Supplementary material

December 5, 2015

Effect of wing center of mass offset on passive pitching

In equation (2.4) and (2.10) we ignore the center of mass offset with respect to the rotational axis. Specifically, accordingly to appendix A, the y-offset to the pitching axis is $0.52mm$. In the experiments and simulations associated with equation (2.4) and equation (2.10), the mean wing chord length is $4mm$. Since the offset is 13% the mean wing chord length, it is relevant to quantify the effect of this offset on the passive pitching dynamics. Here we quantify this effect on equation (2.4) and (2.10).

Equation of motion derivation

Here we consider the effect of center of mass offset on passive pitching. As shown in figure 1a, the wing flapping motion can be decomposed into wing stroke motion $\phi(t)$ and passive pitching motion $\psi(t)$. In the 2D approximation, the wing stroke motion is projected to planar motion. For a particular wing chord located at a distance r from the wing root, the displacement in the 2D plane is approximated as $X(t) = r\phi_{max}\cos(wt)$, where ϕ_{max} is the stroke amplitude and w is the driving angular frequency. We derive the equation of motion for this 2D approximation and quantify the inertial term due to the center of mass offset. An illustration of the set up is shown in figure 1b.

We can derive the equation of motion using the Lagrangian approach. In figure 1b the center of mass of the physical pendulum is marked in red and the rotation axis is green. We assume the horizontal motion X(t) is prescribed and aim to solve for the rotational motion $\psi(t)$. We let l denote the length to the center of mass, m the pendulum mass and I the moment of inertia with respect to the center of mass. The center of mass position is given by:

$$\begin{aligned} x &= l\sin\psi + X \\ y &= -l\cos\psi \end{aligned} \tag{1}$$

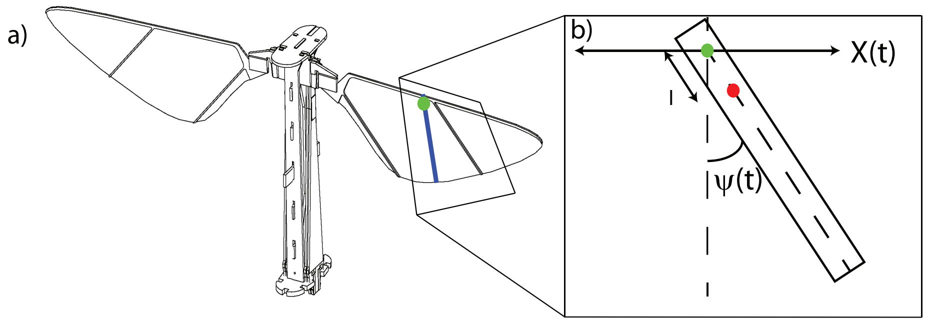

Figure 1: Setup of a horizontally driven physical pendulum

The velocity vector is given by:

$$\begin{aligned} \dot{x} &= l\dot{\psi}\cos\psi + \dot{X} \\ \dot{y} &= l\dot{\psi}\sin\psi \end{aligned} \tag{2}$$

The kinetic energy of the system is given by

$$K = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2. \tag{3}$$

From (2) we find the velocity is given by

$$v^2 = \dot{x}^2 + \dot{y}^2 = l^2\dot{\psi}^2 + \dot{X}^2 + 2l\dot{\psi}\dot{X}\cos\psi \tag{4}$$

The angular velocity w is given by $w = \dot{\psi}$. Consequently, the kinetic energy is given by

$$K = \frac{1}{2}m(l^2\dot{\psi}^2 + \dot{X}^2 + 2l\dot{\psi}\dot{X}\cos\psi) + \frac{1}{2}I\dot{\psi}^2 \tag{5}$$

The potential energy is given by

$$U = -mgl\cos\psi \tag{6}$$

We then obtain the equation of motion from taking the derivative of the Lagrangian:

$$\begin{aligned} \tfrac{\partial L}{\partial \psi} &= -\sin\psi ml\dot{\psi}\dot{X} - mgl\sin\psi \\ \tfrac{d}{dt}\tfrac{\partial L}{\partial \dot{\psi}} &= ml^2\ddot{\psi} + ml\ddot{X}\cos\psi - ml\sin\psi\dot{X}\dot{\psi} + I\ddot{\psi} \end{aligned} \tag{7}$$

Substituting these terms into the equation $\frac{d}{dt}\frac{\partial L}{\partial \dot{\psi}} - \frac{\partial L}{\partial \psi} = \tau_\psi$ we obtain the equation of motion:

$$k\psi + I\ddot{\psi} + ml^2\ddot{\psi} + ml\ddot{X}\cos\psi + mgl\sin\psi = 0 \tag{8}$$

Here $k\psi$ is the spring torque, $(I + ml^2)$ is the effective moment of inertia. Our paper lacks the last 2 terms and here we discuss why they are neglected.

Firstly, our system is driven at $120Hz$ so the contribution of gravity is small. Specifically, the moment arm l is small. Formally we have

$$Iw^2\psi_{max} \gg mgl \tag{9}$$

The term $ml\ddot{X}\cos\psi$ represents the effect of center of mass offset on passive pitching. This term is proportional to the stroke acceleration.

Influence on equation (2.4)

Equation (2.4) does not consider the wing center of mass effect. Although $ml\ddot{X}\cos\psi$ is not included in eqn (2.4), its contribution is 0 at wing midstroke based on the assumption that $\delta = 0°$. Based on our kinematic assumption, $\psi(t)$ and $X(t)$ are purely sinusoidal and the relative phase shift $\delta = 0°$. This assumption is valid in most flapping experiments studied in the paper. Consequently, $\psi(t)$ and $X(t)$ take the form of :

$$\begin{aligned} X(t) &= X_{max}\cos(wt) \\ \psi(t) &= \psi_{max}\sin(wt) \end{aligned} \tag{10}$$

Due to the relative phase, we observe at wing midstroke $\ddot{\psi}(t_{mid})$ is at maximum and $\ddot{X}(t_{mid})$ is zero. Hence, the inertial term derived from the 2D model does not come into eqn (2.4) at midstroke. The error due to this missing inertial term is contained in the assumption that $\delta = 0°$. This inertial term implies that there should be passive rotation in vacuum. However, according to our kinematic assumption (sinusoidal and $\delta = 0$), the contribution of the inertial term is 0 in vacuum at wing mid-stroke even if there is substantial passive rotation. Hence, eqn (2.4) does not lose accuracy and its assumption covers the inertial term's effect. The model prediction fails at large δ, which is when the term involving $\ddot{X}(t_{mid})$ becomes significant. In this respect, this extra term involving $\ddot{X}$ that is responsible for passive pitching in vacuum does not affect equation (2.4) at mid-stroke.

Influence on equation (2.10)

In section 2.4 we proposed a coupled PDE-ODE system to solve for passive pitching. The ODE formulation is given by equation 2.10. The derivation of equation 2.10 is given in this supplement as eqn (8). We ignore the terms $ml\ddot{X}\cos\theta$ and $mgl\sin\theta$ in the simulation.

We ignore the term $ml\ddot{X}\cos\psi$ because the contribution from this term is not significant for the particular simulation in section 4.2.3 . The main discrepancy between the 2D numerical simulation and the 3D experiment comes from the 2D-3D fluid mechanical differences. There is a fitting parameter β that accounts for the 2D-3D discrepancy. The simulation result will be very similar with this added term since the most significant error is taken by the fitting parameter β.

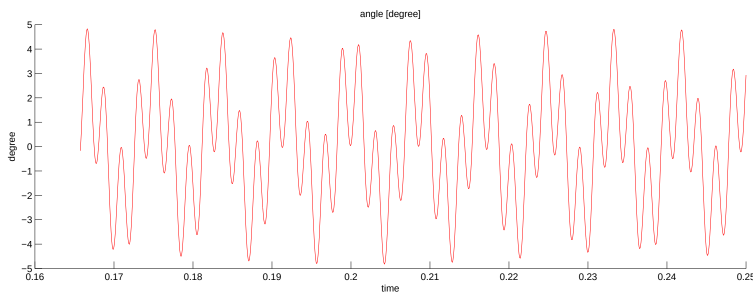

Figure 2: Simulation of the the 2D pendulum in vacuum

To show the inertial term does not have large impact on passive pitching, we run dynamical simulations and flapping experiments in vacuum at 120Hz. We run a simulation using equation (8) with the initial condition to $\psi(0) = 0$ and $\dot{\psi}(0) = 0$. The parameter values are listed in the table below.

ODE parameters	value
k	$1.4\mu Nm/rad$
l	$0.52mm$
g	$9.8m/s^2$
m	0.52mg
I	$1.54mg \cdot mm^2$
A	$3.7mm$

In the dynamical simulation, k and m are normalized by wing span and l is normalized by flexure width. Here we drive the system in vacuum at $120Hz$. A is the leading edge displacement amplitude. We prescribe the driving function using the function

$$X(t) = A\cos(wt).$$

Here we simulate for 30 periods and show the last 10 periods.

Figure 2 shows the oscillation amplitude is less than 5°. Hence the passive pitching in vacuum is not a significant error source. The spurious high frequency signal is due to the initial condition. Equation (8) has no damping term to remove the energy and hence the influence of initial condition persists. If we introduce a small damping term into equation (8), $-b\dot{\psi}$, where b is a small damping coefficient that physically corresponds to the small flexure viscoelastic damping, then the influence from the initial condition can be gradually damped out. Figure 3 shows a similar simulation with this term added. Figure 3 shows the steady state solution. In reality, the damping term b is small and difficult to quantify. The solution should be in between the results shown in figure 2 and 3, depending on the actual value of b.

Physically, the inertial term contribution is not significant because the wing resonance is not deliberately paired with the driving frequency. Here the driving

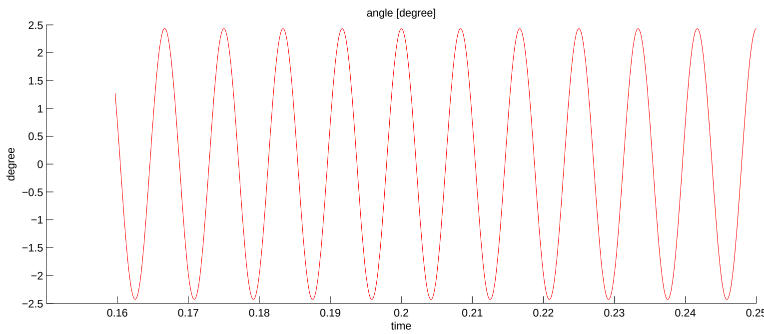

Figure 3: Simulation of the 2D pendulum in vacuum, with a small damping to remove the initial condition influence.

frequency is 120Hz, which corresponds to the resonance of the piezo-electric actuator, not closely paired to the wing. Using equation (8) we can obtain cases in which the passive pitching amplitude is large. However, it requires the natural resonant frequency of the pendulum to be matched with the driving frequency to obtain large passive pitching in vacuum. This can be done by varying K, m, or I. In our case the driving frequency is different from the pendulum resonance, hence the effect of this term is small.

See the online content for a video that compares flapping in air versus in vacuum for 2 wing hinges ($7.5\mu m$ normal and $12.7\mu m$ very stiff). In both cases the passive pitching in vacuum is much smaller than it is in air, and we see the pitching frequency is higher than the driving frequency. The pressure in vacuum is around 0.8 - 1.2 mTorr. These experiments correspond to the simulation shown in section 4.2.3. Consequently, the inertial contribution is small and can be ignored.