

Mass and energy flux coefficients

General Solutions for NSF

NSF equations

$$\gamma = \frac{\sqrt{5} \sqrt{\text{PrM}}}{3};$$

$$v_{\text{NSF}}[r_] := \frac{c1}{r^2}$$

$$\theta_{\text{NSF}}[r_] := \frac{2 c2 \text{Pr}}{5 \text{Kn} r} + c3$$

$$q_{\text{NSF}}[r_] := -\frac{5}{2 \text{Pr}} \text{Kn} D[\theta_{\text{NSF}}[x], x] /. \{x \rightarrow r\}$$

$$\sigma_{\text{NSF}}[r_] := -\frac{4}{3} \text{Kn} \left(D[v_{\text{NSF}}[x], x] - \frac{v_{\text{NSF}}[x]}{x} \right) /. \{x \rightarrow r\}$$

$$\rho_{\text{NSF}}[r_] := -\theta_{\text{NSF}}[r]$$

$$p_{\text{NSF}}[r_] := c6$$

NSF equations with no jump

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c3 = 0;
c6 = 0;
(*ν(=1) and χ(=1) are accomodation coeffcients*)
(*linear Hertz-Knudsen-Schrage relation*)

BC1 =  $\sqrt{\frac{2}{\pi}} \frac{\nu}{2 - \nu} \left( p_{\text{sat}} - p_{\text{NSF}}[1] + \frac{1}{2} (\theta_{\text{NSF}}[1] - \theta_L) \right) = v_{\text{NSF}}[1];$ 

(*θL temperature of liquid and psat is saturation pressure*)
(*No temperature Jump*)
BC2 =  $\theta_{\text{NSF}}[1] = \theta_L;$ 
(*Pressure-driven case*)
{c1NSFNJ, c2NSFNJ} =
  {c1, c2} /. FullSimplify[Solve[FullSimplify[{BC1, BC2} /. {Pr →  $\frac{2}{3}$ , PrM →  $\frac{3}{2}$ ,
    PrR →  $\frac{7}{6}$ , ν → 1, χ → 0, θL → 0, psat → 1}], {c1, c2}]] [[1]];

(*Temperature-driven case*)
{cc1NSFNJ, cc2NSFNJ} =
  {c1, c2} /. FullSimplify[Solve[FullSimplify[{BC1, BC2} /. {Pr →  $\frac{2}{3}$ , PrM →  $\frac{3}{2}$ ,
    PrR →  $\frac{7}{6}$ , ν → 1, χ → 0, θL → 1, psat → 0}], {c1, c2}]] [[1]];

{c1NSFNJ, c2NSFNJ}
{cc1NSFNJ, cc2NSFNJ}

 $\left\{ \sqrt{\frac{2}{\pi}}, 0 \right\}$ 

 $\left\{ 0, \frac{15 \text{ Kn}}{4} \right\}$ 

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NSF equations with temperature jump boundary conditions

$c3 = 0;$

$c6 = 0;$

(* $\nu (=1)$ and $\chi (=1)$ are accommodation coefficients*)

(*Extended Hertz-Knudsen-Schrage relation*)

$$BC1 = \sqrt{\frac{2}{\pi}} \frac{\nu}{2 - \nu} \left(p_{sat} - p_{NSF}[1] - \frac{\sigma_{NSF}[1]}{2} + \frac{1}{2} (\theta_{NSF}[1] - \theta_L) \right) = v_{NSF}[1];$$

(*Temperature Jump Boundary condition*)

$$BC2 = - \frac{\nu + \chi (1 - \nu)}{2 - \nu - \chi (1 - \nu)} \sqrt{\frac{2}{\pi}} \left(2 \theta_{NSF}[1] - 2 \theta_L + \frac{\sigma_{NSF}[1]}{2} \right) - \frac{v_{NSF}[1]}{2} = q_{NSF}[1];$$

(*Pressure-driven case*)

{c1NSFKBC, c2NSFKBC} =

$$\{c1, c2\} /. \text{FullSimplify}\left[\text{Solve}\left[\left(\text{FullSimplify}\left[\{BC1, BC2\} /. \left\{Pr \rightarrow \frac{2}{3}, PrM \rightarrow \frac{3}{2}, PrR \rightarrow \frac{7}{6}, \nu \rightarrow 1, \chi \rightarrow 0, \theta_L \rightarrow 0, p_{sat} \rightarrow 1\right\}\right]\right), \{c1, c2\}\right][[1]]\right];$$

(*Temperature-driven case*)

{cc1NSFKBC, cc2NSFKBC} =

$$\{c1, c2\} /. \text{FullSimplify}\left[\text{Solve}\left[\left(\text{FullSimplify}\left[\{BC1, BC2\} /. \left\{Pr \rightarrow \frac{2}{3}, PrM \rightarrow \frac{3}{2}, PrR \rightarrow \frac{7}{6}, \nu \rightarrow 1, \chi \rightarrow 0, \theta_L \rightarrow 1, p_{sat} \rightarrow 0\right\}\right]\right), \{c1, c2\}\right][[1]]\right];$$

{c1NSFKBC, c2NSFKBC}

{cc1NSFKBC, cc2NSFKBC}

$$\left\{ \frac{16 + 15 \text{Kn} \sqrt{2\pi}}{9 \sqrt{2\pi} + 5 \text{Kn} (8 + 3\pi + 6 \text{Kn} \sqrt{2\pi})}, -\frac{15 \text{Kn} (8 \text{Kn} + \sqrt{2\pi})}{18 \sqrt{2\pi} + 10 \text{Kn} (8 + 3\pi + 6 \text{Kn} \sqrt{2\pi})} \right\}$$

$$\left\{ -\frac{15 \text{Kn} \sqrt{\frac{\pi}{2}}}{9 \sqrt{2\pi} + 5 \text{Kn} (8 + 3\pi + 6 \text{Kn} \sqrt{2\pi})}, \frac{15 \text{Kn} (40 \text{Kn} + 9 \sqrt{2\pi})}{4 (9 \sqrt{2\pi} + 5 \text{Kn} (8 + 3\pi + 6 \text{Kn} \sqrt{2\pi}))} \right\}$$

General Solutions for R13

R13 equations

$$\gamma = \frac{\sqrt{5} \sqrt{\text{PrM}}}{3};$$

$$\text{vR13}[r_]:= \frac{\text{cc1}}{r^2}$$

$$\text{qR13}[r_]:= \frac{\text{cc2}}{r^2}$$

$$\sigma\text{R13}[r_]:= \frac{4 \text{Kn}}{r^3} \frac{(5 \text{cc1} + 2 \text{cc2})}{5} - \text{cc4} \frac{\text{Kn}^3}{\gamma^3 r^3} \text{Exp}\left[-\frac{\gamma}{\text{Kn}} r\right] \left(1 + r \frac{\gamma}{\text{Kn}} + \frac{1}{3} r^2 \frac{\gamma^2}{\text{Kn}^2}\right)$$

$$\text{mR13}[r_]:= -\frac{9}{5} \frac{\text{Kn}}{\text{PrM}} \left(D[\sigma\text{R13}[x], x] - \frac{2}{r} \sigma\text{R13}[x] \right) /. \{x \rightarrow r\}$$

$$\text{RR13}[r_]:= -\frac{56}{15} \frac{\text{Kn}}{\text{PrR}} \left(D[\text{qR13}[x], x] - \frac{1}{r} \text{qR13}[x] \right) /. \{x \rightarrow r\}$$

$$\Delta\text{R13}[r_]:= -\frac{56}{15} \frac{\text{Kn}}{\text{PrR}} \left(D[\text{qR13}[x], x] + \frac{2}{r} \text{qR13}[x] \right) /. \{x \rightarrow r\}$$

$$\theta\text{R13}[r_]:= \text{cc3} + \frac{2}{5} \frac{\text{Pr}}{\text{Kn}} \frac{\text{cc2}}{r} + \frac{2 \text{cc4}}{15} \text{Kn} \frac{\text{Exp}\left[-\frac{\gamma}{\text{Kn}} r\right]}{\gamma r}$$

$$\text{pR13}[r_]:= \frac{1}{3} \frac{\text{cc4} e^{-\frac{\gamma}{\text{Kn}} r} \text{Kn}}{\gamma r} + \text{cc6}$$

R13 equations KBC

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cc3 = 0;
cc5 = 0;
cc6 = 0;
(*ν(=1) and χ(=1) are accommodation coefficients*)
(*θL temperature of liquid and psat is saturation pressure*)
(*Boundary conditions for R13*)

BC1 =  $\sqrt{\frac{2}{\pi}} \frac{\nu}{2 - \nu} \left( \text{psat} - p_{R13}[1] - \frac{\sigma_{R13}[1]}{2} + \frac{RR13[1]}{28} + \frac{1}{2} (\theta_{R13}[1] - \theta_L) \right) == \nu_{R13}[1];$ 

BC2 =
-  $\frac{\nu + \chi (1 - \nu)}{2 - \nu - \chi (1 - \nu)} \sqrt{\frac{2}{\pi}} \left( 2 \theta_{R13}[1] - 2 \theta_L + \frac{\sigma_{R13}[1]}{2} + \frac{5 RR13[1]}{28} \right) - \frac{\nu_{R13}[1]}{2} == q_{R13}[1];$ 

BC3 =  $\frac{\nu + \chi (1 - \nu)}{2 - \nu - \chi (1 - \nu)} \sqrt{\frac{2}{\pi}} \left( \frac{2}{5} \theta_{R13}[1] - \frac{2}{5} \theta_L - \frac{7 \sigma_{R13}[1]}{5} - \frac{RR13[1]}{14} \right) - \frac{2 \nu_{R13}[1]}{5} ==$ 
mR13[1];
(*Pressure-driven case*)
{c1R13KBC, c2R13KBC, c4R13KBC} = {cc1, cc2, cc4} /.
Solve[({BC1, BC2, BC3} /. {Pr →  $\frac{2}{3}$ , PrM →  $\frac{3}{2}$ , PrR →  $\frac{7}{6}$ , ν → 1, χ → 0, θL → 0, psat → 1}),
{cc1, cc2, cc4}][[1]];
(*Temperature-driven case*)
{cc1R13KBC, cc2R13KBC, cc4R13KBC} = {cc1, cc2, cc4} /.
Solve[({BC1, BC2, BC3} /. {Pr →  $\frac{2}{3}$ , PrM →  $\frac{3}{2}$ , PrR →  $\frac{7}{6}$ , ν → 1, χ → 0, θL → 1, psat → 0}),
{cc1, cc2, cc4}][[1]];

Simplify[{c1R13KBC, c2R13KBC}] // Simplify // N // Chop // Simplify
Simplify[{cc1R13KBC, cc2R13KBC}] // Simplify // N // Chop // Simplify

$$\left\{ \frac{2077.01 + 15070.5 \text{ Kn} + 56103.6 \text{ Kn}^2 + 114932. \text{ Kn}^3 + 113459. \text{ Kn}^4 + 62908.7 \text{ Kn}^5}{3098.43 + 26612.6 \text{ Kn} + 107126. \text{ Kn}^2 + 206043. \text{ Kn}^3 + 224887. \text{ Kn}^4 + 86728.8 \text{ Kn}^5}, \right.$$


$$- \left( \left( 0.265152 \text{ Kn} (-0.00633054 - 0.0258131 \text{ Kn} + 0.11573 \text{ Kn}^2 + \right. \right.$$


$$1.27953 \text{ Kn}^3 + 3.56568 \text{ Kn}^4 + 5.63397 \text{ Kn}^5 + 4.16374 \text{ Kn}^6 + 1. \text{ Kn}^7) \Big) /$$


$$(-0.00192962 - 0.00963229 \text{ Kn} + 0.0183969 \text{ Kn}^2 + 0.385012 \text{ Kn}^3 + 1.70572 \text{ Kn}^4 +$$


$$4.332 \text{ Kn}^5 + 6.89591 \text{ Kn}^6 + 7.03614 \text{ Kn}^7 + 4.11512 \text{ Kn}^8 + 1. \text{ Kn}^9) \Big) \Big\}$$


$$\left\{ - \frac{0.072535 \text{ Kn} (0.428444 + 3.56058 \text{ Kn} + 9.26141 \text{ Kn}^2 + 3.77049 \text{ Kn}^3 + 1. \text{ Kn}^4)}{0.0357255 + 0.306849 \text{ Kn} + 1.23519 \text{ Kn}^2 + 2.37572 \text{ Kn}^3 + 2.593 \text{ Kn}^4 + 1. \text{ Kn}^5}, \right.$$


$$(1.19318 \text{ Kn} (-0.00606452 - 0.0175494 \text{ Kn} + 0.115757 \text{ Kn}^2 +$$


$$1.01815 \text{ Kn}^3 + 2.81591 \text{ Kn}^4 + 4.4287 \text{ Kn}^5 + 3.48406 \text{ Kn}^6 + 1. \text{ Kn}^7) \Big) /$$


$$(-0.00192962 - 0.00963229 \text{ Kn} + 0.0183969 \text{ Kn}^2 + 0.385012 \text{ Kn}^3 + 1.70572 \text{ Kn}^4 +$$


$$4.332 \text{ Kn}^5 + 6.89591 \text{ Kn}^6 + 7.03614 \text{ Kn}^7 + 4.11512 \text{ Kn}^8 + 1. \text{ Kn}^9) \Big\}$$


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General Solutions for R26

R26 equations

$$vR26[r_]:= \frac{cc1}{r^2}$$

$$qR26[r_]:= \frac{cc2}{r^2}$$

$$pR26[r_]:= \frac{1}{3} \frac{ccA e^{-\frac{r}{Kn}} Kn}{\gamma_1 r} + \frac{1}{3} \frac{ccB e^{-\frac{r}{Kn}} Kn}{\gamma_2 r} + \frac{1}{3} \frac{ccC e^{-\frac{r}{Kn}} Kn}{\gamma_3 r}$$

$$\sigma R26[r_]:= \frac{(20 cc1 + 8 cc2) Kn}{5 r^3} - \frac{1}{3 r^3 \gamma_1^3 \gamma_2^3 \gamma_3^3} e^{-\frac{r(\gamma_1 + \gamma_2 + \gamma_3)}{Kn}} Kn$$

$$\left(e^{\frac{r}{Kn}} \left(ccA e^{\frac{r}{Kn}} (3 Kn^2 + 3 Kn r \gamma_1 + r^2 \gamma_1^2) \gamma_2^3 + ccB e^{\frac{r}{Kn}} \gamma_1^3 (3 Kn^2 + 3 Kn r \gamma_2 + r^2 \gamma_2^2) \right) \gamma_3^3 + \right.$$

$$\left. ccC e^{\frac{r(\gamma_1 + \gamma_2)}{Kn}} \gamma_1^3 \gamma_2^3 (3 Kn^2 + 3 Kn r \gamma_3 + r^2 \gamma_3^2) \right)$$

$$mR26[r_]:= -\frac{1}{3 r^4 \gamma_1^5 \gamma_2^5 \gamma_3^5} e^{-\frac{r(\gamma_1 + \gamma_2 + \gamma_3)}{Kn}} Kn$$

$$\left(e^{\frac{r}{Kn}} \left(ccA e^{\frac{r}{Kn}} (15 Kn^3 + 15 Kn^2 r \gamma_1 + 6 Kn r^2 \gamma_1^2 + r^3 \gamma_1^3) \gamma_2^5 + \right. \right.$$

$$\left. ccB e^{\frac{r}{Kn}} \gamma_1^5 (15 Kn^3 + 15 Kn^2 r \gamma_2 + 6 Kn r^2 \gamma_2^2 + r^3 \gamma_2^3) \right) \gamma_3^5 +$$

$$\left. ccC e^{\frac{r(\gamma_1 + \gamma_2)}{Kn}} \gamma_1^5 \gamma_2^5 (15 Kn^3 + 15 Kn^2 r \gamma_3 + 6 Kn r^2 \gamma_3^2 + r^3 \gamma_3^3) \right) + \frac{AA}{r^4}$$

$$\phi R26[r_]:= -\frac{1}{21 Pr \phi r^5 \gamma_1^5 \gamma_2^5 \gamma_3^5} 16 e^{-\frac{r(\gamma_1 + \gamma_2 + \gamma_3)}{Kn}} Kn$$

$$\left(e^{\frac{r}{Kn}} \left(e^{\frac{r}{Kn}} \left(-21 AA e^{\frac{r}{Kn}} \gamma_1^5 + ccA (105 Kn^4 + 105 Kn^3 r \gamma_1 + 45 Kn^2 r^2 \gamma_1^2 + 10 Kn r^3 \gamma_1^3 + r^4 \gamma_1^4) \right) \right. \right.$$

$$\left. \gamma_2^5 + ccB e^{\frac{r}{Kn}} \gamma_1^5 (105 Kn^4 + 105 Kn^3 r \gamma_2 + 45 Kn^2 r^2 \gamma_2^2 + 10 Kn r^3 \gamma_2^3 + r^4 \gamma_2^4) \right) \gamma_3^5 +$$

$$\left. ccC e^{\frac{r(\gamma_1 + \gamma_2)}{Kn}} \gamma_1^5 \gamma_2^5 (105 Kn^4 + 105 Kn^3 r \gamma_3 + 45 Kn^2 r^2 \gamma_3^2 + 10 Kn r^3 \gamma_3^3 + r^4 \gamma_3^4) \right)$$

$$RR26[r_]:= -\frac{28 cc1 Kn}{r^3} - \frac{56 cc2 Kn}{5 r^3} + \frac{7 AA PrM}{9 Kn r^3} - \frac{35 ccA e^{\frac{r(\gamma_2 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{9 r^3 \gamma_1^5} Kn^3 PrM -$$

$$\frac{35 ccA e^{\frac{r(\gamma_2 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{9 r^2 \gamma_1^4} Kn^2 PrM + \frac{7 ccA e^{\frac{r(\gamma_2 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{r^3 \gamma_1^3} Kn^3 + \frac{80 ccA e^{\frac{r(\gamma_2 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{9 Pr \phi r^3 \gamma_1^3} Kn^3 -$$

$$\frac{35 ccA e^{\frac{r(\gamma_2 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{27 r \gamma_1^3} Kn PrM + \frac{7 ccA e^{\frac{r(\gamma_2 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{r^2 \gamma_1^2} Kn^2 + \frac{80 ccA e^{\frac{r(\gamma_2 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{9 Pr \phi r^2 \gamma_1^2} Kn^2 +$$

$$\frac{7 ccA e^{\frac{r(\gamma_2 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{3 r \gamma_1} Kn + \frac{80 ccA e^{\frac{r(\gamma_2 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{27 Pr \phi r \gamma_1} Kn - \frac{35 ccB e^{\frac{r(\gamma_1 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{9 r^3 \gamma_2^5} Kn^3 PrM -$$

$$\frac{35 ccB e^{\frac{r(\gamma_1 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{9 r^2 \gamma_2^4} Kn^2 PrM + \frac{7 ccB e^{\frac{r(\gamma_1 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{r^3 \gamma_2^3} Kn^3 + \frac{80 ccB e^{\frac{r(\gamma_1 + \gamma_3)}{Kn}} - r(\gamma_1 + \gamma_2 + \gamma_3)}{9 Pr \phi r^3 \gamma_2^3} Kn^3 -$$

$$\begin{aligned}
& \frac{35 \text{ ccB } e^{\frac{r(\gamma_1+\gamma_3)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn PrM}}{27 r \gamma^2^3} + \frac{7 \text{ ccB } e^{\frac{r(\gamma_1+\gamma_3)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}^2}{r^2 \gamma^2^2} + \frac{80 \text{ ccB } e^{\frac{r(\gamma_1+\gamma_3)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}^2}{9 \text{ Pr}\phi r^2 \gamma^2^2} + \\
& \frac{7 \text{ ccB } e^{\frac{r(\gamma_1+\gamma_3)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}}{3 r \gamma^2} + \frac{80 \text{ ccB } e^{\frac{r(\gamma_1+\gamma_3)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}}{27 \text{ Pr}\phi r \gamma^2} - \frac{35 \text{ ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}^3 \text{PrM}}{9 r^3 \gamma^3^5} - \\
& \frac{35 \text{ ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}^2 \text{PrM}}{9 r^2 \gamma^3^4} + \frac{7 \text{ ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}^3}{r^3 \gamma^3^3} + \\
& \frac{80 \text{ ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}^3}{9 \text{ Pr}\phi r^3 \gamma^3^3} - \frac{35 \text{ ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn PrM}}{27 r \gamma^3^3} + \frac{7 \text{ ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}^2}{r^2 \gamma^3^2} + \\
& \frac{80 \text{ ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}^2}{9 \text{ Pr}\phi r^2 \gamma^3^2} + \frac{7 \text{ ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}}{3 r \gamma^3} + \frac{80 \text{ ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} - \frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}} \text{Kn}}{27 \text{ Pr}\phi r \gamma^3} \\
\psi_{R26}[r_-] := & - \frac{1}{35 \text{ Pr}\phi \text{Pr}\psi r^4 \gamma^1^5 \gamma^2^5 \gamma^3^5} e^{-\frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}}} \\
& \left(\left(\left(21 e^{\frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}}} \left(36 \left(5 \text{ cc}1 + 2 \text{ cc}2 \right) \text{Kn}^2 - 5 \text{AA PrM} \right) \text{Pr}\phi \gamma^1^5 + \text{ccA } e^{\frac{r(\gamma_2+\gamma_3)}{\text{Kn}}} \text{Kn} \left(35 \text{PrM Pr}\phi - \right. \right. \right. \\
& \left. \left. \left(80 + 63 \text{Pr}\phi \right) \gamma^1^2 \right) \left(15 \text{Kn}^3 + 15 \text{Kn}^2 r \gamma^1 + 6 \text{Kn } r^2 \gamma^1^2 + r^3 \gamma^1^3 \right) \right) \gamma^2^5 + \text{ccB } e^{\frac{r(\gamma_1+\gamma_3)}{\text{Kn}}} \text{Kn} \right. \\
& \left. \gamma^1^5 \left(35 \text{PrM Pr}\phi - \left(80 + 63 \text{Pr}\phi \right) \gamma^2^2 \right) \left(15 \text{Kn}^3 + 15 \text{Kn}^2 r \gamma^2 + 6 \text{Kn } r^2 \gamma^2^2 + r^3 \gamma^2^3 \right) \right) \\
& \gamma^3^5 + \text{ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} \text{Kn} \gamma^1^5 \gamma^2^5 \left(35 \text{PrM Pr}\phi - \left(80 + 63 \text{Pr}\phi \right) \gamma^3^2 \right) \\
& \left. \left(15 \text{Kn}^3 + 15 \text{Kn}^2 r \gamma^3 + 6 \text{Kn } r^2 \gamma^3^2 + r^3 \gamma^3^3 \right) \right) \\
\omega_{R26}[r_-] := & \frac{1}{252 \text{Kn}^2 \text{Pr}\phi \text{Pr}\psi r^2 \gamma^1^5 \gamma^2^5 \gamma^3^5} e^{-\frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}}} \\
& \left(\left(\left(-49 e^{\frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}}} \left(5 \left(36 \text{cc}1 \text{Kn}^2 - \text{AA PrM} \right) \text{PrR} + 72 \text{cc}2 \text{Kn}^2 \left(1 + \text{PrR} \right) \right) \text{Pr}\phi \text{Pr}\psi \gamma^1^5 - \right. \right. \right. \\
& \left. \left. \text{ccA } e^{\frac{r(\gamma_2+\gamma_3)}{\text{Kn}}} \text{Kn}^3 \left(\text{Kn} + r \gamma^1 \right) \left(1225 \text{PrM PrR Pr}\phi \text{Pr}\psi - \right. \right. \right. \\
& \left. \left. 35 \left(80 \text{PrR Pr}\psi + 9 \text{Pr}\phi \left(9 \text{PrM} + \left(2 + 7 \text{PrR} \right) \text{Pr}\psi \right) \right) \gamma^1^2 + 81 \left(80 + 63 \text{Pr}\phi \right) \gamma^1^4 \right) \right) \\
& \gamma^2^5 - \text{ccB } e^{\frac{r(\gamma_1+\gamma_3)}{\text{Kn}}} \text{Kn}^3 \gamma^1^5 \left(\text{Kn} + r \gamma^2 \right) \left(1225 \text{PrM PrR Pr}\phi \text{Pr}\psi - \right. \\
& \left. 35 \left(80 \text{PrR Pr}\psi + 9 \text{Pr}\phi \left(9 \text{PrM} + \left(2 + 7 \text{PrR} \right) \text{Pr}\psi \right) \right) \gamma^2^2 + 81 \left(80 + 63 \text{Pr}\phi \right) \gamma^2^4 \right) \right) \gamma^3^5 - \\
& \left. \text{ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} \text{Kn}^3 \gamma^1^5 \gamma^2^5 \left(\text{Kn} + r \gamma^3 \right) \left(1225 \text{PrM PrR Pr}\phi \text{Pr}\psi - \right. \right. \\
& \left. \left. 35 \left(80 \text{PrR Pr}\psi + 9 \text{Pr}\phi \left(9 \text{PrM} + \left(2 + 7 \text{PrR} \right) \text{Pr}\psi \right) \right) \gamma^3^2 + 81 \left(80 + 63 \text{Pr}\phi \right) \gamma^3^4 \right) \right) \\
\Delta_{R26}[r_-] := & - \frac{1}{252 \text{Pr}\Delta \text{Pr}\phi \text{Pr}\psi r \gamma^1^3 \gamma^2^3 \gamma^3^3} e^{-\frac{r(\gamma_1+\gamma_2+\gamma_3)}{\text{Kn}}} \text{Kn} \\
& \left(\left(\left(\text{ccA } e^{\frac{r(\gamma_2+\gamma_3)}{\text{Kn}}} \left(1225 \text{PrM PrR Pr}\phi \text{Pr}\psi - 35 \left(80 \text{PrR Pr}\psi + 9 \text{Pr}\phi \left(9 \text{PrM} + 2 \text{Pr}\psi + 7 \text{PrR Pr}\psi \right) \right) \gamma^1^2 + \right. \right. \right. \right. \\
& \left. \left. 81 \left(80 + 63 \text{Pr}\phi \right) \gamma^1^4 \right) \gamma^2^3 + \text{ccB } e^{\frac{r(\gamma_1+\gamma_3)}{\text{Kn}}} \gamma^1^3 \left(1225 \text{PrM PrR Pr}\phi \text{Pr}\psi - \right. \right. \\
& \left. \left. 35 \left(80 \text{PrR Pr}\psi + 9 \text{Pr}\phi \left(9 \text{PrM} + 2 \text{Pr}\psi + 7 \text{PrR Pr}\psi \right) \right) \gamma^2^2 + 81 \left(80 + 63 \text{Pr}\phi \right) \gamma^2^4 \right) \right) \gamma^3^3 + \\
& \left. \left. \text{ccC } e^{\frac{r(\gamma_1+\gamma_2)}{\text{Kn}}} \gamma^1^3 \gamma^2^3 \left(1225 \text{PrM PrR Pr}\phi \text{Pr}\psi - 35 \left(80 \text{PrR Pr}\psi + 9 \text{Pr}\phi \left(9 \text{PrM} + \left(2 + 7 \text{PrR} \right) \text{Pr}\psi \right) \right) \right. \right. \right. \\
& \left. \left. \gamma^3^2 + 81 \left(80 + 63 \text{Pr}\phi \right) \gamma^3^4 \right) \right) \right)
\end{aligned}$$

$$AA = \frac{1}{5 \text{ PrM PrR}} 36 \left(2 \text{ cc2 Kn}^2 + 5 \text{ cc1 Kn}^2 \text{ PrR} + 2 \text{ cc2 Kn}^2 \text{ PrR} \right);$$

OR26[r_] :=

$$\begin{aligned} & \text{cc3} + \frac{2 \text{ cc2 Pr}}{5 \text{ Kn r}} + \frac{\text{ccA} e^{-\frac{r \gamma_1}{\text{Kn}}} \text{Kn}}{r \gamma_1^3} \frac{1}{3780 \text{ Pr}\Delta \text{ Pr}\phi \text{ Pr}\psi} \left(245 \text{ PrM} (5 \text{ PrR} + 4 \text{ Pr}\Delta) \text{ Pr}\phi \text{ Pr}\psi - 35 \right. \\ & \quad \left. (81 \text{ PrM Pr}\phi + (80 \text{ PrR} + 64 \text{ Pr}\Delta + 9 (2 + 7 \text{ PrR} + 4 \text{ Pr}\Delta) \text{ Pr}\phi) \text{ Pr}\psi) \gamma_1^2 + 81 (80 + 63 \text{ Pr}\phi) \gamma_1^4 \right) + \\ & \quad \frac{1}{3780 \text{ Pr}\Delta \text{ Pr}\phi \text{ Pr}\psi r \gamma_2^3} \text{ccB} e^{-\frac{r \gamma_2}{\text{Kn}}} \text{Kn} \left(245 \text{ PrM} (5 \text{ PrR} + 4 \text{ Pr}\Delta) \text{ Pr}\phi \text{ Pr}\psi - 35 \right. \\ & \quad \left. (81 \text{ PrM Pr}\phi + (80 \text{ PrR} + 64 \text{ Pr}\Delta + 9 (2 + 7 \text{ PrR} + 4 \text{ Pr}\Delta) \text{ Pr}\phi) \text{ Pr}\psi) \gamma_2^2 + 81 (80 + 63 \text{ Pr}\phi) \gamma_2^4 \right) + \\ & \quad \frac{1}{3780 \text{ Pr}\Delta \text{ Pr}\phi \text{ Pr}\psi r \gamma_3^3} \text{ccC} e^{-\frac{r \gamma_3}{\text{Kn}}} \text{Kn} \left(245 \text{ PrM} (5 \text{ PrR} + 4 \text{ Pr}\Delta) \text{ Pr}\phi \text{ Pr}\psi - \right. \\ & \quad \left. 35 (81 \text{ PrM Pr}\phi + (80 \text{ PrR} + 64 \text{ Pr}\Delta + 9 (2 + 7 \text{ PrR} + 4 \text{ Pr}\Delta) \text{ Pr}\phi) \text{ Pr}\psi) \gamma_3^2 + 81 (80 + 63 \text{ Pr}\phi) \gamma_3^4 \right) \end{aligned}$$

(*Knudsen layers coefficients γ_i are zeros (positive) for the following polynomial*)

polynomial = FullSimplify[(3675 PrM PrR Pr Δ Pr ϕ Pr ψ - 35
 (3 Pr Δ (80 PrR + 9 (2 + 7 PrR) Pr ϕ) Pr ψ + PrM Pr ϕ (245 PrR Pr ψ + Pr Δ (243 + 112 Pr ψ))) γ_1^2 +
 (Pr Δ (80 + 63 Pr ϕ) (243 + 112 Pr ψ) + 245 (80 PrR Pr ψ + 9 Pr ϕ (9 PrM + (2 + 7 PrR) Pr ψ))) γ_1^4 -
 567 (80 + 63 Pr ϕ) γ_1^6), Assumptions \rightarrow
 {PrM > 0, PrR > 0, Pr ψ > 0, Pr Δ > 0, Pr ϕ > 0, γ_1 > 0, γ_2 > 0, γ_3 > 0, Kn > 0}];

(*Knudsen layers coefficients γ_i for MM*)

gammaMM =

$$\gamma_1 /. \text{NSolve}\left[\left(\text{polynomial} /. \left\{\text{PrR} \rightarrow \frac{7}{6}, \text{PrM} \rightarrow \frac{3}{2}, \text{Pr}\Delta \rightarrow \frac{2}{3}, \text{Pr}\phi \rightarrow \frac{21}{10}, \text{Pr}\psi \rightarrow \frac{17}{10}\right\}\right) == 0, \gamma_1\right]$$

(*Knudsen layers coefficients γ_i for BGK*)

gammaBGK =

$$\gamma_1 /. \text{NSolve}\left[\left(\text{polynomial} /. \left\{\text{PrR} \rightarrow 1, \text{PrM} \rightarrow 1, \text{Pr}\Delta \rightarrow 1, \text{Pr}\phi \rightarrow 1, \text{Pr}\psi \rightarrow 1\right\}\right) == 0, \gamma_1\right]$$

{-1.16321, -0.677347, -0.452587, 0.452587, 0.677347, 1.16321}

{-0.9427, -0.544841, -0.414501, 0.414501, 0.544841, 0.9427}

R26 equations KBC

cc3 = 0;

(* ν (=1) and χ (=1) are accommodation coefficients*)

(* θ_L temperature of liquid and psat is saturation pressure*)

(*Boundary conditions for R26*)

$$BC1 = \sqrt{\frac{2}{\pi}} \left(psat - pR26[1] - \frac{\sigma R26[1]}{2} + \frac{RR26[1]}{28} + \frac{1}{2} (\theta R26[1] - \theta_L) + \frac{\phi R26[1]}{24} + \frac{\Delta R26[1]}{120} \right) - \nu R26[1];$$

$$BC2 = -\sqrt{\frac{2}{\pi}} \left(2 \theta R26[1] - 2 \theta_L + \frac{\sigma R26[1]}{2} + \frac{5 RR26[1]}{28} + \frac{\Delta R26[1]}{15} - \frac{\phi R26[1]}{12} \right) - \frac{\nu R26[1]}{2} - qR26[1];$$

$$BC3 = \sqrt{\frac{2}{\pi}} \left(\frac{2}{5} \theta R26[1] - \frac{2}{5} \theta_L - \frac{7 \sigma R26[1]}{5} - \frac{RR26[1]}{14} + \frac{\Delta R26[1]}{75} - \frac{13 \phi R26[1]}{30} \right) - \frac{2 \nu R26[1]}{5} - mR26[1];$$

$$BC4 = \sqrt{\frac{2}{\pi}} \left(\frac{6}{5} \theta R26[1] - \frac{6}{5} \theta_L + \frac{9 \sigma R26[1]}{5} - \frac{93 RR26[1]}{70} + \frac{\Delta R26[1]}{5} - \frac{\phi R26[1]}{2} \right) + \frac{6 \nu R26[1]}{5} - \psi R26[1];$$

$$BC5 = \sqrt{\frac{2}{\pi}} \left(8 \theta R26[1] - 8 \theta_L + 2 \sigma R26[1] - RR26[1] - \frac{4 \Delta R26[1]}{3} \right) + 3 \nu R26[1] - \omega R26[1];$$

(*Pressure-driven case*)

(*cc1 \rightarrow c₁, cc2 \rightarrow c₂, ccA \rightarrow a₁, ccB \rightarrow a₂, ccC \rightarrow a₃*)

{cc1AKBC, cc2AKBC, ccAAKBC, ccBAKBC, ccCAKBC} = {cc1, cc2, ccA, ccB, ccC} /.

$$\text{Solve} \left[\left\{ \{BC1 == 0, BC2 == 0, BC3 == 0, BC4 == 0, BC5 == 0\} /. \left\{ PrR \rightarrow \frac{7}{6}, PrM \rightarrow \frac{3}{2}, Pr\Delta \rightarrow \frac{2}{3}, \right. \right. \right. \\ \left. \left. \left. Pr\phi \rightarrow \frac{21}{10}, Pr \rightarrow \frac{2}{3}, Pr\psi \rightarrow \frac{17}{10}, \gamma_1 \rightarrow \frac{4525871}{10^7}, \gamma_2 \rightarrow \frac{6773466}{10^7}, \gamma_3 \rightarrow \frac{11632054}{10^7}, \right. \right. \right. \\ \left. \left. \left. \nu \rightarrow 1, \chi \rightarrow 0, \theta_L \rightarrow 0, psat \rightarrow 1 \right\}, \{cc1, cc2, ccA, ccB, ccC\} \right] [[1]];$$

(*Temperature-driven case*)

{cc1BKBC, cc2BKBC, ccABKBC, ccBBKBC, ccCBKBC} = {cc1, cc2, ccA, ccB, ccC} /.

$$\text{Solve} \left[\left\{ \{BC1 == 0, BC2 == 0, BC3 == 0, BC4 == 0, BC5 == 0\} /. \left\{ PrR \rightarrow \frac{7}{6}, PrM \rightarrow \frac{3}{2}, Pr\Delta \rightarrow \frac{2}{3}, \right. \right. \right. \\ \left. \left. \left. Pr\phi \rightarrow \frac{21}{10}, Pr \rightarrow \frac{2}{3}, Pr\psi \rightarrow \frac{17}{10}, \gamma_1 \rightarrow \frac{4525871}{10^7}, \gamma_2 \rightarrow \frac{6773466}{10^7}, \gamma_3 \rightarrow \frac{11632054}{10^7}, \right. \right. \right. \\ \left. \left. \left. \nu \rightarrow 1, \chi \rightarrow 0, \theta_L \rightarrow 1, psat \rightarrow 0 \right\}, \{cc1, cc2, ccA, ccB, ccC\} \right] [[1]];$$

```
{c1pR26, c2pR26} = {cc1AKBC, cc2AKBC} // Simplify // N // Chop // Simplify;
{c1tR26, c2tR26} = {cc1BKBC, cc2BKBC} // Simplify // N // Chop // Simplify;
```

$$\left\{ \frac{\text{Simplify}\left[\frac{\text{Numerator}[c1pR26]}{10^{95}}\right]}{\text{Simplify}\left[\frac{\text{Denominator}[c1pR26]}{10^{95}}\right]}, \frac{\text{Expand}\left[\text{Simplify}\left[\frac{\text{Numerator}[c2pR26]}{10^{187}}\right]\right]}{\text{Expand}\left[\text{Simplify}\left[\frac{\text{Denominator}[c2pR26]}{10^{187}}\right]\right]} \right\}$$

$$\left\{ \frac{\text{Expand}\left[\text{Simplify}\left[\frac{\text{Numerator}[c1tR26]}{10^{95}}\right]\right]}{\text{Expand}\left[\text{Simplify}\left[\frac{\text{Denominator}[c1tR26]}{10^{95}}\right]\right]}, \frac{\text{Expand}\left[\text{Simplify}\left[\frac{\text{Numerator}[c2pR26]}{10^{188}}\right]\right]}{\text{Expand}\left[\text{Simplify}\left[\frac{\text{Denominator}[c2pR26]}{10^{188}}\right]\right]} \right\}$$

$$\left\{ \begin{aligned} & (2.69257 + 43.9094 \text{ Kn} + 356.175 \text{ Kn}^2 + 1842.67 \text{ Kn}^3 + 6474.32 \text{ Kn}^4 + 15700.4 \text{ Kn}^5 + \\ & 26065.7 \text{ Kn}^6 + 28306. \text{ Kn}^7 + 18370.3 \text{ Kn}^8 + 6499.79 \text{ Kn}^9 + 989.458 \text{ Kn}^{10}) / \\ & (4.02358 + 71.0889 \text{ Kn} + 617.476 \text{ Kn}^2 + 3385.77 \text{ Kn}^3 + 12566.7 \text{ Kn}^4 + 32253.5 \text{ Kn}^5 + \\ & 56661.1 \text{ Kn}^6 + 65440.8 \text{ Kn}^7 + 47404.9 \text{ Kn}^8 + 19296.8 \text{ Kn}^9 + 3255.27 \text{ Kn}^{10}), \\ & (4.27841 \text{ Kn} + 155.807 \text{ Kn}^2 + 2491.79 \text{ Kn}^3 + 23574.2 \text{ Kn}^4 + 144429. \text{ Kn}^5 + 565725. \text{ Kn}^6 + \\ & 1.10855 \times 10^6 \text{ Kn}^7 - 1.79793 \times 10^6 \text{ Kn}^8 - 2.27751 \times 10^7 \text{ Kn}^9 - 9.43704 \times 10^7 \text{ Kn}^{10} - 2.53285 \times 10^8 \\ & \text{Kn}^{11} - 4.91939 \times 10^8 \text{ Kn}^{12} - 7.1446 \times 10^8 \text{ Kn}^{13} - 7.79622 \times 10^8 \text{ Kn}^{14} - 6.31224 \times 10^8 \text{ Kn}^{15} - \\ & 3.68081 \times 10^8 \text{ Kn}^{16} - 1.46015 \times 10^8 \text{ Kn}^{17} - 3.51237 \times 10^7 \text{ Kn}^{18} - 3.83582 \times 10^6 \text{ Kn}^{19}) / \\ & (-5.33762 - 198.481 \text{ Kn} - 3275.23 \text{ Kn}^2 - 32462.7 \text{ Kn}^3 - 213671. \text{ Kn}^4 - 944102. \text{ Kn}^5 - \\ & 2.47579 \times 10^6 \text{ Kn}^6 - 502697. \text{ Kn}^7 + 2.93572 \times 10^7 \text{ Kn}^8 + 1.57513 \times 10^8 \text{ Kn}^9 + \\ & 4.99246 \times 10^8 \text{ Kn}^{10} + 1.11766 \times 10^9 \text{ Kn}^{11} + 1.85439 \times 10^9 \text{ Kn}^{12} + \\ & 2.3143 \times 10^9 \text{ Kn}^{13} + 2.17116 \times 10^9 \text{ Kn}^{14} + 1.51085 \times 10^9 \text{ Kn}^{15} + 7.57155 \times 10^8 \text{ Kn}^{16} + \\ & 2.58207 \times 10^8 \text{ Kn}^{17} + 5.34788 \times 10^7 \text{ Kn}^{18} + 5.04786 \times 10^6 \text{ Kn}^{19}) \end{aligned} \right\}$$

$$\left\{ \begin{aligned} & (-3.24171 \text{ Kn} - 54.5877 \text{ Kn}^2 - 434.887 \text{ Kn}^3 - 2059.37 \text{ Kn}^4 - 6293.88 \text{ Kn}^5 - \\ & 13067.7 \text{ Kn}^6 - 18587.9 \text{ Kn}^7 - 17408.6 \text{ Kn}^8 - 9523.45 \text{ Kn}^9 - 2102.6 \text{ Kn}^{10}) / \\ & (4.02358 + 71.0889 \text{ Kn} + 617.476 \text{ Kn}^2 + 3385.77 \text{ Kn}^3 + 12566.7 \text{ Kn}^4 + 32253.5 \text{ Kn}^5 + \\ & 56661.1 \text{ Kn}^6 + 65440.8 \text{ Kn}^7 + 47404.9 \text{ Kn}^8 + 19296.8 \text{ Kn}^9 + 3255.27 \text{ Kn}^{10}), \\ & (0.427841 \text{ Kn} + 15.5807 \text{ Kn}^2 + 249.179 \text{ Kn}^3 + 2357.42 \text{ Kn}^4 + 14442.9 \text{ Kn}^5 + 56572.5 \text{ Kn}^6 + \\ & 110855. \text{ Kn}^7 - 179793. \text{ Kn}^8 - 2.27751 \times 10^6 \text{ Kn}^9 - 9.43704 \times 10^6 \text{ Kn}^{10} - 2.53285 \times 10^7 \text{ Kn}^{11} - \\ & 4.91939 \times 10^7 \text{ Kn}^{12} - 7.1446 \times 10^7 \text{ Kn}^{13} - 7.79622 \times 10^7 \text{ Kn}^{14} - 6.31224 \times 10^7 \text{ Kn}^{15} - \\ & 3.68081 \times 10^7 \text{ Kn}^{16} - 1.46015 \times 10^7 \text{ Kn}^{17} - 3.51237 \times 10^6 \text{ Kn}^{18} - 383582. \text{ Kn}^{19}) / \\ & (-0.533762 - 19.8481 \text{ Kn} - 327.523 \text{ Kn}^2 - 3246.27 \text{ Kn}^3 - 21367.1 \text{ Kn}^4 - 94410.2 \text{ Kn}^5 - \\ & 247579. \text{ Kn}^6 - 50269.7 \text{ Kn}^7 + 2.93572 \times 10^6 \text{ Kn}^8 + 1.57513 \times 10^7 \text{ Kn}^9 + 4.99246 \times 10^7 \text{ Kn}^{10} + \\ & 1.11766 \times 10^8 \text{ Kn}^{11} + 1.85439 \times 10^8 \text{ Kn}^{12} + 2.3143 \times 10^8 \text{ Kn}^{13} + 2.17116 \times 10^8 \text{ Kn}^{14} + 1.51085 \times 10^8 \\ & \text{Kn}^{15} + 7.57155 \times 10^7 \text{ Kn}^{16} + 2.58207 \times 10^7 \text{ Kn}^{17} + 5.34788 \times 10^6 \text{ Kn}^{18} + 504786. \text{ Kn}^{19}) \end{aligned} \right\}$$