Supplementary data

First-order velocity field

Appendix A. Series representation of the velocity field

The first-order velocity field $\boldsymbol{u}^{(1)}$ can be calculated according to the method by Brenner (1964). However, as pointed out by Dörr & Hardt (2015), corrections to Brenner's method are necessary. Here, we provide all of the expressions required to explicitly compute the velocity field $\boldsymbol{u}^{(1)}$, which is of the form (Brenner 1964)

$$\boldsymbol{u}^{(1)} = \sum_{k=0}^{\infty} \boldsymbol{u}_k^{(1)}. \tag{A 1}$$

To facilitate the explicit evaluation of the velocity field and to simultaneously ensure a high degree of accuracy, we truncate the series after 20 summation terms and arrive at

$$\boldsymbol{u}^{(1)} \approx \sum_{k=0}^{20} \boldsymbol{u}_k^{(1)}. \tag{A 2}$$

The velocity fields $\boldsymbol{u}_k^{(1)}$ can be expressed by

$$\begin{aligned} \boldsymbol{u}_k^{(1)} = \sum_{n=1}^{\infty} \Bigg[& \nabla \times \left(\boldsymbol{r}\, {}_k\chi_{-n-1}^{(1)} \right) + \nabla \left({}_k\phi_{-n-1}^{(1)} \right) - \frac{n-2}{2n(2n-1)\mu_1} r^2 \nabla \left({}_kp_{-n-1}^{(1)} \right) \\ & + \boldsymbol{r} \frac{n+1}{n(2n-1)\mu_1}\, {}_kp_{-n-1}^{(1)} \Bigg], \end{aligned} \tag{A 3}$$

where

$${}_kp_{-n-1}^{(1)} = \frac{(2n-1)\mu_1}{(n+1)a} \left(\frac{a}{r} \right)^{n+1} \left[(n+2)\, {}_kX_n^{(1)} + {}_kY_n^{(1)} \right], \tag{A 4}$$

$${}_k\phi_{-n-1}^{(1)} = \frac{a}{2(n+1)} \left(\frac{a}{r} \right)^{n+1} \left(n\, {}_kX_n^{(1)} + {}_kY_n^{(1)} \right), \text{ and} \tag{A 5}$$

$${}_k\chi_{-n-1}^{(1)} = \frac{1}{n(n+1)} \left(\frac{a}{r} \right)^{n+1} {}_kZ_n^{(1)} \tag{A 6}$$

(Brenner 1964). Through accommodating the velocity field (A 3) to the boundary condition at the particle surface, the functions ${}_kX_n^{(1)}$, ${}_kY_n^{(1)}$ and ${}_kZ_n^{(1)}$ can be specified. One has

$${}_kX_n^{(1)} = 0 \quad \text{for } n \leqslant 1, \tag{A 7}$$

$${}_kY_n^{(1)} = \begin{cases} \dfrac{3}{2} \displaystyle\sum_{m=0}^{k} \frac{c_{km} N_{km} \cos(m\varphi)}{2k+1} (k-1)(k+m) P_{k-1}^m(\cos\theta) & \text{for } n = k-1 \\ -\dfrac{3}{2} \displaystyle\sum_{m=0}^{k} \frac{c_{km} N_{km} \cos(m\varphi)}{2k+1} (2+k)(k-m+1) P_{k+1}^m(\cos\theta) & \text{for } n = k+1 \\ 0 & \text{for all other } n, \end{cases} \tag{A 8}$$

and

$${}_kZ_n^{(1)} = \begin{cases} \frac{3}{2} U \sum_{m=0}^{k} m c_{km} N_{km} P_k^m(\cos\theta) \sin(m\varphi) & \text{for } n = k \\ 0 \quad \text{for } n \neq k. \end{cases} \tag{A 9}$$

In equations (A 7)–(A 9), the P_k^m are associated Legendre polynomials. Expression (A 8), developed by Dörr & Hardt (2015), differs from the corresponding result by Brenner (1964), while equations (A 7) and (A 9) have been adopted from Brenner (1964) without modification. The expansion coefficients c_{km} and N_{km} are given by

$$N_{km} = \begin{cases} \sqrt{2}\sqrt{\frac{2k+1}{4\pi}\frac{(k-m)!}{(k+m)!}} & \text{if } m > 0 \\ \sqrt{\frac{2k+1}{4\pi}} & \text{if } m = 0 \end{cases} \tag{A 10}$$

and

$$c_{km} = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,|\cos\varphi|\,N_{km}P_k^m(\cos\theta)\cos(m\varphi)\sin\theta\mathrm{d}\theta\mathrm{d}\varphi, \tag{A 11}$$

respectively. Equation (A 11) contains information about the particle geometry via the term $\sin\theta\,|\cos\varphi|$, which is equal to the first-order term $\phi^{(1)}$ in the expansion of the particle shape (equation (2.5) in the paper). The velocity field $\boldsymbol{u}^{(1)}$ according to equation (A 1) is thus fully determined. For the sake of convenience, we explicitly display the coefficients c_{km} for $k \leqslant 20$ in appendix B. Due to the symmetry of the particle shape with respect to the plane $x = 0$, coefficients with $m < 0$ and/or m odd vanish.

Appendix B. Coefficients c_{km}

k=0: m=0

$\sqrt{\pi}$

k=2: m=0,2

$-\frac{\sqrt{5\pi}}{8}, \frac{\sqrt{15\pi}}{8}$

k=4: m=0,2,4

$-\frac{3\sqrt{\pi}}{64}, \frac{\sqrt{5\pi}}{32}, -\frac{\sqrt{35\pi}}{64}$

k=6: m=0,2,4,6

$-\frac{5\sqrt{13\pi}}{1024}, \frac{\sqrt{\frac{1365\pi}{2}}}{1024}, -\frac{3\sqrt{91\pi}}{1024}, \frac{\sqrt{\frac{3003\pi}{2}}}{1024}$

k=8: m=0,2,4,6,8

$-\frac{35\sqrt{17\pi}}{16384}, \frac{3\sqrt{\frac{595\pi}{2}}}{4096}, -\frac{3\sqrt{1309\pi}}{8192}, \frac{\sqrt{\frac{7293\pi}{2}}}{4096}, -\frac{3\sqrt{12155\pi}}{16384}$

k=10: m=0,2,...,10

$-\frac{147\sqrt{21\pi}}{131072}, \frac{49\sqrt{385\pi}}{131072}, -\frac{7\sqrt{5005\pi}}{65536}, \frac{21\sqrt{\frac{5005\pi}{2}}}{131072}, -\frac{7\sqrt{\frac{85085\pi}{3}}}{131072}, \frac{7\sqrt{\frac{323323\pi}{6}}}{131072}$

k=12: m=0,2,...,12

$-\frac{3465\sqrt{\pi}}{1048576}, \frac{45\sqrt{3003\pi}}{524288}, -\frac{225\sqrt{\frac{1001\pi}{2}}}{1048576}, \frac{75\sqrt{\frac{2431\pi}{2}}}{524288}, -\frac{15\sqrt{138567\pi}}{1048576}, \frac{15\sqrt{\frac{88179\pi}{2}}}{524288}, -\frac{15\sqrt{\frac{676039\pi}{2}}}{1048576}$

k=14: m=0,2,...,14

$-\frac{14157\sqrt{29\pi}}{33554432}, \frac{1089\sqrt{39585\pi}}{67108864}, -\frac{99\sqrt{\frac{2467465\pi}{2}}}{33554432}, \frac{33\sqrt{46881835\pi}}{67108864}, -\frac{33\sqrt{12785955\pi}}{33554432}, \frac{33\sqrt{58815393\pi}}{67108864},$
$-\frac{165\sqrt{\frac{1508087\pi}{2}}}{33554432}, \frac{495\sqrt{646323\pi}}{67108864}$

k=16: m=0,2,...,16

$$-\frac{306735\sqrt{33\pi}}{1073741824},\ \frac{20449\sqrt{935\pi}}{268435456},\ -\frac{1573\sqrt{\frac{323323\pi}{2}}}{268435456},\ \frac{3003\sqrt{46189\pi}}{268435456},$$
$$-\frac{1001\sqrt{\frac{5311735\pi}{3}}}{536870912},\ \frac{715\sqrt{\frac{2860165\pi}{3}}}{268435456},\ -\frac{2145\sqrt{\frac{245157\pi}{2}}}{268435456},\ \frac{143\sqrt{35547765\pi}}{268435456},\ -\frac{143\sqrt{1101980715\pi}}{1073741824}$$

k=18: m=0,2,...,18

$$-\frac{1738165\sqrt{37\pi}}{8589934592},\ \frac{61347\sqrt{59755\pi}}{8589934592},\ -\frac{5577\sqrt{\frac{920227\pi}{2}}}{2147483648},\ \frac{1001\sqrt{\frac{117920517\pi}{2}}}{4294967296},\ -\frac{15015\sqrt{274873\pi}}{4294967296},$$
$$\frac{2145\sqrt{\frac{28861665\pi}{2}}}{4294967296},\ -\frac{2145\sqrt{\frac{7971317\pi}{2}}}{2147483648},\ \frac{429\sqrt{\frac{3706662405\pi}{2}}}{8589934592},\ -\frac{429\sqrt{2398428615\pi}}{8589934592},\ \frac{715\sqrt{\frac{3357800061\pi}{2}}}{8589934592}$$

k=20: m=0,2,...,20

$$-\frac{10207769\sqrt{41\pi}}{68719476736},\ \frac{48841\sqrt{899745\pi}}{34359738368},\ -\frac{8619\sqrt{117266765\pi}}{68719476736},\ \frac{1105\sqrt{\frac{914680767\pi}{2}}}{17179869184},\ -\frac{9945\sqrt{23453353\pi}}{34359738368},$$
$$\frac{1989\sqrt{\frac{309157835\pi}{2}}}{17179869184},\ -\frac{3315\sqrt{\frac{1916778577\pi}{2}}}{68719476736},\ \frac{3315\sqrt{\frac{531543639\pi}{2}}}{34359738368},\ -\frac{3315\sqrt{1240268491\pi}}{68719476736},\ \frac{1105\sqrt{\frac{7245779079\pi}{2}}}{34359738368},$$
$$-\frac{663\sqrt{\frac{156991880045\pi}{2}}}{68719476736}$$

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