction costs. However, the restriction may originate from regulatory requirements that impose a cap on the leverage ratio of the bank. We have shown in the main paper (see the example in Section 3) that the bank's leverage ratio could spike up drastically during a downturn. Regulators may not allow the bank to wait and continue the operations based on such a risky balance sheet, but instead a prompt deleveraging is required especially during a crisis.

Now, suppose the bank maintains a constant asset to net worth ratio l and its balance sheet prior to a macro-shock consists of A units of asset and N units of equity. A shock brings the bank's asset and equity down to $(1-f)A$ and $N-fA$ units respectively. The bank remains solvent for as long as $N-fA \geq 0$ or equivalently $\frac{A}{N} = l \leq \hat{l} \equiv \frac{1}{f}$. In absence of transaction costs, the bank delevers by selling $fA(l-1)$ units of asset and uses the proceeds to pay off debt to maintain the target asset to net worth ratio of l. See panel A in Figure 1 of the main paper for a recap.

Consider now a proportional transaction cost c that has to be paid by the bank when offloading the asset during a downturn. The amount of loan to be sold, Δ, should now satisfy:

$$\frac{(1-f)A-\Delta}{N-fA-c\Delta} = l$$

which gives $\Delta = \frac{f(l-1)A}{1-cl}$. The net worth after rebalancing is:

$$N - fA - c\Delta = \left[1 - \frac{(1-c)fl}{1-cl}\right] N$$

This quantity is non-negative if $l \leq \frac{1}{c+f-cf} \equiv \hat{l}_c$. Note that $\hat{l}_c \leq \frac{1}{f} = \hat{l}$.

Special care has to be taken when defining 'insolvency' in the presence of transaction costs. If $l \leq \hat{l}_c$, then the bank remains solvent after costly asset sales (rebalancing). Consider the alternative scenario $\hat{l}_c < l \leq \hat{l}$. After the downturn, the bank is still solvent because $l \leq \hat{l}$ and as such $N - fA \geq 0$. However, the bank's leverage level is too high and its entire equity capital will be depleted if it engages in costly delevering. The transaction costs would wipe out the remaining equity, making it impossible for the bank to restore the desired asset to net worth ratio l. If a solvent bank must restore its target balance sheet structure during a downturn, then the transaction costs involved could drive the bank into insolvency post-rebalancing.

We therefore assume that insolvency is triggered during a downturn whenever $l > \hat{l}_c$. The jump size of net worth in the regime of asset sales is now given by $\phi_s(l) = 1 - \frac{(1-c)fl}{1-cl}$. The first order conditions for the optimal investment level l and downturn exposure f can be derived as:

$$\mu + \kappa\lambda f - \rho - \eta\sigma^2 l - \frac{\lambda f(1-c)}{\left(1 - \frac{fl(1-c)}{1-cl}\right)^{\eta}(1-cl)^2} = 0 \tag{A.5}$$

and

$$\kappa - \frac{1-c}{1-cl}\left[1 - \frac{(1-c)fl}{1-cl}\right]^{-\eta} = 0 \tag{A.6}$$

respectively.

Unlike the no-transaction cost case as in the main paper, we no longer have a closed-form expression for the optimal exposure f_s. Nonetheless, we can still infer the effect of transaction cost on the investment level under a fixed f. In particular, note that the left hand side of equation (A.5) is decreasing in c. This implies the solution $l_s(f)$ is decreasing in c. Transaction costs thus reduce the bank's investment and leverage level when the exposure f to downturns is exogenously given.

2.4 Decreasing returns to scale feature of bank's profitability

In our baseline model, the risk premium associated with the crash risk, κ, is assumed to be an exogenously given constant. It is possible to endogenize this quantity by assuming that the risk premium depends on the current leverage level given by:

$$\kappa_t \equiv \kappa_0 \left(\frac{A_t}{N_t}\right)^{\theta} = \kappa_0 l_t^{\theta}$$

where $\kappa_0 > 1$ is a constant and $0 < \theta < 1$ is the Herfindahl index.

Since κ depends on the decision variable l explicitly, the general optimization problem under IRM j can be easily restated as:

$$\max_{l,f} G_j(l,f) \equiv \max_{l,f} \left\{ [\mu + \kappa_0 \lambda f l^\theta - \rho_j(l,f)]l - \frac{\sigma^2\eta}{2} l^2 + \rho_j(l,f) + \frac{\lambda p_j}{1-\eta}[\phi_j(l,f)]^{1-\eta} \right\}$$

It is still relatively straightforward to write down the first order conditions in l and f. For example, in the case of liquidation (with zero bankruptcy cost $c_b = 0$ to simplify the exposition) the objective function becomes:

$$G_b(l,f) = (\mu + (\kappa_0 l^\theta - 1 + \tau)\lambda f - \rho)l - \frac{\sigma^2\eta}{2} l^2 + \rho + \lambda(1-\tau)$$

where the first order conditions in l and f are now given by:

$$\frac{\partial G_b}{\partial l} = \mu - \rho + [\kappa_0(\theta+1)l^\theta - (1-\tau)]\lambda f - \sigma^2 \eta l, \qquad \frac{\partial G_b}{\partial f} = (\kappa_0 l^\theta - 1 + \tau)\lambda l$$

With our standing assumption on the Merton ratio and $\kappa_0 > 1$, we can show that the first order condition in l can give an interior maximizer (there can be up to two roots in l with the equation $\frac{\partial G_b}{\partial l} = 0$. The required maximizer is given by the larger root). The same conditions also allow us to deduce $\frac{\partial G_b}{\partial f} \geq 0$ when evaluated along the optimal choice of l_b and hence $f_b = 1$ is still optimal. We no longer have a simple analytical solution of the optimal l_b and as such numerical studies have to be considered. Similar analysis can be done for the asset sales, bailout and bail-in regimes.

3 Numerical results of equity-conversion bail-in versus debt write-down bail-in

The optimal corporate policies under the equity-conversion bail-in and the debt write-down bail-in are shown in Table 1 for different values of $\xi \equiv \xi_i = \xi_d$. Recall from

Proposition 8 in the main paper that we have $l_i < l_d$, $f_i < f_d$ and $q_i < q_d$ provided that $\frac{\mu+[\kappa-(1-\tau)]-\rho}{\sigma^2\eta} < \frac{l^*}{\xi} \equiv -\frac{1}{h}$. Hence, we conjecture that an increase in ξ is more likely to result in violations of these rankings.

We can see in Table 1 that the rankings of l, f and q hold for small values of ξ up to 0.5. However, we start observing violations in the panel with $\xi = 0.7$ and $\xi = 0.9$ where one can verify that the condition $\frac{\mu+[\kappa-(1-\tau)]-\rho}{\sigma^2\eta} < \frac{l^*}{\xi}$ is not satisfied. For example, when $\xi = 0.7$ and $\lambda = 0.1$, we have $f_i = 96.30\% > 96.19\% = f_d$ and $q_i = 9.21\% > 9.20\% = q_d$. Some further numerical experiments (not reported here) show that under our baseline parameters we require ξ to be at least 0.58 for the rankings to be reversed.

References

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Shreve, S. E., and H. M. Soner (1994): "Optimal investment and consumption with transaction costs," *The Annals of Applied Probability*, pp. 609–692.

		$\xi_i = \xi_d = 0.1$						$\xi_i = \xi_d = 0.3$						$\xi_i = \xi_d = 0.5$						$\xi_i = \xi_d = 0.7$						$\xi_i = \xi_d = 0.9$					
		D_i/A_i	D_d/A_d	f_i	f_d	q_i	q_d	D_i/A_i	D_d/A_d	f_i	f_d	q_i	q_d	D_i/A_i	D_d/A_d	f_i	f_d	q_i	q_d	D_i/A_i	D_d/A_d	f_i	f_d	q_i	q_d	D_i/A_i	D_d/A_d	f_i	f_d	q_i	q_d
Benchmark		82.854	82.906	97.186	97.735	10.967	10.975	82.651	82.742	95.120	95.856	10.883	10.893	82.509	82.622	93.854	94.494	10.826	10.835	82.393	82.519	92.956	93.349	10.782	10.787	82.294	82.426	92.286	92.332	10.745	10.745
μ'	0.08	78.317	78.464	95.962	96.948	11.698	11.709	77.830	78.104	92.884	94.369	11.603	11.619	77.470	77.832	90.928	92.470	11.538	11.554	77.165	77.594	89.492	90.852	11.487	11.500	76.896	77.376	88.383	89.399	11.445	11.453
	0.12	85.789	85.812	97.892	98.232	10.066	10.072	85.687	85.725	96.370	96.780	9.990	9.997	85.617	85.662	95.452	95.735	9.939	9.944	85.561	85.609	94.809	94.862	9.899	9.900	85.513	85.561	94.335	94.092	9.866	9.861
σ'	0.18	86.168	86.193	97.978	98.296	10.493	10.498	86.053	86.095	96.512	96.892	10.418	10.424	85.973	86.023	95.624	95.879	10.368	10.372	85.909	85.962	95.000	95.032	10.329	10.329	85.855	85.908	94.539	94.282	10.296	10.291
	0.22	79.147	79.249	96.197	97.089	11.314	11.325	78.805	78.990	93.360	94.656	11.221	11.236	78.560	78.796	91.594	92.882	11.158	11.172	78.357	78.630	90.324	91.383	11.109	11.120	78.182	78.480	89.363	90.046	11.067	11.073
κ'	1.8	80.813	80.928	95.319	96.533	11.339	11.352	80.468	80.675	91.922	93.633	11.240	11.257	80.223	80.486	89.891	91.516	11.174	11.190	80.021	80.324	88.489	89.726	11.123	11.134	79.848	80.176	87.474	88.128	11.080	11.085
	2.2	84.479	84.505	98.167	98.444	10.547	10.552	84.350	84.395	96.808	97.162	10.474	10.481	84.261	84.314	95.959	96.237	10.425	10.430	84.188	84.246	95.345	95.463	10.386	10.388	84.126	84.186	94.875	94.778	10.353	10.351
ρ'	0.03	85.789	85.812	97.892	98.232	10.206	10.212	85.687	85.725	96.370	96.780	10.130	10.137	85.617	85.662	95.452	95.735	10.079	10.084	85.561	85.609	94.809	94.862	10.039	10.040	85.513	85.561	94.335	94.092	10.006	10.001
	0.07	78.317	78.464	95.962	96.948	11.558	11.569	77.830	78.104	92.884	94.369	11.463	11.479	77.470	77.832	90.928	92.470	11.398	11.414	77.165	77.594	89.492	90.852	11.347	11.360	76.896	77.376	88.383	89.399	11.305	11.313
λ	0	66.200*	66.200*	11.225*	11.225*	11.440*	11.440*	66.200*	66.200*	11.225*	11.225*	11.440*	11.440*	66.200*	66.200*	11.225*	11.225*	11.440*	11.440*	66.200*	66.200*	11.225*	11.225*	11.440*	11.440*	66.200*	66.200*	11.225*	11.225*	11.440*	11.440*
	0.1	88.617	88.634	98.502	98.694	9.501	9.509	88.524	88.551	97.416	97.617	9.366	9.374	88.459	88.491	96.758	96.839	9.276	9.279	88.407	88.440	96.297	96.188	9.205	9.200	88.363	88.394	95.956	95.612	9.146	9.130
τ	0.25	80.216	80.284	97.186	97.812	10.859	10.868	79.982	80.102	95.120	96.000	10.775	10.788	79.818	79.968	93.854	94.688	10.719	10.730	79.684	79.854	92.956	93.586	10.674	10.683	79.570	79.752	92.286	92.609	10.638	10.641
	0.45	85.492	85.530	97.186	97.653	11.074	11.081	85.320	85.386	95.120	95.703	10.990	10.998	85.200	85.279	93.854	94.287	10.934	10.940	85.102	85.189	92.956	93.095	10.890	10.891	85.018	85.107	92.286	92.036	10.853	10.848
η	0.5	86.963	86.970	99.443	99.547	11.672	11.674	86.922	86.936	98.879	99.017	11.641	11.645	86.892	86.908	98.505	98.589	11.619	11.621	86.868	86.882	98.232	98.208	11.601	11.600	86.847	86.859	98.026	97.857	11.586	11.581
	0.8	58.400*	58.400*	11.632*	11.632*	9.631*	9.631*	58.400*	58.400*	11.632*	11.632*	9.631*	9.631*	58.400*	58.400*	11.632*	11.632*	9.631*	9.631*	77.100	58.400*	83.911	11.632*	9.630	9.631*	76.809	77.373	82.525	83.850	9.574	9.580

Table 1: Comparative statics of equity-conversion bail-in and write-down bail-in. Base parameters used are $\mu' = 0.1$, $\sigma' = 0.2$, $\rho' = 0.05$, $\kappa' = 2$, $\tau = 0.35$, $\lambda = 0.05$, $\eta = 0.65$, $\delta = 0.4$, $\alpha = 0.8$, $p_i = p_d = 0.85$, $w^* = 1$ and $l^* = 5$. Numerical results are all expressed in percentages. An asterisk $*$ indicates that the bank is safe and engages in asset sales when a crash arrives.