Supplement to The large-scale flow structure in turbulent rotating Rayleigh-Bénard convection

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(Received 8 September 2011)

1. Prograde and retrograde rotation for larger 1/Ro

Figure 1 shows typical time traces of θ_k for the 1/Ro-range II (a and b) and III (c and d). As one sees in both cases (a and c) the traces for θ_b, θ_m and θ_t are more or less indistinguishable, indicating, that there is no differential rotation for $1/\mathrm{Ro} > 1/\mathrm{Ro}_0$. Instead, the traces show only very slight deviations from each other when one considers small time intervals such as that of Figs. 1b and d.

One also sees from these figures that the slope $\dot{\theta}_k$ changes from positive (prograde rotation) for 1/Ro=0.65 (range II) to negative (retrograde rotation) for 1/Ro=1.44 (range III).

2. The time correlation of sidewall-thermistor temperatures

In this section we present an analysis of the sidewall temperature measurements that avoids a fit of a harmonic function or a Fourier analysis and is based purely on correlation functions of the sidewall temperatures $T_{k,i}$ themselves.

One can get useful information about the dynamics of the temperature field at the side wall by calculating cross- and autocorrelation functions of the thermistor readings as a function of time $T_{i,k}(t)$. We define the cross-correlation function between vertically adjacent thermistors as

$$\tilde{C}^i_{k_1 k_2}(\tau) = \langle T_{i,k_1}(t) \cdot T_{i,k_2}(t+\tau) \rangle_t \,,$$

whereas the subscript "t" denotes that the average is taken over the whole duration time of an experiment, excluding only an initial transient period. We normalized these functions and averaged over all eight azimuthal positions according to

$$C_{k_1 k_2}(\tau) = \frac{1}{8} \sum_i \frac{\tilde{C}^i_{k_1 k_2}(\tau)}{\sqrt{\tilde{C}^i_{k_1 k_1}(0) \cdot \tilde{C}^i_{k_2 k_2}(0)}},$$

where, the index "i" stands for the azimuthal position of a certain thermistor.

Figure 2(a-d) shows $C_{k_1 k_2}$ for four different rotation rates. For no rotation (Fig. 2a), $C_{t,m}$ (blue dashed) and $C_{m,b}$ (red dotted) have both local maxima at $\tau = 15.5\,\mathrm{s}$ and $\tau = -15\,\mathrm{s}$. We believe that these peaks show the time interval that hot (cold) plumes need to travel from the bottom (top) thermistor row to the middle thermistor row. The green solid line (C_{t_b}) shows two peaks ($\tau = -100\,\mathrm{s}$ and $\tau = 80\,\mathrm{s}$) and a dip at $\tau = 0$. These two peaks have their origin in the twisting mode of the LSC as described extensivly for $\Gamma = 1$ (see e.g. Funfschilling & Ahlers (2004); Brown & Ahlers (2009); Xi *et al.* (2009) and recently found also in $\Gamma = 0.50$ by Weiss & Ahlers (2011).

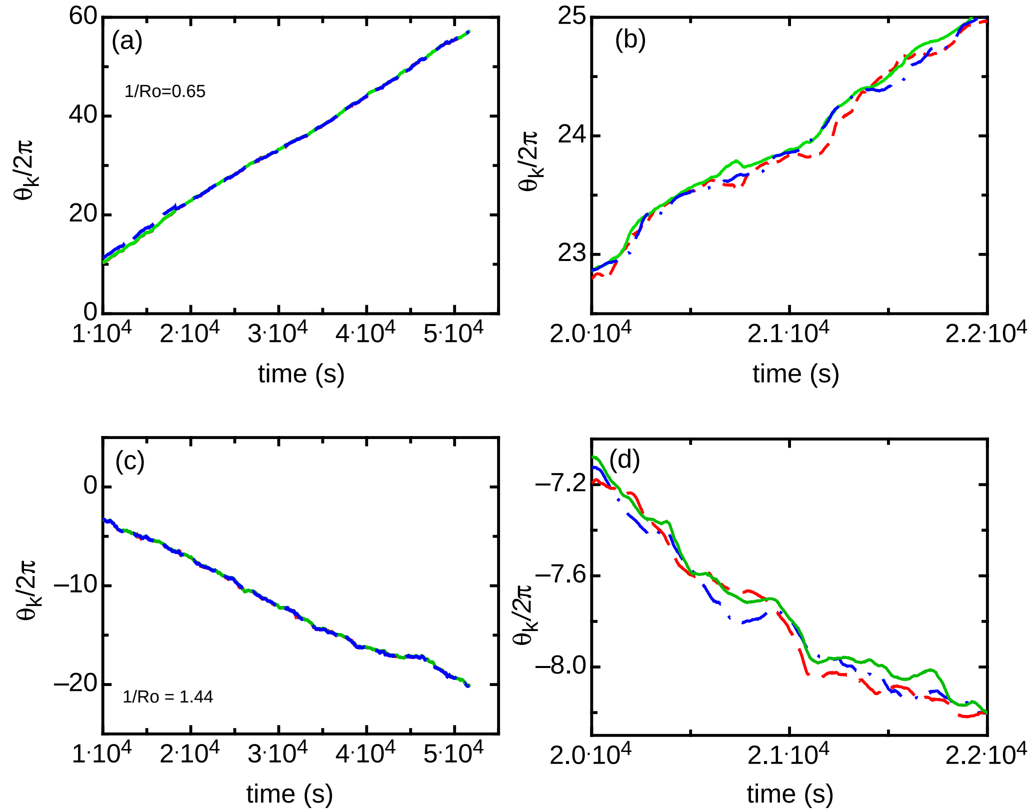

FIGURE 1. The azimuthal orientations θ_k as a function of time for $1/\mathrm{Ro} = 0.65$ (a and b) and $1/\mathrm{Ro} = 1.44$ (c and d). Shown are measurements for the top (dash-dot, blue), the middle (solid, green) and the bottom (dashed, red) thermistor row. While (a) and (c) show a large time range, (b) and (d) show only a section of 2000 s. These experiments were for $\mathrm{Ra} = 1.8 \times 10^{10}$.

If one rotates the container around its cylinder axis, one can see that the corresponding $C_{k_1 k_2}(\tau)$ changes as a function of the rotation rate ($1/\mathrm{Ro}$) in several ways. First, the correlation function shows a periodicity which changes as a function of $1/\mathrm{Ro}$. This periodicity can easily be explained due to the rotation of the LSC with the angular velocity ω as described in Sec. 5.6 of the main document. The periodicity should therefore be similar to $2\pi/\omega$.

The second interesting parameter, that can be read from the correlation functions, is the location of the maxima τ_{max} of $C_{m,b}$ and $C_{t,m}$. It changes for different values of $1/\mathrm{Ro}$. Figures 2e and f show this change for $C_{m,b}$ (red bullets) and $C_{t,m}$ (blue solid squares). For very small rotation rates τ_{max} changes signs and becomes negative (positive) for $C_{t,m}$ ($C_{m,b}$). The location of τ_{max} changes further and reaches a minimum (maximum) at around $1/\mathrm{Ro} \approx 1/\mathrm{Ro}_c$, before it increases (decreases) again. It again changes the sign, reaches a maximum (minimum) and goes than asymptotically to zero for large $1/\mathrm{Ro}$.

This astonishing behaviour is another manifestation of the bend of the LSC. Let us assume a bend of the LSC as shown in Fig. 21 of the main paper, and consider the phase difference between the middle and bottom thermistor row $\Delta\theta_{m,b}$. In addition, the LSC rotates by an angular velocity ω_m as shown in Fig. 13 of the main paper. Out of these assumptions we would expect peaks in the correlation functions $C_{m,b}(\tau)$ to occur at

$$\tilde{\tau}_{max} = \Delta\theta_{m,b}/\omega_m. \tag{2.1}$$

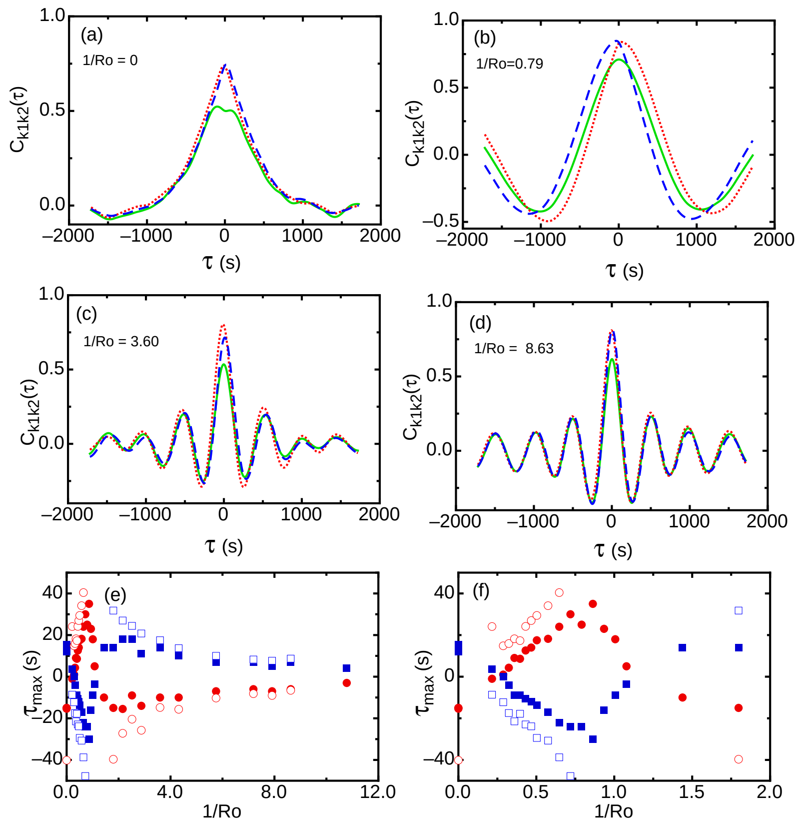

FIGURE 2. Correlation of the thermistors $C_{k1,k2}(\tau)$ of different rows (k_1 and k_2). (a-d) Correlation functions for different 1/Ro. The solid (green online), dotted (red online) and dashed (blue online) lines stand for $C_{t,b}$, $C_{m,b}$, and $C_{t,m}$. (e) and (f) show the location of the maximum in the correlation for $C_{m,b}$ (bullets, red online) and $C_{t,m}$ (solid squares, blue online) as well as for comparison the alternative calculated values of $\tilde{\tau}_{max}$, based on Eq. 2.1. (f) shows the same data as (e) but focuses on a smaller 1/Ro range. All data were taken at $\mathrm{Ra} = 1.8 \times 10^{10}$.

For the calculation we choose ω_m since it is not affected by half-turns and will therefore represent the real rotation rate. We do an analogous calculation for $C_{t,m}(\tau)$ where we need to take into consideration that $\Delta\theta_{t,m} = -\Delta\theta_{m,t}$. We calculated $\tilde{\tau}_{max}$ for both cases and plotted the corresponding values for comparison in the same plots in Fig. 2e and f as open symbols. As can be seen, there is a good agreement between τ_{max} and $\tilde{\tau}_{max}$, at least where the absolute value of τ_{max} is rather small. In the range $0.7 < 1/\mathrm{Ro} < 1.7$, the rotation rate ω is very small. This leads to very large $\tilde{\tau}_{max}$ in this range and a discontinuity at around $1/\mathrm{Ro} \approx 1.1$ where $\omega_m \approx 0$. A small error in ω is therefore strongly amplified. In general $\tilde{\tau}_{max}$ is larger than τ_{max}, which might be due to half turns that increase the average phase difference $\langle \Delta\theta_{m,k} \rangle$.

We see, that by plotting the correlation function as done in Fig. 2, the bend of the LSC and the rotation of the LSC are shown in another alternative way and therefore confirm results gained by the method used in the main document.

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