Supplemental material

Material derivative in added mass force

The form of the Basset-Boussinesq-Oseen equation that considers only drag and added mass is given by:

$$\rho_p \frac{4\pi R_p^3}{3} \frac{\mathrm{d}\boldsymbol{U}_p}{\mathrm{d}t} = \underbrace{6\pi\mu_f R_p \left(\boldsymbol{U}_f - \boldsymbol{U}_p\right)}_{\text{drag}} + \underbrace{\frac{\rho_f}{2} \frac{4\pi R_p^3}{3} \left(\frac{\mathrm{D}\boldsymbol{U}_f}{\mathrm{D}t} - \frac{\mathrm{d}\boldsymbol{U}_p}{\mathrm{d}t}\right)}_{\text{added mass}} \tag{1}$$

This can be rewritten as:

$$\left(\rho_p + \frac{\rho_f}{2}\right) \frac{4\pi R_p^3}{3} \frac{\mathrm{d}\boldsymbol{U}_p}{\mathrm{d}t} = 6\pi\mu_f R_p \left(\boldsymbol{U}_f - \boldsymbol{U}_p\right) + \frac{\rho_f}{2} \frac{4\pi R_p^3}{3} \frac{\mathrm{D}\boldsymbol{U}_f}{\mathrm{D}t} \tag{2}$$

Neglecting the material derivative in the added mass term is reasonable if:

$$6\pi\mu_f R_p \left(\boldsymbol{U}_f - \boldsymbol{U}_p\right) \gg \frac{\rho_f}{2} \frac{4\pi R_p^3}{3} \frac{\mathrm{D}\boldsymbol{U}_f}{\mathrm{D}t} \tag{3}$$

that is, if:

$$\mathcal{F} = \frac{\rho_f R_p^2 \frac{\mathrm{D}\boldsymbol{U}_f}{\mathrm{D}t}}{9\mu_f \left(\boldsymbol{U}_f - \boldsymbol{U}_p\right)} \ll 1 \tag{4}$$

An order of magnitude estimate of $\mathrm{D}\boldsymbol{U}_f/\mathrm{D}t$ is given by:

$$\left|\frac{\mathrm{D}\boldsymbol{U}_f}{\mathrm{D}t}\right| \approx \frac{u'^2}{l} \approx \frac{10 u_\nabla^2}{L} \tag{5}$$

where l is the length scale of the energy-carrying large eddies and L is the largest flow-domain dimension perpendicular to the wall ($2R$ in the pipe and H in the channel).

Since the mean velocity of the particles is of the order of the mean velocity of the fluid, the order of magnitude of $(\boldsymbol{U}_f - \boldsymbol{U}_p)$ is given by u_∇. This is confirmed by the results shown in the paper in Figure 5. We thus find for $\mathcal{F}$:

$$\mathcal{F} \approx \frac{1}{9} \left(\frac{R_p}{L}\right)^2 Re_\nabla \tag{6}$$

which for all cases that are considered is indeed much smaller than unity ($1.0 \cdot 10^{-3}$ for $Re_\nabla = 360$, $2.0 \cdot 10^{-3}$ for $Re_\nabla = 720$). Thus, neglecting the material derivative in the added mass term is reasonable.

Lift force

Considering Saffman-Lift, the analysis for the lift force is as follows:

$$F_s = 6.46 \rho_f R_p^2 \sqrt{\left(\frac{\mu_f}{\rho_f \left|\mathcal{K}\right|}\right)} \left[\mathcal{K} \times \left(\boldsymbol{U}_f - \boldsymbol{U}_p\right)\right] \tag{7}$$

where $\mathcal{K}$ is the curl of the fluid velocity field. Neglecting the lift force is reasonable if:

$$6\pi\mu_f R_p \left(\boldsymbol{U}_f - \boldsymbol{U}_p\right) \gg 6.46 \rho_f R_p^2 \sqrt{\left(\frac{\mu_f}{\rho_f \left|\mathcal{K}\right|}\right)} \left[\mathcal{K} \times \left(\boldsymbol{U}_f - \boldsymbol{U}_p\right)\right] \tag{8}$$

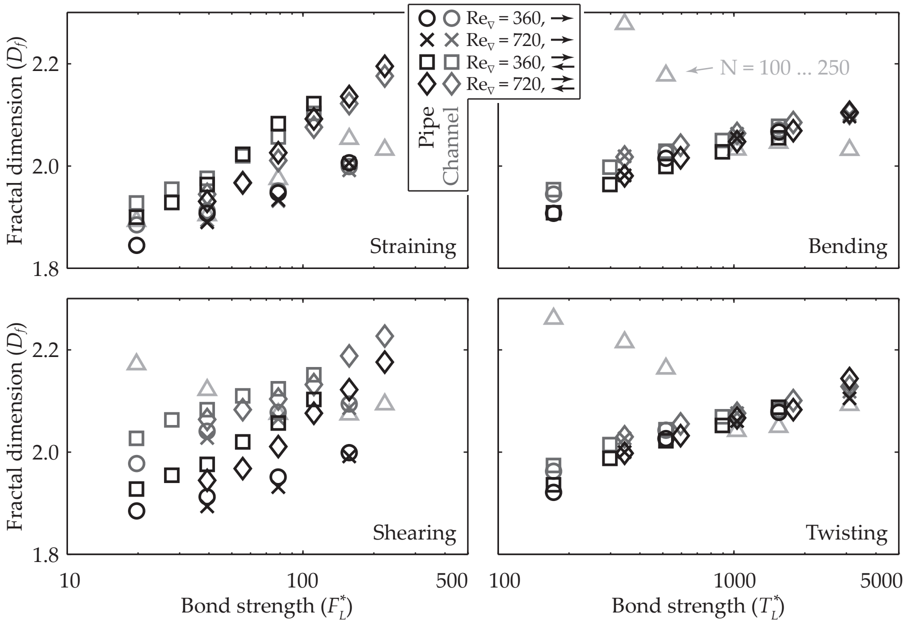

FIGURE 1. Overall fractal dimensions for steady-state agglomerate populations for the different cases considered, amended by average fractal dimensions for a subset of agglomerates (with $N = 100 - 250$).

or

$$\mathcal{F} = \frac{\rho_f R_p \sqrt{\left(\frac{\mu_f}{\rho_f |\mathcal{K}|}\right)} \left[\mathcal{K} \times (\boldsymbol{U}_f - \boldsymbol{U}_p)\right]}{2.92 \mu_f \left(\boldsymbol{U}_f - \boldsymbol{U}_p\right)} \ll 1 \tag{9}$$

By using

$$|\mathcal{K}| \approx \frac{u'}{l} \approx \frac{10 u_\nabla}{L} \tag{10}$$

this leads to:

$$\mathcal{F} \approx 1.08 \frac{R_p}{L} \sqrt{Re_\nabla} \tag{11}$$

which is also smaller than one for the cases considered in the paper (0.10 at $Re_\nabla = 360$ and 0.14 at $Re_\nabla = 720$). Thus, neglecting the lift force is reasonable as well.

Fractal dimension

Figure 1 shows Figure 4 from the paper amended with the results obtained in channel flow.