Supplemental material of "On the influence of viscosity and caustics on acoustic streaming in sessile droplets: an experimental and a numerical study with a cost-effective method."

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CONTENTS

I. PIV EXPERIMENTAL DETAILS

A. material and methods

The experiments were performed with 10 um diameter latex particles, diluted at a concentration of 40 particles/mm^3. The interogation window was 250 um wide with an overlap of 50% between each window. The time lapse depended on the flow velocity in the droplets and is given in table I A.

The 5% less reliable were selected for removal based on a normalized median test (see Raffel et. al. Particle image velocimetry: a practical guide. 2nd ed.)

wt%Glyc	0	10	20	30	40	50	60	70	80	90
Time lapse (ms)	250	250	250	250	250	500	1000	1000	2000	10000

TABLE I. Time laps between two successive images for the PIV experiment.

B. Forces acting on the tracer particles

The beads are exposed to several external forces: the buoyancy and gravity, the drag (due to acoustic streaming), and the acoustic radiation force. Stokes drag dominates over the combined action of gravity and buoyancy provided that $v \gg \frac{d^2(\rho_p - \rho_0)g}{18\mu}$, with d the particle diameter, ρ_p its density and g the acceleration of gravity. This yields a crossover speed of 2.8 μm/s.

The action of acoustic radition pressure is more heterogeneous as it depends on the acoustic field. This field is a combination of caustics (where the power flux is non-null, similar to traveling waves) and background field. We are mostly interested in the overall flow pattern and therefore will be mostly concerned about the background field. Since the particle size is much smaller than the acoustic wavelength ($\frac{1}{2}\frac{d}{\lambda} \simeq 0.07$), simplified analysis in the straightline of Gor'kov model can be applied here [1]. Accordingly, the drag force will tend to dominate at high viscosity and in traveling wave fields. Accordingly, the worst case scenario is the water droplet which has a low viscosity and a standing wave field. In this case, the acoustic radiation force is given by $F \simeq \frac{\pi}{4}\Phi a^3 k\langle\mathcal{E}\rangle$, with Φ the acoustic contrast of the beads in water that ranges near 0.2 [1]. Comparing with the Stokes drag, we get the condition $v \gg \frac{d^2\Phi k\langle\mathcal{E}\rangle}{12\mu}$. In our simulations, the droplet exposed to 10 pm vibrations which yields an acoustic energy density in the background field close to 2.2 mJ/m^3. In our experiments, the vibration amplitude is about 44 pm, which yields a typical acoustic energy density of 0.04 J/m^3. Thus, the associated acoustic radiation force is overcome by a minimum velocity of about 5 μm/s.

II. VALIDATION OF THE CIRCULAR FOURIER TRANSFORM FOR COMPUTING THE ACOUSTIC FIELD

The acoustic field radiated by a SAW incident on a 2 mm diameter sessile droplet was computed by circular Fourier transform and 3D direct numerical simulation. The simulations were carried out at a frequency of 6 MHz on water droplets with an incident wave amplitude of 10 nm. Results are presented in figrue 1.

III. FREEDOM IN THE CHOICE OF THE FILTERING LENGTH k_c

In order to test the validity of the SSSF approach , we reproduced the benchmark simulation in section 4.3.1 (sessile droplet exposed to 6 MHz SAW radiations) with various values of $k_c = \frac{2\pi}{L}$. In all simulations, the acoustic field remains the same (computed by circular Fourier transform) but the streaming force field is filtered. The fluid mechanics are then

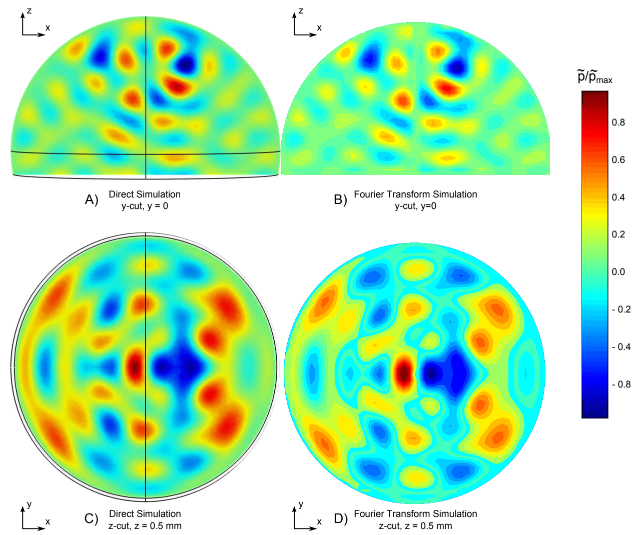

FIG. 1. Acoustic field radiated by a SAW incident on a 2 mm diameter sessile droplet computed by 3D direct numerical simulation (A-C) and circular Fourier transform (B-D). The simulations were carried out at a frequency of 6 MHz on water droplets. A) $-2.63 < \tilde{p} < 2.84$ MPa, B) $-3.8 < \tilde{p} < 4.5$ MPa, C) $-6.90 < \tilde{p} < 5.92$ MPa and D) $-6.2 < \tilde{p} < 5.6$ MPa.

solved on the same mesh. The results are displayed in the table III. It appears clearly that the choice of k_c has very little influence on the flow velocity in the droplet. It is therefore advantageous to remove the smallest scales in order to ease the resolution of the numerical equations.

L	2λ	$D/10$	$D/20$	$D/40$	0 (no filtering)
$\mathcal{U}_{\mathtt{avg}}$ (mm/s)	1.92	1.91	1.98	1.98	1.97

TABLE II. Independence of the average flow velocity with the choice of the filtering length k_c. $D = 2$ mm, $\lambda = 0.25$ mm.

IV. COMPUTATIONAL COST

Computing the acoustic streaming might seem to be an easy task with appropriate softwares. Indeed, many codes are readily available to compute acoustic fields and fluid mechanics. The nonlinear hydrodynamic forcing term can be deduced from the acoustics and computed in a straightforward fashion. Nevertheless, in high frequency regimes (with wavelength much smaller than the characteristic length scale of the flow structure produced), the computation time can become prohibitive. In figure 2, we compare the memory requirements of the direct numerical simulations as already implemented in the commercial software Comsol 4.3b to our more customized implementations. The benchmark test is a 2 mm diameter sessile droplet (water, contact angle 100^o) exposed to an incident SAW radiation for a range of megahertz frequencies.

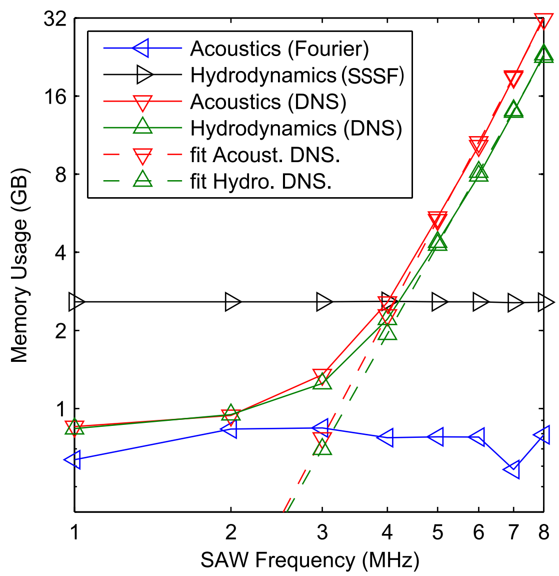

FIG. 2. Memory usage versus SAW frequency for the various parts and methods of the computation. Regression coefficients are $M_{\texttt{AcoustDNS}} = 0.012 \times F^{3.79}$ and $M_{\texttt{HydroDNS}} = 0.014 \times F^{3.54}$ where F is the SAW frequency in MHz and M the memory in GB. Projections at 20 MHz indicate 1.0 TB of RAM of acoustics (DNS) and 580 GB of RAM for the fluidics (DNS).

We first notice that there is some background noise on the memory requirements, which magnitude is about 700 MB, probably related to the OS (Windows 7) and the software. The direct numerical simulations of acoustics and hydrodynamics start consuming a lot of memory after a 4 MHz threshold. This corresponds to a wavelength of 375 μm, which is a third of the droplet radius. After this threshold, the memory requirements grow quickly and extrapolation to 20 MHz excitation estimate the need to 1 TB for the acoustics, and 580 GB for the fluidics. Access to such middle range cluster capabilities being difficult, we used alternative numerical recipes.

The Fourier transform resolves the incident field into azimuthal harmonics to reduce the computation of acoustics to a 2D problem. The memory requirements at these excitation frequencies are so low that they are overwhelmed by the background noise.

The Streaming Source Spatial Filtering always computed with the same grid resolution, which is fixed by the explicit filtering step. The memory needed to compute the force is not shown since it is implementation-dependent. With Matlab, we reconstructed the 3D acoustic variables in multidimensional arrays $\tilde{p}$, $\tilde{v}_r$, $\tilde{v}_\theta$ and $\tilde{v}_z$, each of which weighs about 190 MB for computations of 20 MHz acoustic fields. We used the multidimensional Fourier transform in Matlab to maximize the speed when filtering the forces. Memory requirements were always kept below 32 GB even at 20 MHz. If required, the filtering can be achieved in the real space with low memory consumption by using cross correlation algorithm to smooth directly the force field.

[1] M. Settnes and H. Bruus, Phys. Rev. E **85**, 016327 (2012)