**Supplementary material for: “Stability of a liquid film flowing down an inclined anisotropic and inhomogeneous porous layer: An analytical description”**

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**Appendix A**: **Applicability of Squire’s theorem to inhomogeneous and anisotropic porous layer.**

**Appendix B: Constants appearing in Eqs. (2.36) and (2.37)**.

**Appendix C: Eigenvalue problem ofequations.**

**Appendix D: Solutions ofequations.**

**Appendix E: Eigenvalue problem ofequations.**

**Appendix F: Solutions ofequations.**

**Appendix A**: **Applicability of Squire’s theorem to inhomogeneous and anisotropic porous layer.**

Here we follow the procedure described in Squire (1933) to show that the heterogeneity and anisotropy in the permeability of a porous layer prohibits the application of Squire’s theorem.

Consider an inclined porous layer described in a right-handed coordinate system (, , ), with pointing in the streamwise direction and pointing upwards. Equations governing the Darcian velocity field (, , ) are given by

, (A.1)

, (A.2) . (A.3)

Here is the inhomogeneous function in the direction (assumed to be a function of only , without any loss of generality). All other notations are same as in the main text. Non-dimensionalizing the variables using the respective scales of velocity, length, time and pressure as, ,, , the linearized perturbation equations are

(A.4)

(A.5)

(A.6)

(A.7)

where, = is Reynolds number in the porous layer, and are the anisotropic parameters in the and directions and subscript ‘1’ denotes the perturbation variables.

Eliminating pressure from Eqns. (A.5), (A.6) and (A.7) through cross-differentiation gives

(A.8)

(A.9)

(A.10)

Introducing the following normal modes

(A.11)

(A.12)

, (A.13)

Eqns. (A.4), (A.8), (A.9) and (A.10) reduce to

(A.14)

(+=0 (A.15) (+=0 (A.16)

(+=0. (A.17)

We now eliminate and from equations (A.14) through (A.17) to obtain an equation for the eigenfunction , as given by Eq. (A.22). Below, we present the main steps in the derivation.

Rearranging Eq. (A.17), we get

, (A.18)

where and .

Eliminating from equations (A.14) and (A.18) yields

(*N*+)+=0 (A.19)

which implies

= . (A.20)

Differentiating the above expression with respect to once, we obtain

(A.21)

Rearranging Eq. (A.15), and substituting the expressions for and from Eqns. (A.20) and (A.21) respectively, we get

|  |  |
| --- | --- |
|  | (A.22) |

where *T*= .

**Appendix B: Constants appearing in Eqs. (2.36) and (2.37)**

(B.1)

(B.2)

(B.3)

(B.4)

(B.5)

(B.6)

**Appendix C: Eigenvalue problem of equations**.

|  |  |
| --- | --- |
|  | (C.1) |
|  | (C.2) |
|  | (C.3) |
|  | (C.4) |
|  | (C.5) |
|  | (C.6) |
|  | (C.7) |
|  | (C.8) |

**Appendix D: Solutions of equations**

**Case (a): *A* ≠ 0**

|  |  |
| --- | --- |
|  | (D.1) |
|  | (D.2) |
| Constants appearing in Eqs. (D.1) to (D.3) are given by | (D.3) |

|  |  |
| --- | --- |
|  | (D.4) |
|  | (D.5) |
|  | (D.6) |
|  | (D.7) |
|  | (D.8) |
|  | (D.9) |
|  | (D.10) |

**Case (b): *A* = 0**

|  |  |
| --- | --- |
|  | (D.11) |
|  | (D.12) |
| Constants appearing in Eqs. (D.11) to (D.13) are given by | (D.13) |
|  | (D.14) |
|  | (D.15) |
|  | (D.16) |
|  | (D.17) |
|  | (D.18) |

**Appendix E: Eigenvalue problem of equations**.

|  |  |
| --- | --- |
|  | (E.1) |
|  | (E.2) |
|  | (E.3) |
|  | (E.4) |
|  | (E.5) |
|  | (E.6) |
|  | (E.7) |
|  | (E.8) |

**Appendix F: Solutions of equations**

**Case (a): *A* ≠ 0**

|  |  |
| --- | --- |
|  | (F.1) |
|  | (F.2) |
|  | (F.3) |
| Constants appearing in Eqs. (F.1) to (F.3) are given by |  |
|  | (F.4) |
|  | (F.5) |
|  | (F.6) |
|  | (F.7) |
|  | (F.8) |
|  | (F.9) |
|  | (F.10) |
|  | (F.11) |
| ) | (F.12) |
|  | (F.13) |
| **Case (b): *A* = 0** |  |
|  | (F.14) |
|  | (F.15 ) |
| Constants appearing in Eqs. (F.14) to (F.16) are given by | (F.16) |
|  | (F.17) |
|  | (F.18) |
|  | (F.19) |
|  | (F.20) |
|  | (F.21) |
|  | (F.22) |