

Magnetic field driven micro-convection in the Hele-Shaw cell: Brinkman model and its comparison with experiment

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Supplementary Material

1 Movies

Three movies are added to the supplementary material. First two show numerical simulation results for $t_0 = 0.005$, $Ra_m = 318$. The field of view is 6×5 dimensionless units large and the movie is $t = 0.09$ long. The third movie is the concentration dynamics calculated from the experimental images at $H = 138$ Oe with a 0.36×0.30 mm large field of view. It shows data from the first $t = 0.76$ s and is slowed down ten times for viewing purposes. The magnetic field strength, field of view, length of movies and playing speed are chosen so that dimensional and dimensionless values agree via the introduced scales in § 2 of the paper.

- **Movie1** : Numerical simulation result of the Brinkman model;
- **Movie2** : Numerical simulation result of the Darcy model;
- **Movie3** : Experimental observation.

2 Algorithm for finding characteristic Fourier coefficients

The algorithm is explained graphically in figure SM.1. At first, a fixed line l is defined. For numerical simulations it is a straight line across L_y fixed at x_0 , which is the defined initial concentration interface. In the case of experimental data it is manually defined for each experiment, marking the initial fluid interface (a). If necessary, a segmented line is used, allowing to remove the interface formation effects. Then the concentration information is retrieved from concentration plot on this line l at each time moment t_i ((b)-an example at $t = 3.0$ s, $H = 52$ Oe). This gives a concentration profile $c(l)$ for each t_i .

Fourier coefficients $|\hat{c}(k)|$ of the concentration profile are found with a fast Fourier transform. They are normalized with respect to the maximal value $\max(|\hat{c}(k)|)$ at each t_i . Only Fourier coefficients in the k range $[0; 18]$ for numerical simulations and $[0; 150]$ mm^{-1} for experiments are used (d), as they correspond to realistic finger wavenumber values. Repeating this for each time t_i gives an array of Fourier coefficients $|\hat{c}(k, t)|$, that can be displayed with a color coded grayscale contourplot (as in figure 6 in the paper and figure SM.2).

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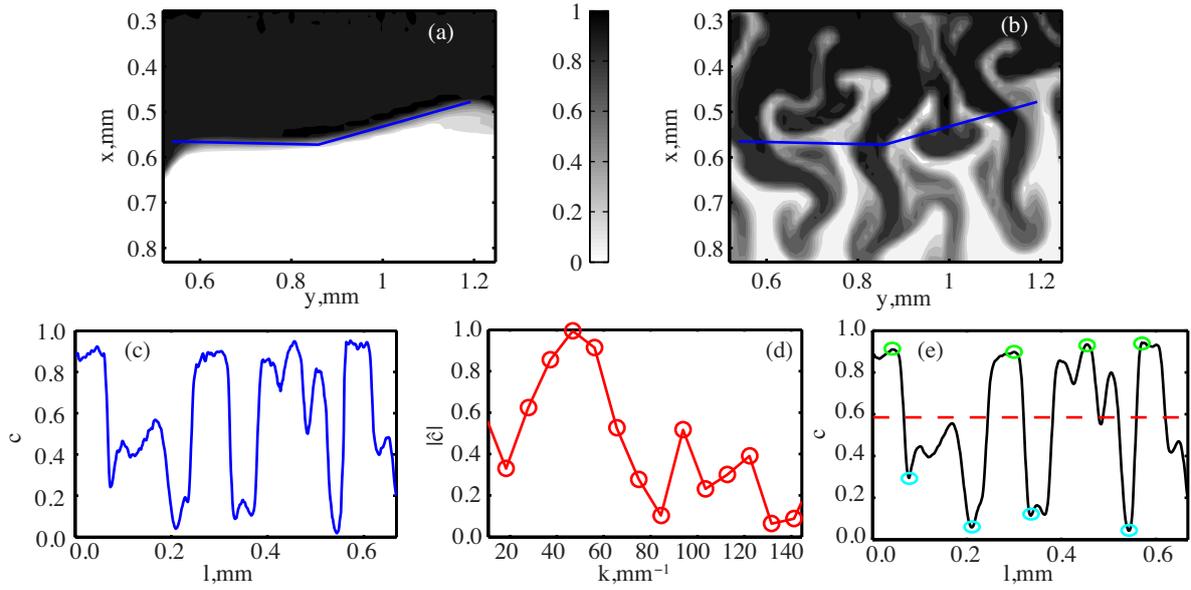


Figure SM.1: Steps for finding the characteristic Fourier coefficients: (a) define a fixed line l (blue), (b) find l (blue) for each t , (c) retrieve concentration profile $c(l)$, (d) calculate normalized Fourier coefficients $|\hat{c}(k)|$. In (e) the characteristic width estimation by the peak finding algorithm is showed. The concentration profile (from (c)) for experimental data is first smoothed (black line). Peaks (maxima - green circles, minima - light blue) are found using two criteria, described in § 3. Mean concentration value is represented with a red dashed line.

3 Algorithm for characteristic wavelength determination using peak finding algorithm

The algorithm is explained in (e) of figure SM.1. For experimental data, the concentration profile is first smoothed to reduce noise. Then a MATLAB built-in function *findpeaks* is used to find peaks. Two peak selection criteria are used. First defines minimum peak height H_p . It must be greater than the difference between mean (\bar{c}) and minimum ($\min(c)$) concentration. For experimental data it also has to be greater than the average concentration noise $\Delta(c_n)$ estimated from the image noise levels, stopping peak finding in low contrast data. Second, the minimum distance between two consecutive peaks ΔH_p is greater than an arbitrary fixed value. For numerical simulations it is fixed at 0.1, which allows to detect the narrow fingers in the Darcy model. For experimental data it is $60 \mu\text{m}$, which is half of the expected width.

By applying these criteria, one finds the maximum and minimum peak positions l_i^{pmax} and l_j^{pmin} . The characteristic wavelength is calculated as the average value of the differences between consecutive maximum and minimum peaks $\lambda^p = \frac{(l_{i+1}^{pmax} - l_i^{pmax}), (l_{j+1}^{pmin} - l_j^{pmin})}{2}$ and its error $\Delta\lambda$ is taken as the standard deviation of these differences. Afterwards, wavenumber is calculated with $k^p = \frac{2\pi}{\lambda^p}$. Repeating this step for each time t_i gives a vector of the average wavenumbers $k^p(t)$ that can be plotted above the contourplots (as in figure 6 in the paper and figure SM.2).

4 Fingering pattern dynamics at various magnetic fields

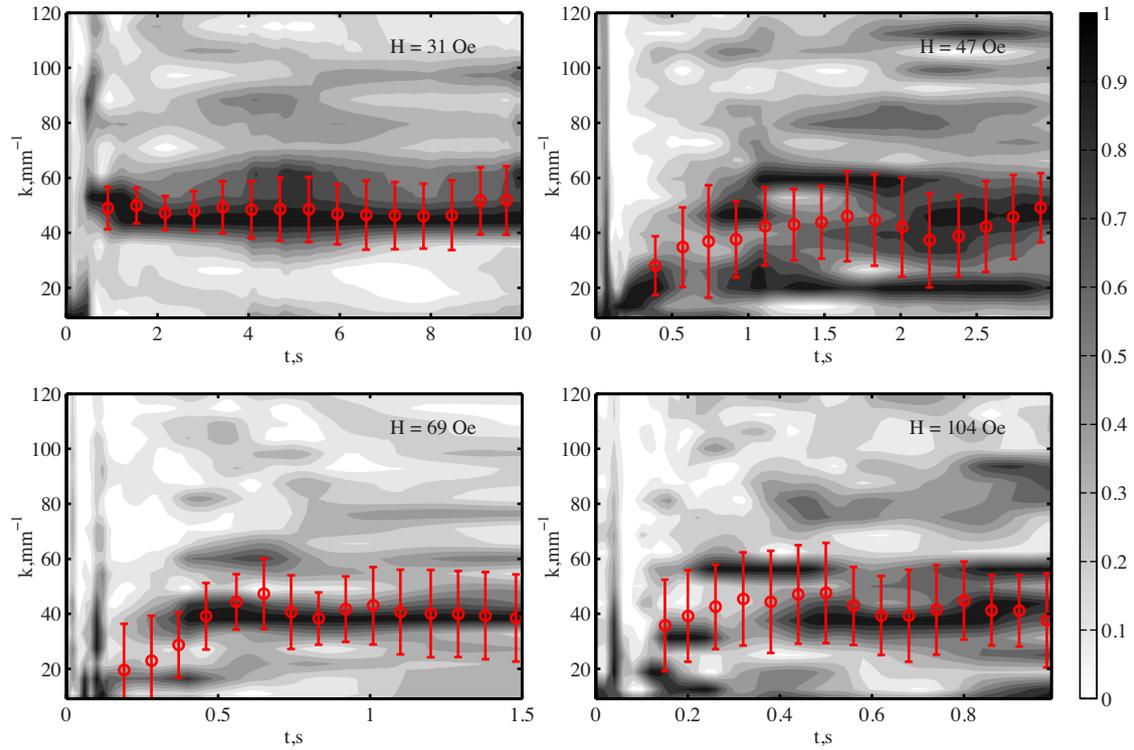


Figure SM.2: Primary finger pattern dynamics for $H = 31, 47, 69$ and 104 Oe. Grayscale contourplot shows dynamics of normalized Fourier coefficients (colorbar on the right). Red circles and error-bars mark the wavenumber values found with the peak finding algorithm.

5 Algorithm for determining finger velocity from experimental images

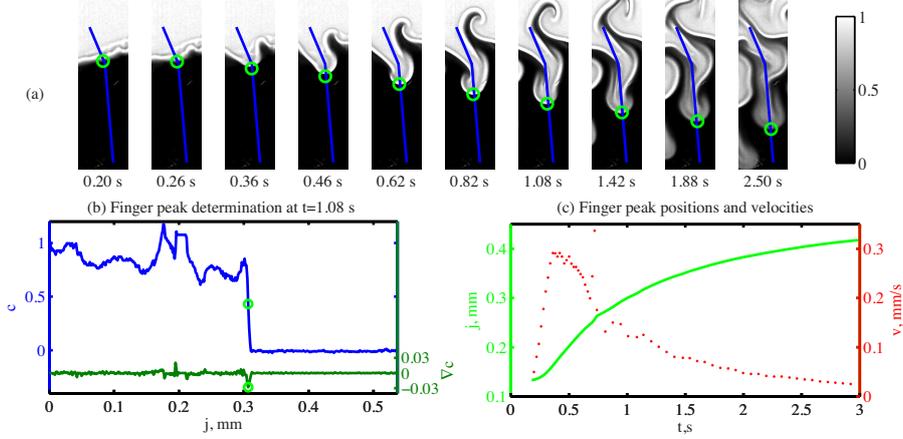


Figure SM.3: Explanation of the algorithm used for finding finger velocity (an example for $H = 69$ Oe.) (a) Concentration plots show finger growth on the manually chosen fixed j line (blue) and the finger peak locations (green circle) over time. (b) The concentration profile (blue) at a time t (here $t = 1.08$ s) is used to calculate the concentration gradient (dark green). The location of its negative maximum marks the finger peak position (green circles). (c) Velocities (red dots) are calculated from the peak positions (green line).

Finger velocities of the instability are found with the following algorithm (for graphic explanation, see figure SM.3). At first, a line j is manually chosen by a visual inspection of the image series, so that it approximates the finger peak trajectory with line segments. For each time moment t_i , the concentration profile along this line j is extracted. Then the concentration gradient is calculated from the concentration profile with the central finite difference method. The finger peak position is set at the the negative maximum of the concentration gradient. Performing this step for each image gives a finger peak position on the line j for each time t_i . As a result, the finger peak position time series $j(t_i)$ is obtained. After a smoothing step that reduces noise, the velocities $v(t)$ are calculated from the position data with $v(t_i) = \frac{j(t_{i+1}) - j(t_{i-1}))}{t_{i+1} - t_{i-1}}$.