Appendix to "Hydrodynamics of granular gases and granular gas mixtures"

By D. Serero, I. Goldhirsch, S. H. Noskowicz & M-L. Tan

Journal of Fluid Mechanics, vol. 554 (2006), pp. 237–258

Explicit expressions for the transport coefficients (to second order in the Sonine polynomial expansions)

In this appendix we present explicit expressions for the transport coefficients pertaining to a binary mixture of spheres, whose collisions are characterized by constant coefficients of normal restitution, as explained in detail in the article. These expressions are correct to second order in the Sonine polynomial expansions. We do possess expressions to third order in these expansions but they are too cumbersome to present here. In the interest of conciseness of the presentation, we first define certain matrices and vectors and then present expressions for the transport coefficients in terms of these entities.

Recall the following definitions from the main text: the particle masses are m_A and m_B, the number densities are n_A and n_B respectively, the diameters are σ_A and σ_B respectively, $\sigma_{AB} \equiv \frac{\sigma_A+\sigma_B}{2}$, $M_A \equiv \frac{m_A}{m_A+m_B}$, and $M_B \equiv \frac{m_B}{m_A+m_B}$.

I. Preliminary definitions

Define the following four-component vectors :

$$\mathbf{R}^{K,T} = \left(\frac{\sqrt{6}}{2} \frac{1}{\sqrt{M_A}} \frac{(M_B - M_A)}{\frac{n_A}{n} M_A + \frac{n_B}{n} M_B}, -\frac{5\sqrt{6}}{4} \frac{n}{n_B} \frac{1}{\sqrt{M_A}}, 0, -\frac{5\sqrt{6}}{4} \frac{1}{\sqrt{M_B}} \frac{n}{n_A} \right)^t \tag{1}$$

$$\mathbf{R}^{K,V} = \left(\left(5 + \frac{5n_A}{n_B} \right), 0, \left(5 + \frac{5n_B}{n_A}, \right), 0 \right)^t \tag{2}$$

$$\mathbf{R}^{K,n_A} = \left(-\frac{\sqrt{6}}{2} \frac{\sqrt{M_A}}{\frac{n_A}{n} M_A + \frac{n_B}{n} M_B}, 0, 0, 0 \right)^t \tag{3}$$

$$\mathbf{R}^{K,n_B} = \left(0, 0, 0, -\frac{\sqrt{6}}{2} \frac{\sqrt{M_B}}{\frac{n_A}{n} M_A + \frac{n_B}{n} M_B} \right)^t \tag{4}$$

where the superscript t denotes the transpose of a vector or matrix. Define next the 4×4 matrix $\underline{\underline{\mathcal{L}}}^T$ by its nonzero elements:

$$L_{11}^T = \frac{4\sqrt{6\pi}}{9\sqrt{M_A M_B}} \left(4M_A^2 + 2M_A - 6 - 3\sqrt{2M_B} \left(\frac{n_A}{n_B} \right) \left(\frac{\sigma_A}{\sigma_{AB}} \right)^2 \right)$$

$$L_{12}^T = L_{21}^T = \frac{\sqrt{6\pi}}{9\sqrt{M_A M_B}} \left(16M_A^3 - 20M_A^2 - 8M_A + 12 + 3\sqrt{2}\sqrt{M_B} \left(\frac{n_A}{n_B} \right) \left(\frac{\sigma_A}{\sigma_{AB}} \right)^2 \right)$$

$$L_{13}^T = L_{31}^T = \frac{16\sqrt{6\pi}}{9}\sqrt{M_A M_B}$$

$$L_{14}^T = \frac{16\sqrt{6\pi}}{9} M_A^{\frac{3}{2}} M_B^{-\frac{3}{2}} \left(M_A^3 - 3M_A M_B - 1\right)$$

$$L_{22}^T = \frac{205\sqrt{3\pi}}{18}\frac{1}{\sqrt{M_A}}\left(\frac{n_A}{n_B}\right)\left(\frac{\sigma_A}{\sigma_{AB}}\right)^2 + \frac{2\sqrt{6\pi}}{9}\left(120M_A^3 - 63M_A^2 + 32M_A + 51\right)\sqrt{\frac{M_B}{M_A}}$$

$$L_{23}^T = -\frac{16\sqrt{6\pi}}{9} M_B^{\frac{3}{2}}\sqrt{M_A}$$

$$L_{41}^T = -\frac{16\sqrt{6\pi}}{9} M_A^{\frac{3}{2}}\sqrt{M_B}$$

$$L_{24}^T = L_{42}^T = 160\sqrt{\pi} M_B^{\frac{3}{2}} M_A^{\frac{3}{2}}$$

$$L_{32}^T = \widehat{L_{14}^T}$$

$$L_{33}^T = \widehat{L_{11}^T}$$

$$L_{34}^T = L_{43}^T = \widehat{L_{12}^T}$$

$$L_{44}^T = \widehat{L_{22}^T}$$

where an overhat denotes here and below the exchange $A \leftrightarrow B$. Next define the 4×4 matrix $\underline{\underline{\mathcal{L}}}^V$ by its nonzero elements:

$$L_{11}^V = -\frac{4\sqrt{6\pi}}{3}\sqrt{\frac{M_B}{M_A}}$$

$$L_{12}^V = \frac{2\sqrt{6\pi}}{3}\frac{M_B^{\frac{3}{2}}}{\sqrt{M_A}}$$

$$L_{13}^V = \frac{4\sqrt{6\pi}}{3}$$

$$L_{14}^V = -\frac{2\sqrt{6\pi}}{3} M_A$$

$$L_{21}^V = \frac{2\sqrt{6\pi}}{3}\frac{M_B^{\frac{3}{2}}}{\sqrt{M_A}}$$

$$L_{23}^V = -\frac{2\sqrt{6\pi}}{3} M_B$$

$$L_{24}^V = 9\sqrt{6\pi} M_A M_B$$

$$L^V_{22} = -\frac{\sqrt{6\pi}}{3\sqrt{M_B M_A}}\left(-27M_A^3 + 37M_A^2 - 23M_A + 4\sqrt{2}\sqrt{M_B}\left(\frac{n_A}{n_B}\right)\left(\frac{\sigma_A}{\sigma_{AB}}\right)^2 + 13\right)$$

$$L^V_{31} = 1$$

$$L^V_{33} = \frac{n_B}{n_A}\sqrt{\frac{M_B}{M_A}}$$

$$L^V_{41} = -\frac{2\sqrt{6\pi}}{3}M_A$$

$$L^V_{42} = \widehat{L^V_{24}}$$

$$L^V_{43} = \widehat{L^V_{21}}$$

$$L^V_{44} = \widehat{L^V_{22}}$$

Using the matrices $\underline{\underline{\mathcal{L}}}^T$ and $\underline{\underline{\mathcal{L}}}^V$ and the vectors defined in Eqs. (1- 4), define the following vectors:

$$\underline{\Psi}^{K,V} = \left(\underline{\underline{\mathcal{L}}}^T\right)^{-1}\cdot \mathbf{R}^{K,V} \tag{5}$$

$$\underline{\Psi}^{K,T} = \left(\underline{\underline{\mathcal{L}}}^V\right)^{-1}\cdot \mathbf{R}^{K,T} \tag{6}$$

$$\underline{\Psi}^{K,n_\alpha} = \left(\underline{\underline{\mathcal{L}}}^V\right)^{-1}\cdot \mathbf{R}^{K,n_\alpha} \tag{7}$$

whose elements (which are coefficients in the Sonine polynomial expansion) are denoted by:

$$\underline{\Psi}^{K,V} \equiv \left(\psi_A^{K,V,(0)}, \psi_A^{K,V,(1)}, \psi_B^{K,V,(0)}, \psi_B^{K,V,(1)}\right)^t$$

$$\underline{\Psi}^{K,T} \equiv \left(\psi_A^{K,T,(0)}, \psi_A^{K,T,(1)}, \psi_B^{K,T,(0)}, \psi_B^{K,T,(1)}\right)^t$$

$$\underline{\Psi}^{K,n_\alpha} \equiv \left(\psi_A^{K,n_\alpha,(0)}, \psi_A^{K,n_\alpha,(1)}, \psi_B^{K,n_\alpha,(0)}, \psi_B^{K,n_\alpha,(1)}\right)^t$$

Let the following entities (which are also coefficients in the Sonine polynomial expansion) be defined, for $\alpha, \beta \in \{A, B\}$, as:

$$\psi_\alpha^{\varepsilon_{\alpha\alpha},(1)} \equiv \frac{\sqrt{2}}{8}\frac{n_A n_B}{n^2}\frac{\sigma_\alpha^2}{\sigma_{AB}^2}\frac{1}{M_\alpha\sqrt{M_\beta}} \tag{8}$$

$$\psi_\alpha^{\varepsilon_{AB},(1)} \equiv \frac{1}{4}\left(\frac{n_A n_B}{n^2}\frac{1}{M_\beta} - \left(\frac{n_\beta}{n}\right)^2\frac{1}{M_\alpha}\right) \tag{9}$$

and for $\alpha \neq \beta$

$$\psi_\alpha^{\varepsilon_{\beta\beta},(1)} \equiv \frac{\sqrt{2}}{8}\left(\frac{n_\beta}{n}\right)^2\frac{\sigma_\beta^2}{\sigma_{AB}^2}\frac{1}{M_\beta\sqrt{M_\alpha}} \tag{10}$$

II. Definitions of certain matrices

In this subsection a set of 4×4 matrices needed for the calculation of the transport coefficients, namely the matrices $\underline{\underline{\mathcal{R}}}^V_{\alpha\beta}$, $\underline{\underline{\mathcal{R}}}^T_{\alpha\beta}$, $\underline{\underline{\mathcal{K}}}_{\alpha\beta}$, $\underline{\underline{\mathcal{R}}}^{1,n_A}_{\alpha\beta}$, $\underline{\underline{\mathcal{R}}}^{1,n_B}_{\alpha\beta}$, $\underline{\underline{\mathcal{R}}}^{2,n_A}_{\alpha\beta}$ and $\underline{\underline{\mathcal{R}}}^{2,n_B}_{\alpha\beta}$, are defined by their nonzero matrix elements. Notice that the subscripts do not denote matrix elements but rather different matrices. In the expressions for the matrix elements, an overhat denotes, as before, an exchange $A \leftrightarrow B$, $\tilde{\delta}_{AB} \equiv \delta_{\alpha A}\delta_{\beta B} + \delta_{\alpha B}\delta_{\beta A}$; $\tilde{\delta}_{\nu\nu} = \delta_{\alpha\nu}\delta_{\beta\nu}$ where $\delta_{\alpha\beta}$ is the Kronecker delta, the coefficients $\psi_A^{\varepsilon_{\alpha\beta},(1)}$ are given in Eqs. (8- 10), and the rescaled energy sink terms $\widetilde{\Gamma^{\varepsilon_{\alpha\beta}}}$ are given by:

$$\begin{aligned} \widetilde{\Gamma^{\varepsilon_{\nu\nu}}} &= \frac{4\sqrt{3\pi}}{9}\frac{n_\nu^2}{n^2}\frac{\sigma_\nu^2}{\sigma_{AB}^2}\frac{1}{\sqrt{M_\nu}} \\ \widetilde{\Gamma^{\varepsilon_{AB}}} &= \frac{4\sqrt{6\pi}}{9}\frac{n_A n_B}{n^2}\frac{1}{\sqrt{M_A M_B}} \end{aligned} \tag{11}$$

The nonzero elements of the matrices $\underline{\underline{\mathcal{R}}}^V_{\alpha\beta}$ are:

$$R^V_{\alpha\beta,11} = -\frac{15}{8}\frac{\sqrt{6}}{\sqrt{\pi}}\frac{n}{n_B}\frac{\widetilde{\Gamma^{\varepsilon_{\alpha\beta}}}}{\sqrt{M_B M_A}} - \left(10\tilde{\delta}_{AB} + \left(\frac{3\sqrt{2}\left(\frac{\sigma_A}{\sigma_{AB}}\right)^2}{M_A\sqrt{M_B}} - 4\left(4M_A+1\right)\right)\frac{n_A}{n_B}\psi_A^{\varepsilon_{\alpha\beta},(1)}\right)$$

$$R^V_{\alpha\beta,12} = 7M_B\tilde{\delta}_{AB} - \left(\frac{3\sqrt{2}\left(\frac{\sigma_A}{\sigma_{AB}}\right)^2}{4M_A\sqrt{M_B}} - 2M_B\left(6M_A+1\right)\right)\frac{n_A}{n_B}\psi_A^{\varepsilon_{\alpha\beta},(1)}$$

$$R^V_{\alpha\beta,13} = 10\tilde{\delta}_{AB} - 8\left(2M_A - 1\right)\psi_A^{\varepsilon_{\alpha\beta},(1)}$$

$$R^V_{\alpha\beta,14} = -7M_A\tilde{\delta}_{AB} - 4\left(3M_A-2\right)M_A\psi_A^{\varepsilon_{\alpha\beta},(1)}$$

$$\begin{aligned} R^V_{\alpha\beta,21} = &\frac{105}{8}\frac{\sqrt{6}}{\sqrt{\pi}}\frac{n}{n_B}\frac{\widetilde{\Gamma^{\varepsilon_{\alpha\beta}}}}{\sqrt{M_B M_A}} - \frac{\sqrt{2}}{4}\left(35\tilde{\delta}_{AA} + 19\psi_A^{\varepsilon_{\alpha\beta},(1)}\right)\frac{n_A}{n_B}\frac{\left(\frac{\sigma_A}{\sigma_{AB}}\right)^2}{M_A\sqrt{M_B}} \\ &- 35\left(M_A - M_B\right)\tilde{\delta}_{AB} + 2\left(120M_A^2 - 51M_A + 1\right)\frac{n_A}{n_B}\psi_A^{\varepsilon_{\alpha\beta},(1)} \end{aligned}$$

$$\begin{aligned} R^V_{\alpha\beta,22} = &-\frac{315}{16}\frac{\sqrt{6}}{\sqrt{\pi}}\frac{n}{n_B}\frac{\widetilde{\Gamma^{\varepsilon_{\alpha\beta}}}}{\sqrt{M_B M_A}} + \left(245\tilde{\delta}_{AA} - 89\psi_A^{\varepsilon_{\alpha\beta},(1)}\right)\frac{\sqrt{2}\left(\frac{\sigma_A}{\sigma_{AB}}\right)^2}{16M_A\sqrt{M_B}}\frac{n_A}{n_B} \\ &- \tilde{\delta}_{AB}\frac{35}{2}\left(18M_A^2 - 21M_A + 7\right) + \left(420M_A^3 - 435M_A^2 + 142M_A + 13\right)\frac{n_A}{n_B}\psi_A^{\varepsilon_{\alpha\beta},(1)} \end{aligned}$$

$$\begin{aligned} R^V_{\alpha\beta,23} &= -70M_B\tilde{\delta}_{AB} - 8\left(30M_A^2 - 33M_A + 10\right) \\ R^V_{\alpha\beta,24} &= \left(315M_A M_B\tilde{\delta}_{AB} + 4M_A\left(105M_A^2 - 135M_A + 44\right)\psi_A^{\varepsilon_{\alpha\beta},(1)}\right) \end{aligned}$$

$$R^V_{\alpha\beta,31} = \widehat{R^V_{\alpha\beta,13}}$$

$$R^V_{\alpha\beta,32} = \widehat{R^V_{\alpha\beta,14}}$$

$$R^V_{\alpha\beta,33} = \widehat{R^V_{\alpha\beta,11}}$$

$$R^V_{\alpha\beta,34} = \widehat{R^V_{\alpha\beta,12}}$$

$$R^V_{\alpha\beta,41} = \widehat{R^V_{\alpha\beta,23}}$$

$$R^V_{\alpha\beta,42} = \widehat{R^V_{\alpha\beta,24}}$$

$$R^V_{\alpha\beta,43} = \widehat{R^V_{\alpha\beta,21}}$$

$$R^V_{\alpha\beta,44} = \widehat{R^V_{\alpha\beta,22}}$$

The nonzero elements of the matrices $\underline{\underline{\mathcal{R}}}^T_{\alpha\beta}$ are given by:

$$R^T_{\alpha\beta,11} = -\frac{3}{2}\frac{n}{n_B}$$

$$R^T_{\alpha\beta,21} = \frac{15}{4}\frac{n}{n_B}$$

$$R^T_{\alpha\beta,22} = -\frac{15}{2}\frac{n}{n_B}$$

$$R^T_{\alpha\beta,33} = -\frac{3}{2}\frac{n}{n_A}$$

$$R^T_{\alpha\beta,43} = \frac{15}{4}\frac{n}{n_A}$$

$$R^T_{\alpha\beta,44} = -\frac{15}{2}\frac{n}{n_A}$$

The nonzero elements of the matrices $\underline{\underline{\mathcal{K}}}_{\alpha\beta}$ are:

$$K_{\alpha\beta,11} = \tilde{\delta}_{AB}\frac{2}{M_A}\left(1+\frac{\rho_A}{\rho_B}\right)$$

$$K_{\alpha\beta,12} = -2M_B\frac{n_A}{n_B}\psi_A^{\varepsilon_{\alpha\beta},(1)} - \tilde{\delta}_{AB}\frac{M_B}{M_A}$$

$$K_{\alpha\beta,14} = -2\sqrt{M_A M_B}\psi_A^{\varepsilon_{\alpha\beta},(1)} + \tilde{\delta}_{AB}\sqrt{\frac{M_A}{M_B}}$$

$$K_{\alpha\beta,21} = -4\left(\frac{2M_A+3}{M_B} - \frac{2\sqrt{2}\left(\frac{\sigma_A}{\sigma_{AB}}\right)^2}{M_A\sqrt{M_B}}\right)\frac{n_A}{n_B}\psi_A^{\varepsilon_{\alpha\beta},(1)} + \tilde{\delta}_{AB}\left(15-\frac{5}{M_A}-15\frac{n_A}{n_B}\right)$$

$$+\tilde{\delta}_{AA}\frac{5\sqrt{2}}{M_A\sqrt{M_B}}\frac{n_A}{n_B}\left(\frac{\sigma_A}{\sigma_{AB}}\right)^2$$

$$K_{\alpha\beta,22}=-\left(75M_A^2-62M_A+17\right)\frac{n_A}{n_B}\psi_A^{\varepsilon_{\alpha\beta},(1)}+\tilde{\delta}_{AB}\left(\frac{105}{2}M_A-50+\frac{25}{2M_A}\right)$$

$$-\tilde{\delta}_{AA}\frac{25\sqrt{2}}{4M_A\sqrt{M_B}}\frac{n_A}{n_B}\left(\frac{\sigma_A}{\sigma_{AB}}\right)^2$$

$$K_{\alpha\beta,24}=-\sqrt{\frac{M_A}{M_B}}\left(75M_A^2-102M_A+37\right)\psi_A^{\varepsilon_{\alpha\beta},(1)}-\frac{105}{2}\tilde{\delta}_{AB}\sqrt{M_AM_B}$$

$$K_{\alpha\beta,31}=\widehat{K_{\alpha\beta,13}}$$

$$K_{\alpha\beta,32}=\widehat{K_{\alpha\beta,14}}$$

$$K_{\alpha\beta,33}=\widehat{K_{\alpha\beta,11}}$$

$$K_{\alpha\beta,34}=\widehat{K_{\alpha\beta,12}}$$

$$K_{\alpha\beta,41}=\widehat{K_{\alpha\beta,23}}$$

$$K_{\alpha\beta,42}=\widehat{K_{\alpha\beta,24}}$$

$$K_{\alpha\beta,43}=\widehat{K_{\alpha\beta,21}}$$

$$K_{\alpha\beta,44}=\widehat{K_{\alpha\beta,22}}$$

The nonzero elements of the matrices $\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{1n_A}$ are

$$R_{\alpha\beta,11}^{1n_A}=-\frac{2}{5}\frac{n}{n_B}\widetilde{\delta_A\Gamma^{\varepsilon_{\alpha\beta}}}$$

$$R_{\alpha\beta,22}^{1n_A}=-\frac{n}{n_B}\widetilde{\delta_A\Gamma^{\varepsilon_{\alpha\beta}}}$$

$$R_{\alpha\beta,31}^{1n_A}=\frac{2}{5}\frac{n}{n_B}\sqrt{\frac{M_A}{M_B}}\widetilde{\delta_A\Gamma^{\varepsilon_{\alpha\beta}}}$$

$$R_{\alpha\beta,44}^{1n_A}=-\frac{n}{n_A}\widetilde{\delta_A\Gamma^{\varepsilon_{\alpha\beta}}}$$

where $\widetilde{\delta_\alpha\Gamma^{\varepsilon_{\alpha\alpha}}}=\left(2-\frac{n_\alpha}{n}\right)\widetilde{\Gamma^{\varepsilon_{\alpha\alpha}}}$, $\widetilde{\delta_\alpha\Gamma^{\varepsilon_{AB}}}=\frac{n_\beta}{n}\widetilde{\Gamma^{\varepsilon_{AB}}}$, and $\widetilde{\delta_\alpha\Gamma^{\varepsilon_{\beta\beta}}}=-\frac{n_\alpha}{n}\widetilde{\Gamma^{\varepsilon_{\beta\beta}}}$. The nonzero elements of the matrices $\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{1n_B}$ are given by:

$$R_{\alpha\beta,33}^{1n_B}=\widehat{R_{\alpha\beta,11}^{1n_A}}$$

$$R_{\alpha\beta,44}^{1n_B}=\widehat{R_{\alpha\beta,22}^{1n_A}}$$

$$R^{1n_B}_{\alpha\beta,13} = \widehat{R^{1n_A}_{\alpha\beta,31}}$$

$$R^{1n_B}_{\alpha\beta,22} = \widehat{R^{1n_A}_{\alpha\beta,44}}$$

The nonzero elements of the matrices $\underline{\underline{\mathcal{R}}}^{2n_A}_{\alpha\beta}$ are given by:

$$R^{2n_A}_{\alpha\beta,11} = -\frac{1}{5}\frac{n}{n_B}$$

$$R^{2n_A}_{\alpha\beta,21} = \frac{n}{n_B}$$

$$R^{2n_A}_{\alpha\beta,22} = -\frac{3}{2}\frac{n}{n_B}$$

$$R^{2n_A}_{\alpha\beta,31} = \frac{1}{5}\frac{n}{n_B}\sqrt{\frac{M_A}{M_B}}$$

$$R^{2n_A}_{\alpha\beta,41} = -\frac{n}{n_B}\sqrt{\frac{M_A}{M_B}}$$

$$R^{2n_A}_{\alpha\beta,44} = -\frac{3}{2}\frac{n}{n_A}$$

Finally, the nonzero matrix elements of the matrices $\underline{\underline{\mathcal{R}}}^{2n_B}_{\alpha\beta}$ are:

$$R^{2n_B}_{\alpha\beta,33} = \widehat{R^{2n_A}_{\alpha\beta,11}}$$

$$R^{2n_B}_{\alpha\beta,43} = \widehat{R^{2n_A}_{\alpha\beta,21}}$$

$$R^{2n_B}_{\alpha\beta,44} = \widehat{R^{2n_A}_{\alpha\beta,22}}$$

$$R^{2n_B}_{\alpha\beta,13} = \widehat{R^{2n_A}_{\alpha\beta,31}}$$

$$R^{2n_B}_{\alpha\beta,23} = \widehat{R^{2n_A}_{\alpha\beta,41}}$$

$$R^{2n_B}_{\alpha\beta,22} = \widehat{R^{2n_A}_{\alpha\beta,44}}$$

III. Transport coefficients

III-1. Shear viscosity

The shear viscosity, μ, is expanded to linear order in the degrees of inelasticity, as follows:

$$\mu = \mu^{(0)} + \sum_{\alpha,\beta\in\{A,B\}} \varepsilon_{\alpha\beta}\mu^{\varepsilon_{\alpha\beta}}$$

where $\mu^{(0)}$ is given by:

$$\mu^{(0)} = -\frac{\sqrt{Tm_0}}{6\sigma_{AB}^2}\left(\frac{n_A}{n}\psi_A^{K,V,(0)} + \frac{n_B}{n}\psi_B^{K,V,(0)}\right)$$

and $\mu^{\varepsilon_{\alpha\beta}}$, for $\{\alpha\beta\} \in \{AA, AB, BB\}$, reads:

$$\mu^{\varepsilon_{\alpha\beta}} = -\frac{\sqrt{Tm_0}}{30}\frac{n_A n_B}{n^2\sigma_{AB}^2}\left[\frac{\sqrt{6\pi}}{9}\sqrt{M_B M_A}\underline{\Psi}^{K,V}\cdot\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{V}\cdot\underline{\Psi}^{K,V} + 5\sum_{\nu,\eta\,\in\,\{A,B\};\,\nu\neq\eta}\frac{n}{n_\eta}\left(\frac{7}{2}\psi_\nu^{K,V,(1)} - \psi_\nu^{K,V,(0)}\right)\psi_\nu^{\varepsilon_{\alpha\beta},(1)}\right]$$

where the vector $\underline{\Psi}^{K,V}$ is given by Eq. (5), the matrix $\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{V}$ is given in section II of this appendix, the coefficients $\psi_\nu^{\varepsilon_{\alpha\beta},(1)}$ are given by Eqs. (8- 10), and the sink terms $\widetilde{\Gamma^{\varepsilon_{\alpha\beta}}}$ are given by Eq. (11).

III-2. Diffusion coefficients

Similarly, the diffusion coefficients in Eq. (4.17) of the article are expressed as follows.

$$\kappa_A^X = \kappa_A^{X,(0)} + \sum_{\alpha\beta}\varepsilon_{\alpha\beta}\kappa_A^{X,\varepsilon_{\alpha\beta}}$$

for $X \in \{T, n_A, n_B\}$, with $\kappa_A^{X,(0)} = \psi_A^{K,X,(0)}$, and $\kappa_A^{\varepsilon_{\alpha\beta},T}$ and $\kappa_A^{\varepsilon_{\alpha\beta},n_\gamma}$, for $\gamma \in \{A, B\}$ given by:

$$\kappa_A^{\varepsilon_{\alpha\beta},T} = \frac{\sqrt{6}}{6}\frac{n_B}{n}\sqrt{M_A}\left[\frac{\sqrt{6}}{2}\sum_{\nu,\eta\,\in\,\{A,B\};\,\nu\neq\eta}\frac{n\rho}{n_\eta\rho_\eta\sqrt{M_\nu}}\left(-\psi_\nu^{K,n_A,(0)} + \frac{5}{2}\left(3 - \frac{n\rho_\nu}{n_\nu\rho}\right)\psi_\nu^{K,n_A,(1)}\right)\right.$$
$$\left.\times\psi_\nu^{\varepsilon_{\alpha\beta},(1)} + \widetilde{\underline{\Psi}^{K,n_A}}\cdot\left(\widetilde{\Gamma^{\varepsilon_{\alpha\beta}}}\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{T} - \frac{\sqrt{6\pi}}{6}\sqrt{M_A M_B}\underline{\underline{\mathcal{K}}}_{\alpha\beta}\right)\cdot\underline{\Psi}^{K,T}\right]$$

$$\kappa_A^{\varepsilon_{\alpha\beta},n_\gamma} = \frac{\sqrt{6}}{6}\frac{n_B}{n}\sqrt{M_A}\left\{\frac{\sqrt{6}}{2}\sum_{\nu,\eta=A,B;\nu\neq\eta}\frac{\rho}{\rho_\eta\sqrt{M_\nu}}\frac{n}{n_\eta}(2\delta_{\gamma\nu} - 1)\left[-\psi_\nu^{K,n_A,(0)}(\delta_{\gamma\nu} + \partial_\nu)\right.\right.$$
$$\left. + \frac{5}{2}\psi_\nu^{K,n_A,(1)}\left(\partial_\nu + \delta_{\gamma\nu}\frac{\rho_\eta}{\rho}\left(1 - \frac{n_\eta\rho_\nu}{n_\nu\rho}\right) + \frac{\rho_\nu n_\eta}{\rho n_\nu}\right)\right]\psi_\nu^{\varepsilon_{\alpha\beta},(1)}$$
$$\left. + \frac{15}{4}\widetilde{\underline{\Psi}^{K,n_A}}\cdot\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{1n_\gamma}\cdot\underline{\Psi}^{K,T} + \widetilde{\underline{\Psi}^{K,n_A}}\cdot\left(\frac{15\widetilde{\Gamma^{\varepsilon_{\alpha\beta}}}}{4}\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{2n_\gamma} - \frac{\sqrt{6\pi}}{6}\sqrt{M_A M_B}\,\underline{\underline{\mathcal{K}}}_{\alpha\beta}\right)\cdot\underline{\Psi}^{K,n_\gamma}\right\}$$

where $\partial_\nu \equiv n_\nu\frac{\partial}{\partial n_\nu}$, $\widetilde{\underline{\Psi}^{K,n_A}} \equiv \left(\frac{\rho}{\rho_B}\psi_A^{K,n_A,(0)}, \frac{\rho}{\rho_B}\psi_A^{K,n_A,(1)}, \frac{\rho}{\rho_A}\psi_B^{K,n_A,(0)}, \frac{\rho}{\rho_A}\psi_B^{K,n_A,(1)}\right)^t$, the vectors $\underline{\Psi}^{K,T}$ and $\underline{\Psi}^{K,n_\alpha}$ are given by Eqs. (6- 7), the matrices $\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{T}$, $\underline{\underline{\mathcal{K}}}_{\alpha\beta}$, $\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{1n_\gamma}$ and $\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{2n_\gamma}$ are given in section II of this appendix, the coefficients $\psi_\nu^{\varepsilon_{\alpha\beta},(1)}$ are given by Eqs. (8- 10), and the rescaled sink terms are given by Eq. (11).

III-3. Heat flux transport coefficients

The coefficients λ^T, λ^{n_A} and λ^{n_B} in the expression for the heat flux, Eq. (4.18) of the article, are expanded as follows:

$$\lambda^X = \lambda^{X,(0)} + \sum_{\alpha\beta}\varepsilon_{\alpha\beta}\lambda^{X,\varepsilon_{\alpha\beta}}$$

for $X \in \{T, n_A, n_B\}$, where

$$\lambda^{X,(0)} = \frac{n_A}{n\sqrt{M_A}}\left(\psi_A^{K,X,(0)} - \psi_A^{K,X,(1)}\right) + \frac{n_B}{n\sqrt{M_B}}\left(\psi_B^{K,X,(0)} - \psi_B^{K,X,(1)}\right),$$

and the coefficients $\lambda^{T,\varepsilon_{\alpha\beta}}$, $\lambda^{n_A,\varepsilon_{\alpha\beta}}$, and $\lambda^{n_B,\varepsilon_{\alpha\beta}}$ for $\{\alpha\beta\} \in \{AA, AB, BB\}$ read:

$$\lambda^{\varepsilon_{\alpha\beta},T} = \frac{2\sqrt{6}}{15}\frac{n_A n_B}{n^2}\left[\frac{\sqrt{6}}{2}\sum_{\nu,\eta=A,B;\nu\neq\eta}\frac{n}{n_\eta\sqrt{M_\nu}}\left(-\psi_\nu^{K,T,(0)} + \frac{5}{2}\left(3 - \frac{n\rho_\nu}{n_\nu\rho}\right)\psi_\nu^{K,T,(1)}\right)\right.$$

$$\left.\times\psi_\nu^{\varepsilon_{\alpha\beta},(1)} + \underline{\Psi}^{K,T}\cdot\left(\widetilde{\Gamma^{\varepsilon_{\alpha\beta}}}\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{T} - \frac{\sqrt{6\pi}}{6}\sqrt{M_A M_B}\underline{\underline{\mathcal{K}}}_{\alpha\beta}\right)\cdot\underline{\Psi}^{K,T}\right]$$

$$+\frac{3}{5}\left(1 - \frac{m_A}{m_B}\right)\frac{n_A}{n}\frac{1}{\sqrt{M_A}}\kappa_A^{\varepsilon_{\alpha\beta},T}$$

$$\lambda^{\varepsilon_{\alpha\beta},n_\gamma} = \frac{2\sqrt{6}}{15}\frac{n_A n_B}{n^2}\left\{\frac{\sqrt{6}}{2}\sum_{\nu,\eta\ \in\ \{A,B\}\ ;\nu\neq\eta}\frac{1}{\sqrt{M_\nu}}\frac{n}{n_\eta}(2\delta_{\gamma\nu} - 1)\left[-\psi_\nu^{K,T,(0)}(\delta_{\gamma\nu} + \partial_\nu)\right.\right.$$
$$\left.+\frac{5}{2}\psi_\nu^{K,T,(1)}\left(\partial_\nu + \delta_{\gamma\nu}\frac{\rho_\eta}{\rho}\left(1 - \frac{n_\eta\rho_\nu}{n_\nu\rho}\right) + \frac{\rho_\nu n_\eta}{\rho n_\nu}\right)\right]\psi_\nu^{\varepsilon_{\alpha\beta},(1)}$$
$$\left.+\frac{15}{4}\underline{\Psi}^{K,T}\cdot\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{1n_\gamma}\cdot\underline{\Psi}^{K,T} + \underline{\Psi}^{K,T}\cdot\left(\frac{15\widetilde{\Gamma^{\varepsilon_{\alpha\beta}}}}{4}\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{2n_\gamma} - \frac{\sqrt{6\pi}}{6}\sqrt{M_A M_B}\underline{\underline{\mathcal{K}}}_{\alpha\beta}\right)\cdot\underline{\Psi}^{K,n_\gamma}\right\}$$
$$+\frac{3}{5}\left(1 - \frac{m_A}{m_B}\right)\frac{n_A}{n}\frac{1}{\sqrt{M_A}}\kappa_A^{\varepsilon_{\alpha\beta},T}$$

where $\partial_\nu \equiv n_\nu\frac{\partial}{\partial n_\nu}$, the vectors $\underline{\Psi}^{K,T}$ and $\underline{\Psi}^{K,n_\alpha}$ are given by Eqs. (6-7), the matrices $\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{T}$, $\underline{\underline{\mathcal{K}}}_{\alpha\beta}$, $\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{1n_\gamma}$ and $\underline{\underline{\mathcal{R}}}_{\alpha\beta}^{2n_\gamma}$ are given in section II of this appendix, the coefficients $\psi_\nu^{\varepsilon_{\alpha\beta},(1)}$ are given by Eqs. (8-10), and the rescaled sink terms $\widetilde{\Gamma^{\varepsilon_{\alpha\beta}}}$ are given by Eq. (11).