Appendix to "Three-dimensional instabilities of periodic gravity waves in shallow water"

By M. Francius and C. Kharif

Journal of Fluid Mechanics, vol. 561 (2006), pp. 417–437

In order to illustrate briefly the accuracy of our numerical results, we consider the convergence of the eigenvalues only for the steepest waves we have studied for $kh = 0.5$ and $kh = 0.3$. Tables 1 and 2 show the convergence of the most unstable modes of class II (n=3, 5) and class I (n=4, 6), respectively, when $ak = 0.170$ $kh = 0.5$. Tables 3 and 4 show the convergence of the most unstable modes of class II (n=3, 5, 7) and class I (n=4, 6), respectively, when $ak = 0.100$ $kh = 0.3$.
In these tables, M is the number of Fourier modes which are used to represent the perturbations. N is the number of points which are used to discretize the finite amplitude wave over one wavelength. As explained by Zhang & Melville (1989), N and M can be chosen independently but with the weak constraint $M < \frac{1}{2}N$. For $kh = 0.5$, tables 1 and 2 show that $N = 512$ and $M = 60$ is sufficient to get accurate eigenvalues up to the six decimal places. As the water depth decreases, the stability calculations become more expensive. For $kh = 0.3$, tables 3 and 4 show that for the same value of N the number of Fourier modes for the perturbations must be increased to get the same precision. For $kh = 0.1$, the speed of convergence in the method used to determine the basic nonlinear wave slows down. This, in turn, determines the speed of convergence of the eigenvalues and causes serious difficulties when the steepness becomes moderate. For $ak = 0.040$, the accuracy of the eigenvalues is very poor with only three significant figures. For $ak = 0.005$ which is the only case presented in this work when $kh = 0.1$, the accuracy of the eigenvalues is still good with six significant figures (not shown here).

N	M	Class II(n=3) Re(σ)	$p=\frac{1}{2}$ $q=0.475$ Im(σ)	Class II(n=5) Re(σ)	$p=\frac{1}{2}$ $q=1.973$ Im(σ)
256	20	0.12816706×10^{-3}	0.51798173×10^{-1}	0.16318794×10^{-3}	0.66781740×10^{-1}
	40	0.12518863×10^{-5}	0.51824354×10^{-1}	0.15825710×10^{-5}	0.67250086×10^{-1}
	60	0.17728192×10^{-7}	0.51825854×10^{-1}	0.22641074×10^{-7}	0.67256528×10^{-1}
512	20	0.12816706×10^{-3}	0.51798173×10^{-1}	0.16318794×10^{-3}	0.66781740×10^{-1}
	40	0.12518888×10^{-5}	0.51824354×10^{-1}	0.15825757×10^{-5}	0.67250086×10^{-1}
	60	0.17600063×10^{-7}	0.51825854×10^{-1}	0.21603753×10^{-7}	0.67256530×10^{-1}
	80	$-.36046026\times10^{-8}$	0.51825888×10^{-1}	$-.19370318\times10^{-6}$	0.67256274×10^{-1}

Table 1: Convergence of the eigenvalues for the most unstable modes of Class II(n=3, 5) when $ak=0.170$ and $d=0.5$.

N	M	Class I(n=4) Re(σ)	$p=0$ $q=1.102$ Im(σ)	Class I(n=6) Re(σ)	$p=0$ $q=3.169$ Im(σ)
256	20	$-.15966383\times10^{-13}$	0.68187098×10^{-1}		
	40	0.23742909×10^{-12}	0.68495707×10^{-1}		
	60	0.73243788×10^{-10}	0.68500341×10^{-1}	0.10779661×10^{-9}	0.56942654×10^{-1}
	70	0.88186381×10^{-9}	0.68500412×10^{-1}	$-.11626527\times10^{-8}$	0.56942761×10^{-1}
	80	0.65382363×10^{-8}	0.68500371×10^{-1}	$-.32770236\times10^{-7}$	0.56942741×10^{-1}
	90	$-.51671551\times10^{-5}$	0.68501907×10^{-1}	$-.13494966\times10^{-5}$	0.56945767×10^{-1}
512	20	$-.90991633\times10^{-14}$	0.68187098×10^{-1}		
	40	$-.30797635\times10^{-12}$	0.68495707×10^{-1}		
	60	0.34908174×10^{-11}	0.68500341×10^{-1}	0.27182642×10^{-11}	0.56942654×10^{-1}
	70	$-.10575059\times10^{-8}$	0.68500416×10^{-1}	$-.26964063\times10^{-9}$	0.56942758×10^{-1}
	80	$-.46505968\times10^{-7}$	0.68500402×10^{-1}	0.15278876×10^{-8}	0.56942764×10^{-1}
	90	$-.13758546\times10^{-5}$	0.68503155×10^{-1}	0.78667591×10^{-7}	0.56942744×10^{-1}

Table 2: Convergence of the eigenvalues for the most unstable modes of Class I (n=4, 6) when $ak=0.170$ and $d=0.5$.

		Class II(n=3)	$p=\frac{1}{2}$ $q=0.447$	Class II(n=5)	$p=\frac{1}{2}$ $q=1.560$	Class II(n=7)	$p=\frac{1}{2}$ $q=3.220$
N	M	Re(σ)	Im(σ)	Re(σ)	Im(σ)	Re(σ)	Im(σ)
256	20	0.13231160×10^{-3}	0.21904618×10^{-1}	0.15608924×10^{-3}	0.40145632×10^{-1}	0.19225226×10^{-3}	0.38412599×10^{-1}
	60	0.17721901×10^{-7}	0.21490760×10^{-1}	0.21301846×10^{-7}	0.40404570×10^{-1}	0.25624473×10^{-7}	0.38900220×10^{-1}
	70	0.20855068×10^{-8}	0.21490724×10^{-1}	0.25063852×10^{-8}	0.40404624×10^{-1}	0.30018048×10^{-8}	0.38900306×10^{-1}
	80	0.59391200×10^{-9}	0.21490721×10^{-1}	0.29852119×10^{-9}	0.40404631×10^{-1}	0.35978161×10^{-9}	0.38900317×10^{-1}
	90	0.24737089×10^{-9}	0.21490723×10^{-1}	$-.92747107\times10^{-10}$	0.40404633×10^{-1}	$-.14132412\times10^{-10}$	0.38900319×10^{-1}
	100	$-.15457872\times10^{-8}$	0.21490722×10^{-1}	$-.74510770\times10^{-9}$	0.40404635×10^{-1}	$-.61341352\times10^{-9}$	0.38900321×10^{-1}
512	60	0.17721389×10^{-7}	0.21490760×10^{-1}	0.21301490×10^{-7}	0.40404570×10^{-1}	0.25629071×10^{-7}	0.38900220×10^{-1}
	70	0.20856352×10^{-8}	0.21490724×10^{-1}	0.25049022×10^{-8}	0.40404624×10^{-1}	0.29951646×10^{-8}	0.38900306×10^{-1}
	80	0.25784181×10^{-9}	0.21490720×10^{-1}	0.30042177×10^{-9}	0.40404631×10^{-1}	0.35958037×10^{-9}	0.38900316×10^{-1}
	90	0.47711012×10^{-8}	0.21490721×10^{-1}	$-.14185160\times10^{-9}$	0.40404632×10^{-1}	$-.11473251\times10^{-9}$	0.38900317×10^{-1}
	100	$-.15409227\times10^{-8}$	0.21490720×10^{-1}	$-.12019617\times10^{-8}$	0.40404635×10^{-1}	$-.35531630\times10^{-10}$	0.38900317×10^{-1}

Table 3: Convergence of the eigenvalues for the Class II (n=3, 5, 7) instabilities when $ak = 0.100$ and $d = 0.3$.

N	M	Class I(n=4) Re(σ)	$p=0$ $q=0.954$ Im(σ)	Class I(n=6) Re(σ)	$p=0$ $q=2.302$ Im(σ)
256	20	0.39543507×10^{-15}	0.34592729×10^{-1}		
	40	$-.37735076\times10^{-14}$	0.34648393×10^{-1}		
	50	$-.10805212\times10^{-12}$	0.34649758×10^{-1}	$-.31477312\times10^{-13}$	0.41303945×10^{-1}
	60	0.24825657×10^{-11}	0.34649951×10^{-1}	0.42392833×10^{-13}	0.41304532×10^{-1}
	70	$-.60531274\times10^{-11}$	0.34649977×10^{-1}	$-.10237330\times10^{-12}$	0.41304604×10^{-1}
	80	$-.20048873\times10^{-10}$	0.34649980×10^{-1}	0.23669347×10^{-11}	0.41304613×10^{-1}
	90	0.25146550×10^{-9}	0.34649981×10^{-1}	0.24525362×10^{-8}	0.41304616×10^{-1}
512	50	0.32739377×10^{-13}	0.34649758×10^{-1}	$-.26869924\times10^{-14}$	0.41303945×10^{-1}
	60	0.43177769×10^{-11}	0.34649951×10^{-1}	$-.36780054\times10^{-12}$	0.41304532×10^{-1}
	70	$-.25517602\times10^{-11}$	0.34649976×10^{-1}	$-.23870495\times10^{-12}$	0.41304604×10^{-1}
	80	$-.31049614\times10^{-10}$	0.34649980×10^{-1}	0.33906538×10^{-11}	0.41304613×10^{-1}
	90	0.36451404×10^{-9}	0.34649980×10^{-1}	$-.12376523\times10^{-10}$	0.41304614×10^{-1}

Table 4: Convergence of the eigenvalues for the Class I (n=4, 6) instabilities when $ak = 0.100$ and $d = 0.3$.