Internet Appendix to
"The design and welfare implications
of mandatory pension plans"

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This Internet Appendix supplements the main paper in the following way. Section IA.1 provides details of the modeling of the pension fund's payout policy. Section IA.2 explains how we can reduce the dimensionality and solve an individual's lifetime utility maximization problem both with and without a mandatory pension scheme, and the numerical implementation of the solution is also described. Section IA.3 explains how we handle the optional features discussed in the main paper and provides detailed numerical results for welfare gains with a premature payout option. Finally, Section IA.4 considers the case with a cut in Social Security benefits and provides more detailed results for this case than in the main paper.

IA.1 Details of the model specification

In retirement, m_t denotes the fraction of the pension account balance paid out to the member at the beginning of year t. In monetary units, the payout is $m_t A_t$. We fix $m_{t_M} = 1$ so when the individual turns t_M years old, the fund pays out the remaining balance A_{t_M}. The payout pattern before t_M is controlled by the so-called *assumed interest rate* schedule $\tilde{r}_{t_R}, \dots, \tilde{r}_{t_M-1}$. We define the payout rates recursively by

$$m_t = \left(1 + m_{t+1}^{-1} e^{-\tilde{r}_t} (1 + d_t)^{-1}\right)^{-1}, \quad t = t_R, t_R + 1, \dots, t_M - 1, \tag{IA.1}$$

which implies that

$$m_{t+1} = e^{-\tilde{r}_t} (1 + d_t)^{-1} m_t / (1 - m_t). \tag{IA.2}$$

The recursion is solved by

$$
\begin{aligned}
m_t = \Big(&1 + e^{-\tilde{r}_t}(1+d_t)^{-1} + e^{-(\tilde{r}_t+\tilde{r}_{t+1})}(1+d_t)^{-1}(1+d_{t+1})^{-1} + \dots \\
&+ e^{-(\tilde{r}_t+\tilde{r}_{t+1}+\dots+\tilde{r}_{t_M-1})}(1+d_t)^{-1}(1+d_{t+1})^{-1}\dots(1+d_{t_M-1})^{-1}\Big)^{-1}.
\end{aligned}
\tag{IA.3}
$$

Using $\alpha_t = 0$ for $t \geq t_R$, the pension balance dynamics in Eq. (5) simplifies to $A_{t+1} = (1-m_t)A_t R_{At}(1+d_t)$, so

$$
m_{t+1}A_{t+1} = e^{-\tilde{r}_t}(1+d_t)^{-1}\frac{m_t}{1-m_t}(1-m_t)A_t R_{At}(1+d_t) = m_t A_t e^{-\tilde{r}_t} R_{At}.
$$

Therefore, the payout is increasing [decreasing] if the realized log after-tax return $\ln R_{At}$ is greater [smaller] than the assumed interest rate $\tilde{r}_t$ at age t. We assume throughout the paper that

$$
\tilde{r}_t = \ln \mathrm{E}_t[R_{At}] \equiv \ln\big(\tau_A + (1-\tau_A)\exp\{r + w(t)\mu_S\}\big),
\tag{IA.4}
$$

so that payouts are constant in expectation through retirement. By substituting (IA.4) into (IA.2), we obtain the recursion (6) for m.

Expected payouts are increasing [decreasing] through retirement if $\tilde{r}_t$ is smaller [larger] than $\ln \mathrm{E}_t[R_{At}]$. We find that welfare gains are only marginally different with non-constant expected payouts than with constant expected payouts (details are available upon request). In addition, non-constant expected payouts complicate the plan design, whereas plan simplicity facilitates transparency and, probably, public support. Hence, we stick to plans with constant expected payouts.

IA.2 Solving the utility maximization problem

IA.2.1 The case with a mandatory pension scheme

The problem involves the three state variables F_t, Y_t, and A_t, but is formulated so that the dimensionality can be reduced by one. Different choices of scaling are possible, but they are not equally convenient for the numerical solution approach, which involves a grid for the scaled state variables. We define the scaled state variables

$$
y_t = \frac{\bar{Y}_t}{F_t + \bar{A}_t}, \qquad a_t = \frac{\bar{A}_t}{F_t + \bar{A}_t},
$$

where $\bar{Y}_t = (1-\tau_Y)Y_t$ and $\bar{A}_t = (1-\tau_Y)A_t$. Here, we are using total (after-tax) savings $F_t + \bar{A}_t$ as denominator, which is going to be relatively independent of the assumed pension scheme since larger contribution rates that generate larger values of $\bar{A}$ tend to be partially compensated by lower private savings and thus lower values of F. We are solving the problem on a grid of points (t, y, a). Note that by definition $a \in [0,1]$ and $y > 0$, and we impose a small lower bound $y^\ell > 0$ and a suitable upper bound $y^u > y^\ell$ on y_t, and solve the problem on a grid in the space $[t_1, t_M] \times [y^\ell, y^u] \times [0,1]$; see Appendix IA.2.3 for more information on the numerical implementation.

An alternative would be to use disposable wealth $F_t + \bar{Y}_t$ as the denominator and the scaled state variables $y'_t = \bar{Y}_t/(F_t + \bar{Y}_t)$, $a'_t = \bar{A}_t/(F_t + \bar{Y}_t)$. However, in this case the denominator and the relevant range for a' would depend heavily on the pension scheme design, which complicates an appropriate definition of a grid for (y', a'). Yet another alternative would be to use $F_t + \bar{Y}_t + \bar{A}_t$ and the scaled state variables $y''_t = \bar{Y}_t/(F_t + \bar{Y}_t + \bar{A}_t)$, $a''_t = \bar{A}_t/(F_t + \bar{Y}_t + \bar{A}_t)$. However, in this case $y'' + a'' \le 1$, which hinders the use of a square grid for (y'', a'').

Final year. Death is certain at the end of the period ($p_{t_M} = 0$). Since $m_{t_M} = 1$, the bequest is

$$B_{t_M+1} = F_{t_M+1} = (1 - c_{t_M})\tilde{F}_{t_M} R_{F,t_M} = (1 - c_{t_M})(F_{t_M} + \bar{Y}_{t_M} + \bar{A}_{t_M})R_{F,t_M}.$$

The certainty equivalent is

$$\begin{aligned}
\mathrm{CE}_{t_M} &= \left(\mathrm{E}_{t_M}\left[\xi^{\frac{1-\gamma}{\psi-1}} B_{t_M+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} = \xi^{\frac{1}{\psi-1}}\left(1 - c_{t_M}\right)\left(F_{t_M} + \bar{Y}_{t_M} + \bar{A}_{t_M}\right)\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \\
&= \xi^{\frac{1}{\psi-1}}(1 - c_{t_M})(F_{t_M} + \bar{A}_{t_M})(1 + y_{t_M})\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}},
\end{aligned}$$

and the indirect utility is

$$\begin{aligned}
J_{t_M} &= \max_{c_{t_M}, \pi_{t_M}} \left\{\left(c_{t_M}\tilde{F}_{t_M}\right)^{1-\frac{1}{\psi}} + \beta\, \mathrm{CE}_{t_M}^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
&= (F_{t_M} + \bar{A}_{t_M})(1 + y_{t_M}) \max_{c_{t_M}, \pi_{t_M}} \left\{c_{t_M}^{1-\frac{1}{\psi}} + \beta\xi^{\frac{1}{\psi}}\left(1 - c_{t_M}\right)^{1-\frac{1}{\psi}}\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}.
\end{aligned}$$

Here, the optimal stock weight $\pi^*_{t_M}$ is determined by maximizing $\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{1/(1-\gamma)}$, and then the optimal consumption rate and the indirect utility are given by

$$c^*_{t_M} = \left(1 + \xi\beta^{\psi}\left(\mathrm{E}_{t_M}\left[R^{1-\gamma}_{F,t_M}\right]\right)^{\frac{\psi-1}{1-\gamma}}\right)^{-1}, \tag{IA.5}$$

$$J_{t_M} = (F_{t_M} + \bar{A}_{t_M})G_{t_M}(y_{t_M}, a_{t_M}), \tag{IA.6}$$

where $\pi^*_{t_M}$ is applied for generating the return and

$$G_{t_M}(y_{t_M}, a_{t_M}) = (1 + y_{t_M})\left\{\left(c^*_{t_M}\right)^{1-\frac{1}{\psi}} + \beta\xi^{\frac{1}{\psi}}\left(1 - c^*_{t_M}\right)^{1-\frac{1}{\psi}}\left(\mathrm{E}_{t_M}\left[R^{1-\gamma}_{F,t_M}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{IA.7}$$

which, in fact, does not depend on a_{t_M}. Note that $c^*_{t_M} \approx 1/(1+\xi)$ so the fraction of disposable wealth not consumed, and thus used for bequest, is approximately $\xi/(1+\xi)$. Hence, the bequest is approximately ξ times the amount consumed in the final year.

Non-final years, $t = t_1, \ldots, t_M - 1$. The bequest next year is

$$B_{t+1} = F_{t+1} + (1-\tau_Y)(1-I)[(1-m_t)A_t + \alpha_t Y_t]R_{At} = F_{t+1} + \frac{(1-I)\bar{A}_{t+1}}{1+d_t}.$$

For an induction argument, we assume that $J_{t+1} = (F_{t+1} + \bar{A}_{t+1})G_{t+1}(y_{t+1}, a_{t+1})$ which implies that the certainty equivalent is

$$\begin{aligned}
\mathrm{CE}_t &= \Big(p_t\mathrm{E}_t\left[(F_{t+1} + \bar{A}_{t+1})^{1-\gamma}G_{t+1}(y_{t+1}, a_{t+1})^{1-\gamma}\right] \\
&\qquad + (1-p_t)\mathrm{E}_t\left[\xi^{\frac{1-\gamma}{\psi-1}}(F_{t+1} + \frac{1-I}{1+d_t}\bar{A}_{t+1})^{1-\gamma}\right]\Big)^{\frac{1}{1-\gamma}} \\
&= (F_t + \bar{A}_t)\Bigg(p_t\mathrm{E}_t\Bigg[\left(\tfrac{F_{t+1}+\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma}G_{t+1}(y_{t+1}, a_{t+1})^{1-\gamma}\Bigg] \\
&\qquad + (1-p_t)\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_t\Bigg[\left(\tfrac{F_{t+1}+\frac{1-I}{1+d_t}\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma}\Bigg]\Bigg)^{\frac{1}{1-\gamma}} \\
&\equiv (F_t + \bar{A}_t)\mathcal{C}_t(y_t, a_t).
\end{aligned}$$

Here, the expectation is over the shock ε_{St} to stock prices and (before retirement) the shock ε_{Yt} to income, and we use that (for $k = 1$ or $k = (1-I)/(1+d_t)$)

$$\frac{F_{t+1} + k\bar{A}_{t+1}}{F_t + \bar{A}_t} = (1-c_t)\left(1 + (1-\alpha_t)y_t - (1-m_t)a_t\right) R_{Ft} + k\left((1-m_t)a_t + \alpha_t y_t\right) R_{At}(1+d_t),$$

$$y_{t+1} = \frac{y_t R_{Yt}}{(1-c_t)\left(1 + (1-\alpha_t)y_t - (1-m_t)a_t\right) R_{Ft} + \left((1-m_t)a_t + \alpha_t y_t\right) R_{At}(1+d_t)},$$

$$a_{t+1} = \frac{\left((1-m_t)a_t + \alpha_t y_t\right) R_{At}(1+d_t)}{(1-c_t)\left(1 + (1-\alpha_t)y_t - (1-m_t)a_t\right) R_{Ft} + \left((1-m_t)a_t + \alpha_t y_t\right) R_{At}(1+d_t)}.$$

The utility recursion (8) from the main paper implies that

$$\begin{aligned}
J_t &= \max_{c_t,\pi_t} \left\{ c_t^{1-\frac{1}{\psi}} \left(F_t + (1-\alpha_t)\bar{Y}_t + m_t \bar{A}_t\right)^{1-\frac{1}{\psi}} + \beta \left(F_t + \bar{A}_t\right)^{1-\frac{1}{\psi}} \mathcal{C}_t(y_t, a_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
&= \left(F_t + \bar{A}_t\right) \max_{c_t,\pi_t} \left\{ c_t^{1-\frac{1}{\psi}} \left(1 + (1-\alpha_t)y_t - (1-m_t)a_t\right)^{1-\frac{1}{\psi}} + \beta \mathcal{C}_t(y_t, a_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
&\equiv \left(F_t + \bar{A}_t\right) G_t\left(y_t, a_t\right).
\end{aligned}$$

Since the expectation in $\mathcal{C}_t$ involves G_{t+1}, we get the recursion

$$\begin{aligned}
G_t\left(y_t, a_t\right) = \max_{c_t,\pi_t} \Bigg\{ & c_t^{1-\frac{1}{\psi}} \left(1 + (1-\alpha_t)y_t - (1-m_t)a_t\right)^{1-\frac{1}{\psi}} \\
& + \beta \Bigg(p_t \mathrm{E}_t\left[\left(\tfrac{F_{t+1}+\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma} G_{t+1}(y_{t+1}, a_{t+1})^{1-\gamma} \right] \\
& + (1-p_t)\xi^{\frac{1-\gamma}{\psi-1}} \mathrm{E}_t\left[\left(\frac{F_{t+1} + \frac{1-I}{1+d_t}\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma} \right] \Bigg)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \Bigg\}^{\frac{1}{1-\frac{1}{\psi}}}
\end{aligned}$$

We solve this backwards starting with $t = t_M - 1$ in which case G_{t_M} is known from (IA.7).

IA.2.2 The case without a mandatory pension scheme

Suppose the individual spends a fraction $\eta \in [0,1]$ of his disposable wealth $F_{t_R-1} + \bar{Y}_{t_R-1}$ on purchasing a life-long annuity which provides a payment at the beginning of each year $t_R, t_R+1, \ldots, t_M$ conditional on being alive. The annuity stipulates an assumed interest rate $\tilde{r}$ and a benchmark portfolio. As annuity payments are made, the value of the portfolio is reduced correspondingly. Let V_t denote the value of the portfolio at the beginning of year t, just before the annuity payment is made. We let m_t denote the fraction of the portfolio value paid out, i.e. the annuity pays $m_t V_t$ at the beginning of year t, where

$$m_t = \left(\sum_{s=t}^{t_M} P_{t,s} e^{-\tilde{r}(s-t)} \right)^{-1}, \quad t = t_R, \ldots, t_M.$$

We assume that the amount $\eta \left(F_{t_R-1} + \bar{Y}_{t_R-1} \right) / (1+k)$ is invested in the portfolio immediately after the annuitization decision at the beginning of year $t_R - 1$, where $k \geq 0$ represents costs and profits of the issuer making the annuity less-than-fair for the individual. The payment from the annuity is not taxed as it is financed by the savings of after-tax labor income. The portfolio value at the beginning of year t_R is then

$$V_{t_R} = \frac{\eta}{1+k} \left(F_{t_R-1} + \bar{Y}_{t_R-1} \right) R_{V,t_R-1} p_{t_R-1}^{-1},$$

and the dynamics of the portfolio value are

$$V_{t+1} = V_t (1 - m_t) R_{Vt} p_t^{-1}, \quad t = t_R + 1, \ldots, t_M,$$

where R_{Vt} is the gross after-tax return on the portfolio, and the term p_t^{-1} represents a transfer of value from deceased customers in year t. We assume the same tax rate τ_F applies to the returns on the annuity portfolio as the returns on non-annuitized private investments so

$$R_{Vt} = 1 + (1 - \tau_F) \left[\exp \left\{ r + w_t \mu_S - \frac{1}{2} w_t^2 \sigma_S^2 + w_t \sigma_S \varepsilon_{St} \right\} - 1 \right],$$

where w_t is the annuity portfolio weight of the stock in year t. As explained in Section IA.1, the payout is increasing [decreasing] if the realized log after-tax return $\ln R_{Vt}$ is greater [smaller] than the assumed interest rate $\tilde{r}$. A fixed annuity is a special case where $\tilde{r} = \ln \left(\tau_F + (1 - \tau_F)(e^r - 1) \right)$.

The dynamics of private, non-annuitized wealth are

$$F_{t+1} = \begin{cases} (1 - c_t) \left(F_t + \bar{Y}_t + m_t V_t \right) R_{Ft} & \text{for } t = t_R, \ldots, t_M, \\ (1 - c_t)(1 - \eta) \left(F_t + \bar{Y}_t \right) R_{Ft} & \text{for } t = t_R - 1, \\ (1 - c_t) \left(F_t + \bar{Y}_t \right) R_{Ft} & \text{for } t = t_1, \ldots, t_R - 2. \end{cases}$$

Here $\bar{Y}_t = (1 - \tau_Y) Y_t$ and the dynamics of Y are given by Eqs. (1) and (2) in the main paper.

The state variables for this problem are $F_t, \bar{Y}_t$ before retirement and $F_t, \bar{Y}_t, V_t$ in retirement. The problem set up allows us to reduce the dimensionality by one through a scaling. We show below that the indirect utility has the form

$$J_t = \begin{cases} (F_t + V_t) G_t(y_t, v_t) & \text{for } t = t_R, \ldots, t_M, \\ F_t G_t(y_t), & \text{for } t = t_1, \ldots, t_R - 1, \end{cases}$$

where

$$y_t = \frac{\bar{Y}_t}{F_t + V_t}, \quad v_t = \frac{V_t}{F_t + V_t},$$

with $V_t = v_t = 0$ for $t = t_1, \ldots, t_R - 1$, and G_t and the optimal decisions are determined by backward recursion.

Final year, $t = t_M$. The individual is sure to die at the end of the period, leaving a bequest of

$$B_{t_M+1} = F_{t_M+1} = (1 - c_{t_M}) \left(F_{t_M} + \bar{Y}_{t_M} + V_{t_M}\right) R_{F,t_M},$$

where we have applied $m_{t_M} = 1$. The certainty equivalent is therefore

$$\mathrm{CE}_{t_M} = \left(\mathrm{E}_{t_M}\left[\xi^{\frac{1-\gamma}{\psi-1}} B_{t_M+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} = \xi^{\frac{1}{\psi-1}} (1 - c_{t_M}) \left(F_{t_M} + V_{t_M}\right) (1 + y_{t_M}) \left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}.$$

The indirect utility is

$$\begin{aligned} J_{t_M} &= \max_{c_{t_M}, \pi_{t_M}} \left\{ \left(c_{t_M}[F_{t_M} + \bar{Y}_{t_M} + V_{t_M}]\right)^{1-\frac{1}{\psi}} + \beta\, \mathrm{CE}_{t_M}^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &= \left(F_{t_M} + V_{t_M}\right) G_{t_M}(y_{t_M}, v_{t_M}), \end{aligned}$$

where

$$G_{t_M}(y_{t_M}, v_{t_M}) = (1 + y_{t_M}) \max_{c_{t_M}, \pi_{t_M}} \left\{ c_{t_M}^{1-\frac{1}{\psi}} + \beta \xi^{\frac{1}{\psi}} (1 - c_{t_M})^{1-\frac{1}{\psi}} \left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}},$$

which in fact is independent of v_{t_M}. The optimal portfolio weight $\pi^*_{t_M}$ is determined numerically by maximizing $\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$. The optimal consumption rate is

$$c^*_{t_M} = \left(1 + \xi \beta^{\psi} \left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{\psi-1}{1-\gamma}}\right)^{-1}.$$

In retirement, $t = t_R, t_R+1, \dots, t_M-1$. The bequest if dying at the beginning of year $t+1$ is $B_{t+1} = F_{t+1}$. With $J_{t+1} = (F_{t+1}+V_{t+1})\, G_{t+1}(y_{t+1}, v_{t+1})$, the certainty equivalent is

$$\begin{aligned}
\mathrm{CE}_t &= \Big(p_t \mathrm{E}_t\left[(F_{t+1}+V_{t+1})^{1-\gamma} G_{t+1}(y_{t+1}, v_{t+1})^{1-\gamma}\right] + (1-p_t)\mathrm{E}_t\left[\xi^{\frac{1-\gamma}{\psi-1}} F_{t+1}^{1-\gamma}\right]\Big)^{\frac{1}{1-\gamma}} \\
&= (F_t+V_t)\Bigg(p_t \mathrm{E}_t\Bigg[\left(\tfrac{F_{t+1}+V_{t+1}}{F_t+V_t}\right)^{1-\gamma} G_{t+1}(y_{t+1}, v_{t+1})^{1-\gamma}\Bigg] + (1-p_t)\xi^{\frac{1-\gamma}{\psi-1}} \mathrm{E}_t\Bigg[\left(\tfrac{F_{t+1}}{F_t+V_t}\right)^{1-\gamma}\Bigg]\Bigg)^{\frac{1}{1-\gamma}} \\
&\equiv (F_t+V_t)\mathcal{C}_t(y_t, v_t).
\end{aligned}$$

Here, the expectation is over the shock ε_{St} to stock prices, and we use that

$$\begin{aligned}
\frac{F_{t+1}+V_{t+1}}{F_t+V_t} &= (1-c_t)\,(1+y_t-(1-m_t)v_t)\,R_{Ft} + (1-m_t)v_t R_{Vt} p_t^{-1}, \\
\frac{F_{t+1}}{F_t+V_t} &= (1-c_t)\,(1+y_t-(1-m_t)v_t)\,R_{Ft}, \\
y_{t+1} &= \frac{y_t R_{Yt}}{(1-c_t)\,(1+y_t-(1-m_t)v_t)\,R_{Ft} + (1-m_t)v_t R_{Vt} p_t^{-1}}, \\
v_{t+1} &= \frac{(1-m_t)v_t R_{Vt} p_t^{-1}}{(1-c_t)\,(1+y_t-(1-m_t)v_t)\,R_{Ft} + (1-m_t)v_t R_{Vt} p_t^{-1}}
\end{aligned}$$

with $R_{Yt} = 1$. The indirect utility is

$$\begin{aligned}
J_t &= \max_{c_t, \pi_t}\left\{c_t^{1-\frac{1}{\psi}}(F_t + \bar{Y}_t + m_t V_t)^{1-\frac{1}{\psi}} + \beta\,(F_t+V_t)^{1-\frac{1}{\psi}}\,\mathcal{C}_t(y_t, v_t)^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
&= (F_t+V_t)\max_{c_t, \pi_t}\left\{c_t^{1-\frac{1}{\psi}}(1+y_t-(1-m_t)v_t)^{1-\frac{1}{\psi}} + \beta\mathcal{C}_t(y_t, v_t)^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
&\equiv (F_t+V_t) G_t(y_t, v_t).
\end{aligned}$$

Note that G satisfies the recursion

$$\begin{aligned}
&G_t\,(y_t, v_t) = \max_{c_t, \pi_t}\Bigg\{c_t^{1-\frac{1}{\psi}}(1+y_t-(1-m_t)v_t)^{1-\frac{1}{\psi}} \\
&+\beta\Bigg(p_t \mathrm{E}_t\Bigg[\left(\tfrac{F_{t+1}+V_{t+1}}{F_t+V_t}\right)^{1-\gamma} G_{t+1}(y_{t+1}, v_{t+1})^{1-\gamma}\Bigg] + (1-p_t)\xi^{\frac{1-\gamma}{\psi-1}} \mathrm{E}_t\Bigg[\left(\tfrac{F_{t+1}}{F_t+V_t}\right)^{1-\gamma}\Bigg]\Bigg)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\Bigg\}^{\frac{1}{1-\frac{1}{\psi}}}.
\end{aligned}$$

The optimal consumption rate c_t^* and the optimal portfolio weight π_t^* are determined numerically.

At retirement, $t = t_R - 1$. The bequest if dying just before year t_R begins is $B_{t_R} = F_{t_R}$. With $J_{t_R} = \big(F_{t_R} + V_{t_R}\big)\, G_{t_R}(y_{t_R}, v_{t_R})$, the certainty equivalent is

$$\begin{aligned}
\mathrm{CE}_{t_R-1} &= \Big(p_{t_R-1}\mathrm{E}_{t_R-1}\left[(F_{t_R} + V_{t_R})^{1-\gamma} G_{t_R}(y_{t_R}, v_{t_R})^{1-\gamma}\right] + (1-p_{t_R-1})\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_{t_R-1}\left[F_{t_R}^{1-\gamma}\right]\Big)^{\frac{1}{1-\gamma}} \\
&= F_{t_R-1}\Bigg(p_{t_R-1}\mathrm{E}_{t_R-1}\left[\left(\tfrac{F_{t_R}+V_{t_R}}{F_{t_R-1}}\right)^{1-\gamma} G_{t_R}(y_{t_R}, v_{t_R})^{1-\gamma}\right] \\
&\qquad\qquad + (1-p_{t_R-1})\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_{t_R-1}\left[\left(\tfrac{F_{t_R}}{F_{t_R-1}}\right)^{1-\gamma}\right]\Bigg)^{\frac{1}{1-\gamma}} \\
&\equiv F_{t_R-1}\mathcal{C}_{t_R-1}(y_{t_R-1}).
\end{aligned}$$

Here, the expectation is over the shock ε_{St} to stock prices, and we use that

$$\begin{aligned}
\frac{F_{t_R} + V_{t_R}}{F_{t_R-1}} &= (1 + y_{t_R-1})\left[(1 - c_{t_R-1})(1-\eta)R_{F,t_R-1} + \frac{\eta}{1+k}R_{V,t_R-1}p_{t_R-1}^{-1}\right], \\
\frac{F_{t_R}}{F_{t_R-1}} &= (1 - c_{t_R-1})(1-\eta)(1 + y_{t_R-1})R_{F,t_R-1}, \\
y_{t_R} &= \frac{\zeta y_{t_R-1}}{(1 + y_{t_R-1})\left[(1 - c_{t_R-1})(1-\eta)R_{F,t_R-1} + \frac{\eta}{1+k}R_{V,t_R-1}p_{t_R-1}^{-1}\right]}, \\
v_{t_R} &= \frac{\frac{\eta}{1+k}R_{V,t_R-1}p_{t_R-1}^{-1}}{(1 - c_{t_R-1})(1-\eta)R_{F,t_R-1} + \frac{\eta}{1+k}R_{V,t_R-1}p_{t_R-1}^{-1}}.
\end{aligned}$$

The indirect utility is

$$\begin{aligned}
J_{t_R-1} &= \max_{c_{t_R-1},\pi_{t_R-1},\eta}\left\{c_{t_R-1}^{1-\frac{1}{\psi}}(1-\eta)^{1-\frac{1}{\psi}}(F_{t_R-1} + \bar{Y}_{t_R-1})^{1-\frac{1}{\psi}} + \beta F_{t_R-1}^{1-\frac{1}{\psi}}\mathcal{C}_{t_R-1}(y_{t_R-1})^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
&= F_{t_R-1}\max_{c_{t_R-1},\pi_{t_R-1},\eta}\left\{c_{t_R-1}^{1-\frac{1}{\psi}}(1-\eta)^{1-\frac{1}{\psi}}(1 + y_{t_R-1})^{1-\frac{1}{\psi}} + \beta\mathcal{C}_{t_R-1}(y_{t_R-1})^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
&\equiv F_{t_R-1}G_{t_R-1}(y_{t_R-1}).
\end{aligned}$$

Note that G satisfies the recursion

$$G_{t_R-1}(y_{t_R-1}) = \max_{\substack{\eta, c_{t_R-1} \\ \pi_{t_R-1}}} \Bigg\{ c_{t_R-1}^{1-\frac{1}{\psi}} (1-\eta)^{1-\frac{1}{\psi}} (1+y_{t_R-1})^{1-\frac{1}{\psi}} + \beta \Bigg(p_{t_R-1} \mathrm{E}_{t_R-1} \left[\left(\tfrac{F_{t_R}+V_{t_R}}{F_{t_R-1}} \right)^{1-\gamma} G_{t_R}(y_{t_R}, v_{t_R})^{1-\gamma} \right] + (1-p_{t_R-1}) \xi^{\frac{1-\gamma}{\psi-1}} \mathrm{E}_{t_R-1} \left[\left(\tfrac{F_{t_R}}{F_{t_R-1}} \right)^{1-\gamma} \right] \Bigg)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \Bigg\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

The optimal annuitization rate η^*, the optimal consumption rate $c^*_{t_R-1}$, and the optimal portfolio weight $\pi^*_{t_R-1}$ are determined by numerical maximization.

Before retirement, $t = t_1, t_1+1, \ldots, t_R - 2$. The bequest if dying at the beginning of year $t+1$ is $B_{t+1} = F_{t+1}$. With $J_{t+1} = F_{t+1} G_{t+1}(y_{t+1})$, the certainty equivalent is

$$\begin{aligned} \mathrm{CE}_t &= \left(p_t \mathrm{E}_t \left[F_{t+1}^{1-\gamma} G_{t+1}(y_{t+1})^{1-\gamma} \right] + (1-p_t) \mathrm{E}_t \left[\xi^{\frac{1-\gamma}{\psi-1}} F_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} \\ &= F_t \Bigg(p_t \mathrm{E}_t \Bigg[\left(\tfrac{F_{t+1}}{F_t} \right)^{1-\gamma} G_{t+1}(y_{t+1})^{1-\gamma} \Bigg] + (1-p_t) \xi^{\frac{1-\gamma}{\psi-1}} \mathrm{E}_t \Bigg[\left(\tfrac{F_{t+1}}{F_t} \right)^{1-\gamma} \Bigg] \Bigg)^{\frac{1}{1-\gamma}} \\ &\equiv F_t \mathcal{C}_t(y_t). \end{aligned}$$

Here, the expectation is over the shock ε_{St} to stock prices and the shock ε_{Yt} to income, and we use that

$$\frac{F_{t+1}}{F_t} = (1-c_t)(1+y_t) R_{Ft}, \quad y_{t+1} = \frac{y_t R_{Yt}}{(1-c_t)(1+y_t) R_{Ft}}.$$

The indirect utility is

$$\begin{aligned} J_t &= \max_{c_t, \pi_t} \left\{ c_t^{1-\frac{1}{\psi}} (F_t + \bar{Y}_t)^{1-\frac{1}{\psi}} + \beta F_t^{1-\frac{1}{\psi}} \mathcal{C}_t(y_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &= F_t \max_{c_t, \pi_t} \left\{ c_t^{1-\frac{1}{\psi}} (1+y_t)^{1-\frac{1}{\psi}} + \beta \mathcal{C}_t(y_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &\equiv F_t G_t(y_t). \end{aligned}$$

Note that G satisfies the recursion

$$G_t(y_t) = \max_{c_t,\pi_t}\left\{c_t^{1-\frac{1}{\psi}}(1+y_t)^{1-\frac{1}{\psi}} + \beta\left(p_t \mathrm{E}_t\left[\left(\frac{F_{t+1}}{F_t}\right)^{1-\gamma} G_{t+1}(y_{t+1})^{1-\gamma}\right]\right.\right.$$
$$\left.\left. + (1-p_t)\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_t\left[\left(\frac{F_{t+1}}{F_t}\right)^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

The optimal consumption rate c_t^* and the optimal portfolio weight π_t^* are determined numerically.

In the situation where the individual does not have access to annuitization at retirement, then $\eta = 0$ in the above derivations and the state variable $v_t \equiv 0$ drops out.

IA.2.3 The numerical solution approach

Both in the case with and the case without a mandatory pension plan, the scaled state variable y is varying considerably over the working phase of life. With typical values of parameters and initial conditions, y starts out very high as annual income tends to be large relative to financial wealth for young individuals. As wealth accumulates over life, the value of y typically drops considerably approaching retirement. Such variations can be problematic when implementing the dynamic programming approach to utility maximization on an equidistant and relatively sparse grid.[1] Hence, we define

$$\hat{y}_t = y_t \exp\{-k_y(t_R-1-t)^+ + k_y^{\mathrm{ret}}(t-t_R)^+\}/\left(1-(1-\zeta)\mathbf{1}_{\{t\geq t_R\}}\right)$$

and form the grid using $\hat{y}$ instead of y. The scaled variable $\hat{y}$ is more stable over life than y with an appropriate choice of k_y, which depends on the values of parameters, initial conditions, the contribution rate and starting age, as well as the assumed degree of financial sophistication. Denote the initial income-wealth ratio by y_0, a "target" income-wealth ratio at retirement by y_R, and a "target" terminal income-wealth ratio by y_M. Then we let $k_y = -\ln(y_R/y_0)/(t_R-1-t_1)$ and $k_y^{\mathrm{ret}} = \ln(y_R/y_M)/(t_M-t_R)$. Clearly, y_M should be decreasing in the bequest weight ξ and $y_M = 1/(0.1+\xi)$ works well. Moreover, y_R should be higher for low savers and thus decreasing in γ and higher for procrastinators than rational individuals.

After extensive experimentation, we have settled on a 21×21 grid which produces robust and precise results with decent computation time. With our Matlab implementation

[1] In retirement, both wealth and net income decrease, leaving less systematic variation in y.

on a PC with an Intel Core i7-8550U 1.8GHz processor, the calculation of the optimal private decisions and indirect utility for a given pension design takes about eight minutes. The grid is an equidistant grid on $(\hat{y}, a) \in [\hat{y}_{\min}, \hat{y}_{\min} + (\hat{y}_{t_1} - \hat{y}_{\min})/0.7] \times [0, 1]$, where $\hat{y}_{\min} = \min\{0.001, 0.1 \times \hat{y}_{t_1}\}$. We first solve backwards in time for optimal decisions and the indirect utility on the grid, and then we simulate forward in time and make sure that the scaled state variables along almost all paths stay nicely within the grid boundaries, if not we vary y_R and, possibly, the grid spacing.

The backward recursions for the function G_t involve an expectation of a function of one or two standard normally distributed random variables. We approximate this expectation by the use of Gauss-Hermite quadrature. The continuous distribution of each standard normal random variable is approximated by a discrete distribution involving some number n of possible values and associated weights. In the one-dimensional case, we would like to calculate $\mathrm{E}[h(\varepsilon_1)]$ where $\varepsilon_1 \sim N(0, 1)$. Then the approximation is

$$\mathrm{E}[h(\varepsilon_1)] \approx \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} w_i h(\sqrt{2} x_i),$$

where x_i is node i, w_i the associated weight, and π is the constant 3.14159.... The ratio $p_i = w_i/\sqrt{\pi}$ is effectively the probability assigned to $z_i = \sqrt{2} x_i$. In the two-dimensional case, we would like to calculate $\mathrm{E}[h(\varepsilon_1, \varepsilon_2)]$ where $\varepsilon_1, \varepsilon_2 \sim N(0, 1)$ are independent. Then the approximation is

$$\mathrm{E}[h(\varepsilon_1, \varepsilon_2)] \approx \frac{1}{\pi} \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j h(\sqrt{2} x_i, \sqrt{2} x_j),$$

The nodes and weights can be calculated with the socalled Golub-Welsch method. We use $n = 9$ nodes as, e.g., Blake, Wright, and Zhang (2014, App. A), and in this case the numbers are:

i	x_i	w_i	y_i	p_i
1	-3.19099	3.96070×10^{-5}	-4.51274	2.23459×10^{-5}
2	-2.26658	0.00494362	-3.20543	0.00278914
3	-1.46855	0.0884745	-2.07684	0.0499164
4	-0.723551	0.432652	-1.02326	0.244098
5	0	0.720235	0	0.406349
6	0.723551	0.432652	1.02326	0.244098
7	1.46855	0.088475	2.07684	0.0499164
8	2.26658	0.00494362	3.20543	0.00278914
9	3.19099	3.96070×10^{-5}	4.51274	2.23459×10^{-5}

The values of the state variables corresponding to $\sqrt{2}x_i$ and $\sqrt{2}x_j$ do typically not match points in the grid for the state variable. If outside the grid boundaries, we use the value at the nearest boundary. If inside the grid boundaries, we use linear interpolation.

IA.3 Sweetening options

IA.3.1 Some tractable option features

Option to pay out fraction of pension savings without penalty. Suppose the individual at a prespecified age t can choose to pay out a fraction $\nu \in [0, \bar{\nu}]$ of the pension savings to the private, liquid account without paying a penalty. The payout is taxed at the income tax rate, just as the regular retirement payouts are. Then

$$\begin{aligned}
J_{t-}(F_t, Y_t, A_t) &= \max_{\nu} J_t\left(F_t + \nu[1-\tau_Y]A_t, Y_t, [1-\nu]A_t\right) \\
&= \max_{\nu} \left(F_t + \nu[1-\tau_Y]A_t + [1-\tau_Y][1-\nu]A_t\right) G_t\left(y_t, [1-\nu]a_t\right) \\
&= \max_{\nu} \left(F_t + [1-\tau_Y]A_t\right) G_t\left(y_t, [1-\nu]a_t\right) \\
&= \left(F_t + [1-\tau_Y]A_t\right) \max_{\nu} G_t\left(y_t, [1-\nu]a_t\right) \\
&= \left(F_t + [1-\tau_Y]A_t\right) G_t\left(y_t, [1-\nu^*(y_t, a_t)]a_t\right) \\
&\equiv \left(F_t + [1-\tau_Y]A_t\right) G_{t-}(y_t, a_t).
\end{aligned}$$

This option can be handled with a single "grid."

Option to change contribution rate from a certain age. Suppose the individual at a prespecified age $t < t_R - 1$ has the option to change the fixed, original contribution rate α to another value α' from some set $\mathcal{A}$ of possible values. Then

$$\begin{aligned} J_{t-}(F_t, Y_t, A_t) &= \max_{\alpha' \in \mathcal{A}} J_t(F_t, Y_t, A_t; \alpha') \\ &= \{(F_t + [1 - \tau_Y] A_t) \max_{\alpha' \in \mathcal{A}} G_t(y_t, a_t; \alpha') \\ &= \{(F_t + [1 - \tau_Y] A_t)\, G_t \left(y_t, a_t; \alpha^*(y_t, a_t)\right) \\ &\equiv (F_t + [1 - \tau_Y] A_t)\, G_{t-}(y_t, a_t). \end{aligned}$$

Since the optimally chosen contribution rate α^* is depending only on y_t and a_t, the separation is maintained. In an implementation, we need to calculate and store the values $G_t(y, a; \alpha')$ in all grid points (y, a) for all possible choices of α' from time t and on. With $\alpha' = 0$ being the only alternative, this option is a halt option, i.e., an option to terminate contributions prematurely.

IA.3.2 Numerical results with sweetening options

Table 5 in the main paper shows that rational individuals with low RRA (and low-to-modest EIS and bequest weight) dislike the plan with 10% contributions from age 30, even if they can choose the full-stock strategy IP5 and their favorite solidarity factor. Of course, they would prefer contributions at a lower rate or starting later or both, but such a change would reduce the welfare gains for procrastinators and for more risk-averse rational individuals. An alternative strategy is to maintain the 10%/30y plan but add options that are valuable primarily to the low-RRA rational individuals and do not allow procrastinators to reduce their savings substantially in the early part of the accumulation phase. One option found in some mandatory pension plans is to shift between investment policies throughout the contribution period. However, in our setting the RRA 2 individual would hardly ever want to deviate from the IP5 policy, so this option has little value to the critical type of participants given that they can already choose their preferred policy at enrollment.

We consider introducing a "payout option" allowing the individual to pay out a fraction of the pension account, up to some specified limit, at some specified age close to retirement. The payout is taxed at the income tax rate, but there is no additional penalty for the premature payout. As shown above, such an option respects the separation in Eq. (10) in the main paper and is thus numerically tractable. Table IA.1 shows how individuals with various preference parameters use and benefit from selected payout options. For a

γ	ψ	ξ	No option	10% at 60		10% at 66		12% at 55		19% at 60		30% at 66	
Rational													
4	1/4	2	2.06	2.26	(3.8)	2.10	(2.6)	2.25	(4.8)	2.41	(6.3)	2.15	(3.0)
6	1/4	2	2.50	2.63	(0.9)	2.58	(0.8)	2.61	(0.7)	2.70	(1.4)	2.58	(0.8)
2	1/4	2	-0.21	0.18	(6.8)	-0.09	(2.3)	0.24	(9.7)	0.38	(12.0)	-0.09	(2.7)
2	1/6	2	-0.53	-0.14	(7.4)	-0.43	(5.5)	-0.07	(10.4)	0.04	(13.1)	-0.42	(10.8)
2	1/4	1/2	-0.72	-0.15	(8.2)	-0.52	(3.5)	-0.06	(11.1)	0.11	(14.9)	-0.52	(3.9)
2	1/6	1/2	-1.35	-0.83	(8.7)	-1.21	(6.4)	-0.72	(11.6)	-0.61	(15.9)	-1.21	(8.3)
Procrastinator													
4	1/4	2	30.34	31.10	(10.0)	30.45	(10.0)	31.03	(12.0)	31.18	(19.0)	29.24	(29.1)
6	1/4	2	45.69	46.37	(9.8)	45.38	(9.1)	45.98	(12.0)	45.85	(18.4)	43.36	(20.1)
2	1/4	2	12.32	12.88	(10.0)	12.39	(10.0)	12.81	(12.0)	13.02	(19.0)	11.98	(29.4)

Table IA.1: **Welfare gains with a payout option.** The table shows percentage welfare gains when a payout option is added to the 10%/30y plan. The header '10% at 60' means that the individual can choose to pay out up to 10% of the pension wealth at age 60. Similarly for the other headings. The number in parenthesis shows the percentage of pension wealth that the individual would choose on average to pay out across the 10,000 simulated paths.

rational individual with the base case parameters (top row) we see that the option to pay out up to 10% of pension savings at age 60 increases the welfare gain induced by the mandatory plan from 2.06% to 2.26%. The individual exercises the option in states with high a_t (high pension wealth relative to liquid wealth) or low y_t (low income relative to total wealth). On average across all states, the individual chooses to pay out 3.8% of the pension balance. If the same option is available at age 66, it is less valuable with a gain of only 2.10%.

The payout option is relatively more valuable to the risk-tolerant individuals who dislike accumulating high wealth. With RRA 2 (and still EIS 1/4 and bequest weight 2), the '10% at age 60' option turns the welfare loss of 0.21% into a gain of 0.18% with an average payout of 6.8% of wealth. The RRA 6 individual pays out 0.9% on average with the welfare gain increasing marginally from 2.50% to 2.63%.

The rational individuals with RRA 2 choose the full-stock policy IP5 instead of the default IP3, and this leads to a significantly larger pension wealth being build up: about 12% at age 55, 19% at age 60, and 30% at age 66, for example (using $I = 1$ both for IP3 and IP5). One idea is that the individuals choosing IP5 can be allowed to pay out the excess pension savings. In fact, we see from Table IA.1 that the option to pay out up to 19% at age 60 leads to a gain for the risk-tolerant, rational individuals, except for the one with low EIS and low bequest weight who incurs a loss of 0.61% instead of the 1.35%

without the option and who would need an option to pay out 35% at age 50, for example, to reach a gain.

Procrastinators choose to pay out a larger fraction of the savings when possible. Given the option to pay out up to 10% at age 60, we find that the procrastinator always pays out 10% with RRA 2 or 4 and, on average, 9.8% with RRA 6. This premature payout facilitates an even better consumption plan so that the welfare gain increases slightly, for example from 30.34% to 31.10% for RRA 4. However, options allowing a significant premature payout decrease the welfare gain of the pension plan as seen in Table IA.1 for the '30% at 66' option. Overall, an option to pay out 19% at age 60 seems valuable to a broad range of individuals and particularly to the risk-tolerant rational individuals who would lose from the mandatory plan without such an option.

Another option tractable in our setting is a "halt option" allowing the individual to terminate contributions at some specified age before retirement. This is just as special case of the option to change the contribution rate from a certain age that was shown to respect the indirect utility separation in the previous subsection. As the payout option, individuals would tend to exercise the halt option if their pension wealth is a large share of total wealth or if income is low relative to total wealth. However, terminating contributions, say, at age 60 reduces the accumulated savings at retirement rather modestly which results in smaller welfare improvements than found for the payout option. For example, a rational individual with RRA 2 (and EIS 1/4, bequest weight 2) would choose to halt contributions at age 60 in 31% of the states, which reduces the loss from 0.21% to only 0.11%.

IA.4 Reduction in Social Security benefits

Our main analysis assumes Social Security retirement benefits at the current level. As explained in the main paper, the Social Security system lacks funding and is further stressed by the projected increased budget deficit and debt over the coming decades. To avoid the need for direct government subsidies or substantial increases in the retirement age or the pay-roll tax rate financing the system, benefits have to be cut to make the system sustainable. The April 2020 report of the Social Security and Medicare Boards of Trustees estimates that a 24% cut in benefits is necessary.[2]

In our model, the parameter ζ represents the level of Social Security benefits to pre-

[2]Sources: http://www.crfb.org/papers/updated-budget-projections-show-fiscal-toll-covid-19-pandemic and https://www.ssa.gov/OACT/TR/2020/index.html, accessed on August 7, 2020.

retirement income. Hence, we consider the implications of a 24% reduction in ζ from 0.45 to 0.342 so that Social Security benefits are reduced from 45% to 34.2% of pre-retirement income. We keep the retirement age and the tax rates unchanged, and repeat the analysis from Section 4.1 with the new parameter values. Since we have modeled the possible out-of-pocket medical expenses as fractions $h = 0.03$, $H = 0.85$ of Social Security benefits, we multiply these parameters by $1/(1 - 0.24) \approx 1.316$ to keep expenses fixed in dollar terms. However, to avoid costs exceeding income, we cap H at 0.95, effectively assuming that a positive net income is ensured through other welfare programs or support from family and friends.

Table IA.2 shows the welfare implications selected pension plans with the reduced Social Security benefits. When comparing with the base case in Table 3, we see that individuals appreciate a given pension plan more when Social Security is cut. The optimal contribution rate increases. Based on the weighted average gains, the best plans still have contributions from age 30, but now contribution rates of 11% or 12% provide the largest average gain of about 22.4% (or \$193,000), whereas the largest average gain of around 20.3% (\$175,000) in the base case was obtained with a contribution rate of 10% and 11%. As in the base case, the lower contribution rate has the benefit of causing only marginal losses for the most risk-tolerant rational individuals, losses that can be eliminated or reduced by adding a payout option as discussed in Section 4.4. To sum up, with a 24% cut in Social Security benefits, a well-designed mandatory pension scheme is even more appreciated, and the optimal contribution rate increases from 10% to 11%.

References

Blake, D., Wright, D., Zhang, Y., 2014. Age-dependent investing: Optimal funding and investment strategies in defined contribution pension plans when members are rational life cycle financial planners. Journal of Economic Dynamics and Control 38, 105–124.

	25-66y				30-66y				35-66y			
	8%	9%	10%	11%	10%	11%	12%	13%	13%	14%	15%	16%
Panel A: Rational												
RRA 2												
Default	-2.52	-3.27	-4.07	-4.91	-1.31	-1.77	-2.27	-2.80	**-0.80**	-1.10	-1.43	-1.79
With IP/SF option	-0.29	-0.80	-1.39	-2.05	0.20	-0.15	-0.56	-1.02	0.52	0.27	-0.01	-0.32
Best IP/SF	5/0.2	5/0.1	5/0.1	5/0.1	5/0.4	5/0.3	5/0.3	5/0.3	5/0.5	5/0.4	5/0.4	5/0.4
RRA 4												
Default	1.53	1.22	0.77	0.19	2.35	2.30	2.13	1.86	2.47	2.50	2.45	2.34
With IP/SF option	1.58	1.37	1.01	0.53	2.35	2.31	2.22	2.04	2.47	2.50	2.48	2.44
Best IP/SF	3/0.9	3/0.9	3/0.9	3/0.9	3/1.0	3/0.9	3/0.9	3/0.9	3/1.0	3/1.0	3/0.9	3/0.9
RRA 6												
Default	2.22	2.37	2.33	2.13	2.43	2.74	2.95	3.04	2.29	2.59	2.83	3.01
With IP/SF option	2.22	2.37	2.33	2.13	2.43	2.74	2.95	3.04	2.29	2.59	2.83	3.01
Best IP/SF	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0
Panel B: Procrastinator												
RRA 2												
Default	9.91	9.34	8.65	7.87	10.90	10.55	10.11	9.60	10.98	10.71	10.39	10.00
With IP/SF option	16.28	16.13	15.75	15.20	15.86	15.78	15.55	15.19	15.23	15.13	14.93	14.63
Best IP/SF	5/0.1	5/0.1	5/0.1	5/0.1	5/0.2	5/0.2	5/0.2	5/0.2	5/0.2	5/0.2	5/0.2	5/0.2
RRA 4												
Default	32.64	32.65	32.33	31.76	33.34	33.49	33.39	33.09	32.79	32.89	32.81	32.58
With IP/SF option	32.64	32.65	32.33	31.76	33.34	33.49	33.39	33.09	32.79	32.89	32.81	32.58
Best IP/SF	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0
RRA 6												
Default	47.43	48.73	49.36	49.50	46.80	48.07	48.81	49.13	45.03	46.03	46.67	47.00
With IP/SF option	47.43	48.73	49.36	49.50	46.80	48.07	48.81	49.13	45.03	46.03	46.67	47.00
Best IP/SF	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0	3/1.0
Panel C: Average gains												
Equal weights												
Default	15.20	15.17	14.89	14.42	15.75	15.90	15.85	15.65	15.46	15.60	15.62	15.52
All optimal IP/SF	16.64	16.74	16.57	16.18	16.83	17.04	17.06	16.91	16.39	16.57	16.62	16.56
Rational opt. IP/SF	15.58	15.61	15.38	14.96	16.00	16.17	16.15	15.98	15.68	15.83	15.86	15.79
Non-equal weights												
Default	20.67	20.69	20.43	19.95	21.21	21.40	21.36	21.15	20.80	20.96	20.97	20.85
All optimal IP/SF	21.92	22.05	21.88	21.46	22.17	22.41	22.43	22.26	21.62	21.81	21.85	21.76
Rational opt. IP/SF	20.86	20.92	20.69	20.24	21.34	21.53	21.52	21.33	20.91	21.08	21.10	20.99

Table IA.2: **Welfare implications of selected pension plans: Social Security cuts.** Percentage welfare gains are shown for rational individuals in Panel A and procrastinators in Panel B, in each case for RRA 2, 4, and 6. Gains are listed for plans with selected contribution rates from age 25, 30, or 35. The default is investment strategy IP3 and full solidarity ($I = 1$). For each individual and plan, we report the optimal IP and I and the corresponding welfare gain. Panel C reports the average gain across the six participant types, both an equally weighted average and an average assuming 2/3 procrastinators and 1/3 rational and with 25% in each group having RRA 2, 50% RRA 4, and 25% RRA 6. Each average is calculated assuming either that all individuals follow the default choice or that either all or only the rational optimally choose the investment policy and the solidarity factor. Losses are written in red. The maximum in each row is in blue. Social Security benefits are cut by 24% relative to the base case and medical costs are adjusted accordingly. Except for these parameters and the RRA, the baseline parameter values from Table 1 are used.