

Subdomain model (SK)_1D

In[809]:=

```
Remove["Global`*"]

(* Derivatives of the coordinate transformation *)
sigmat = 1/H[x, t] * (D[h[x, t], t] -  $\sigma$  * Ht);
sigmax = 1/H[x, t] * (D[h[x, t], x] -  $\sigma$  * D[H[x, t], x]);
sigmaz = 1/H[x, t];
(*****)
```



```
(* Horizontal velocity assumption
(Velocity unknowns are denoted as  $U_i$ ) *)
K = 2; (*K needs to be specified*)
 $c_0 = 0$ ;
 $c_K = 1$ ;
Do[ $N_i = \sigma^i$ , {i, 0, 5}];
Do[ $UC_i = U_i[x, t] * N_i$ , {i, 0, K - 1, 1}];
u = Sum[ $UC_i$ , {i, 0, K - 1}];
(*****)
```



```
(* Vertical velocity expression*)
ub = u /.  $\sigma \rightarrow 0$ ;
wb = -D[h[x, t], t] - ub * D[h[x, t], x];
w = wb - 1/sigmaz * Integrate[D[u, x] + D[u,  $\sigma$ ] * sigmax, { $\sigma$ , 0,  $\sigma$ ]];
(*****)
```



```
(* Depth-integaretd continuity equation *)
fluxx = Integrate[u, { $\sigma$ , 0, 1}]/sigmaz;
Ht = -D[fluxx, x];
(*****)
```

In[824]:=

```

Derivative[0, 1][H][x, t] = -D[fluxx, x];
Derivative[1, 1][H][x, t] = D[-D[fluxx, x], x];
Derivative[2, 1][H][x, t] = D[D[-D[fluxx, x], x], x];

(* Vertical momentum equation to get pressure field*)
wm = D[w, t] + D[w, σ] * sigmat + u * (D[w, x] + D[w, σ] * sigmax) + w * D[w, σ] * sigmaz;
ps = (-ρ * g - ρ * wm) / sigmaz;
cf = CoefficientList[wm, σ];
order = Length[cf];
pnh = Sum[Part[cf, i] * ((1 - σi) / i), {i, 1, order}];
p = (ρ * g * (1 - σ) + ρ * pnh) / sigmaz;    (* PRESSURE FIELD*)
(*****)

(* LHS of the horizontal momentum equation to form weighted residuals *)
LHS = D[u, t] + D[u, σ] * sigmat + u * (D[u, x] + D[u, σ] * sigmax) + w * D[u, σ] * sigmaz;
lcf = CoefficientList[LHS, σ];
lorder = Length[lcf];
Do[MomLhsi = Sum[Part[lcf, j] * ((cij - ci-1j) / j), {j, 1, lorder}], {i, 1, K}];
(*****)

(* RHS of the horizontal momentum equation to form weighted residuals *)
RHS = -1 / ρ * (D[p, x] + ps * (D[h[x, t], x] - σ * D[H[x, t], x]) / H[x, t]);
cf = CoefficientList[RHS, σ];
order = Length[cf];
Do[MomRhsi = Sum[Part[cf, j] * ((cij - ci-1j) / j), {j, 1, order}], {i, 1, K}];
(*****)

(* Full equation *)
Do[XEQi = MomLhsi - MomRhsi, {i, 1, K}];
(*****)

```

In[842]:= (* Print out the full equation*)

```

(* 1. Continuity equation with ContinuityLHS=H_t *)
Print[Ht]

```

$$-\left(u_0[x, t] + \frac{1}{2} u_1[x, t]\right) H^{(1,0)}[x, t] - H[x, t] \left(u_0^{(1,0)}[x, t] + \frac{1}{2} u_1^{(1,0)}[x, t]\right)$$

In[843]:=

(* 2. K number of weighted horizontal momentum equations *)

Do[Print[Collect[-XEQ_i, _[x, t], Simplify]], {i, 1, K, 1}]

(* ***** THE END ***** *)

$$\begin{aligned}
& c_1 h^{(0,2)}[x, t] H^{(1,0)}[x, t] + \frac{c_1^4 (-5 + 4 c_1) U_1[x, t]^2 H^{(1,0)}[x, t]^3}{40 H[x, t]} + \\
& U_0[x, t] \left(-c_1 U_0^{(1,0)}[x, t] - \frac{1}{2} c_1^2 U_1^{(1,0)}[x, t] \right) + \\
& U_1[x, t] \left(-\frac{1}{2} c_1^2 U_0^{(1,0)}[x, t] - \frac{1}{12} c_1^2 (3 + 2 c_1) U_1^{(1,0)}[x, t] \right) + \\
& H^{(1,0)}[x, t]^2 \left(-\frac{1}{12} c_1 (2 + c_1^3) U_0[x, t] U_1^{(1,0)}[x, t] + \right. \\
& \quad \left. U_1[x, t] \left(\frac{1}{4} c_1 (2 + c_1^3) U_0^{(1,0)}[x, t] + \frac{1}{40} (5 - 2 c_1) c_1^4 U_1^{(1,0)}[x, t] \right) \right) + h^{(1,0)}[x, t]^2 \\
& \left(\frac{c_1^2 (-3 + 2 c_1) U_1[x, t]^2 H^{(1,0)}[x, t]}{12 H[x, t]} + U_0[x, t] \left(-c_1 U_0^{(1,0)}[x, t] - \frac{1}{2} c_1^2 U_1^{(1,0)}[x, t] \right) + \right. \\
& \quad \left. U_1[x, t] \left(-\frac{1}{2} c_1^2 U_0^{(1,0)}[x, t] - \frac{1}{12} c_1^2 (3 + 2 c_1) U_1^{(1,0)}[x, t] \right) \right) + U_0^{(0,1)}[x, t] \\
& \left(-c_1 - c_1 h^{(1,0)}[x, t]^2 + c_1 h^{(1,0)}[x, t] H^{(1,0)}[x, t] - \frac{1}{2} H[x, t] (-2 + c_1) c_1 h^{(2,0)}[x, t] \right) + \\
& U_1^{(0,1)}[x, t] \left(-\frac{c_1^2}{2} - \frac{1}{2} c_1^2 h^{(1,0)}[x, t]^2 + \frac{1}{6} c_1 (3 + 2 c_1^2) h^{(1,0)}[x, t] H^{(1,0)}[x, t] - \frac{1}{12} c_1 (2 + c_1^3) \right. \\
& \quad \left. H^{(1,0)}[x, t]^2 + H[x, t] \left(-\frac{1}{6} c_1 (-3 + c_1^2) h^{(2,0)}[x, t] + \frac{1}{24} c_1 (-4 + c_1^3) H^{(2,0)}[x, t] \right) \right) + \\
& h^{(1,0)}[x, t] \left(g c_1 - c_1 h^{(0,2)}[x, t] + \frac{(4 - 3 c_1) c_1^3 U_1[x, t]^2 H^{(1,0)}[x, t]^2}{12 H[x, t]} + \right. \\
& \quad H^{(1,0)}[x, t] \left(U_0[x, t] \left(c_1 U_0^{(1,0)}[x, t] + \frac{1}{6} c_1 (3 + 2 c_1^2) U_1^{(1,0)}[x, t] \right) + \right. \\
& \quad \left. U_1[x, t] \left(\frac{1}{6} c_1 (3 - 2 c_1^2) U_0^{(1,0)}[x, t] + \frac{1}{12} c_1 (7 - 2 c_1^2 + 2 c_1^3) U_1^{(1,0)}[x, t] \right) \right) - \\
& 2 c_1 U_0[x, t] h^{(1,1)}[x, t] - c_1 U_0[x, t]^2 h^{(2,0)}[x, t] + \\
& U_1[x, t] (-c_1^2 h^{(1,1)}[x, t] - c_1^2 U_0[x, t] h^{(2,0)}[x, t]) + \\
& U_1[x, t]^2 \left(-\frac{1}{3} c_1^3 h^{(2,0)}[x, t] + \frac{1}{12} (c_1 - 2 c_1^3 + 2 c_1^4) H^{(2,0)}[x, t] \right) + \\
& H[x, t] \left(c_1 U_0^{(1,0)}[x, t]^2 + c_1 U_0^{(1,0)}[x, t] U_1^{(1,0)}[x, t] + \frac{1}{12} c_1 (5 - 2 c_1^2 + c_1^3) U_1^{(1,0)}[x, t]^2 - \right. \\
& \quad (-1 + c_1) c_1 U_0^{(1,1)}[x, t] + \frac{1}{6} c_1 (3 - 2 c_1^2) U_1^{(1,1)}[x, t] + \\
& \quad U_0[x, t] \left(-(-1 + c_1) c_1 U_0^{(2,0)}[x, t] + \frac{1}{6} c_1 (3 - 2 c_1^2) U_1^{(2,0)}[x, t] \right) + \\
& \quad \left. U_1[x, t] \left(\frac{1}{6} c_1 (3 - 4 c_1^2) U_0^{(2,0)}[x, t] + \frac{1}{12} (5 c_1 - 2 c_1^3 - 2 c_1^4) U_1^{(2,0)}[x, t] \right) \right) + \\
& H^{(1,0)}[x, t] \left(-g c_1 + \frac{c_1^2 (-3 + 2 c_1) U_1[x, t]^2}{12 H[x, t]} + 2 c_1 U_0[x, t] h^{(1,1)}[x, t] + \right. \\
& \quad c_1 U_0[x, t]^2 h^{(2,0)}[x, t] + U_1[x, t] \left(\left(c_1 + \frac{c_1^3}{3} \right) h^{(1,1)}[x, t] + \left(c_1 + \frac{c_1^3}{3} \right) U_0[x, t] h^{(2,0)}[x, t] \right) + \\
& \quad \left. U_1[x, t]^2 \left(\frac{1}{24} c_1 (10 - 2 c_1^2 + 5 c_1^3) h^{(2,0)}[x, t] + \frac{1}{8} (-c_1 + c_1^4 - c_1^5) H^{(2,0)}[x, t] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& H[x, t] \left(-c_1 U_0^{(1,0)}[x, t]^2 - \frac{1}{6} c_1 (2 + c_1^3) U_0^{(1,0)}[x, t] U_1^{(1,0)}[x, t] - \right. \\
& \quad \frac{1}{20} c_1^5 U_1^{(1,0)}[x, t]^2 + c_1 U_0^{(1,1)}[x, t] + \frac{1}{12} c_1 (2 + c_1^3) U_1^{(1,1)}[x, t] + \\
& \quad U_0[x, t] \left(c_1 U_0^{(2,0)}[x, t] + \frac{1}{12} c_1 (2 + c_1^3) U_1^{(2,0)}[x, t] \right) + \\
& \quad \left. U_1[x, t] \left(\frac{3}{2} c_1 U_0^{(2,0)}[x, t] + \frac{1}{20} c_1 (10 + c_1^4) U_1^{(2,0)}[x, t] \right) \right) + \\
& H[x, t] \left(-(-2 + c_1) c_1 U_0^{(1,0)}[x, t] h^{(1,1)}[x, t] + \left(c_1 - \frac{c_1^3}{3} \right) U_1^{(1,0)}[x, t] h^{(1,1)}[x, t] - \right. \\
& \quad \frac{1}{2} (-2 + c_1) c_1 h^{(1,2)}[x, t] + U_0[x, t] \left(-\frac{3}{2} (-2 + c_1) c_1 U_0^{(1,0)}[x, t] h^{(2,0)}[x, t] + \right. \\
& \quad U_1^{(1,0)}[x, t] \left(-\frac{1}{2} c_1 (-3 + c_1^2) h^{(2,0)}[x, t] + \frac{1}{24} c_1 (-4 + c_1^3) H^{(2,0)}[x, t] \right) - \\
& \quad \left. (-2 + c_1) c_1 h^{(2,1)}[x, t] \right) - \frac{1}{2} (-2 + c_1) c_1 U_0[x, t]^2 h^{(3,0)}[x, t] + \\
& \quad U_1[x, t] \left(U_1^{(1,0)}[x, t] \left(\frac{1}{24} (26 c_1 - 2 c_1^3 - 5 c_1^4) h^{(2,0)}[x, t] + \frac{1}{80} c_1^4 (-5 + 4 c_1) H^{(2,0)}[x, t] \right) + \right. \\
& \quad U_0^{(1,0)}[x, t] \left(-\frac{1}{2} c_1 (-3 + c_1^2) h^{(2,0)}[x, t] - \frac{1}{8} c_1 (-4 + c_1^3) H^{(2,0)}[x, t] \right) + \\
& \quad \left(c_1 - \frac{c_1^3}{3} \right) h^{(2,1)}[x, t] + \left(c_1 - \frac{c_1^3}{3} \right) U_0[x, t] h^{(3,0)}[x, t] \Big) + \\
& \quad U_1[x, t]^2 \left(-\frac{1}{12} c_1 (-4 + c_1^3) h^{(3,0)}[x, t] + \frac{1}{240} c_1 (-10 - 5 c_1^3 + 6 c_1^4) H^{(3,0)}[x, t] \right) \Big) + \\
& H[x, t]^2 \left(U_0^{(1,0)}[x, t] \left(\frac{1}{6} c_1 (-3 + c_1^2) U_0^{(2,0)}[x, t] + \frac{1}{24} c_1 (-4 + c_1^3) U_1^{(2,0)}[x, t] \right) + \right. \\
& \quad U_1^{(1,0)}[x, t] \left(-\frac{1}{24} c_1 (-4 + c_1^3) U_0^{(2,0)}[x, t] + \frac{1}{80} c_1 (10 - 5 c_1^3 + 2 c_1^4) U_1^{(2,0)}[x, t] \right) - \\
& \quad \frac{1}{6} c_1 (-3 + c_1^2) U_0^{(2,1)}[x, t] - \frac{1}{24} c_1 (-4 + c_1^3) U_1^{(2,1)}[x, t] + \\
& \quad U_0[x, t] \left(-\frac{1}{6} c_1 (-3 + c_1^2) U_0^{(3,0)}[x, t] - \frac{1}{24} c_1 (-4 + c_1^3) U_1^{(3,0)}[x, t] \right) + \\
& \quad \left. U_1[x, t] \left(-\frac{1}{8} c_1 (-4 + c_1^3) U_0^{(3,0)}[x, t] + \frac{1}{240} (50 c_1 - 5 c_1^4 - 6 c_1^5) U_1^{(3,0)}[x, t] \right) \right) \Big) \\
& (1 - c_1) h^{(0,2)}[x, t] H^{(1,0)}[x, t] + \frac{(-1 + 5 c_1^4 - 4 c_1^5) U_1[x, t]^2 H^{(1,0)}[x, t]^3}{40 H[x, t]} + \\
& U_0[x, t] \left((-1 + c_1) U_0^{(1,0)}[x, t] + \frac{1}{2} (-1 + c_1^2) U_1^{(1,0)}[x, t] \right) + \\
& U_1[x, t] \left(\frac{1}{2} (-1 + c_1^2) U_0^{(1,0)}[x, t] + \frac{1}{12} (-5 + 3 c_1^2 + 2 c_1^3) U_1^{(1,0)}[x, t] \right) + \\
& h^{(1,0)}[x, t]^2 \left(-\frac{(-1 + c_1)^2 (1 + 2 c_1) U_1[x, t]^2 H^{(1,0)}[x, t]}{12 H[x, t]} + \right. \\
& \quad U_0[x, t] \left((-1 + c_1) U_0^{(1,0)}[x, t] + \frac{1}{2} (-1 + c_1^2) U_1^{(1,0)}[x, t] \right) + \\
& \quad \left. U_1[x, t] \left(\frac{1}{2} (-1 + c_1^2) U_0^{(1,0)}[x, t] + \frac{1}{12} (-5 + 3 c_1^2 + 2 c_1^3) U_1^{(1,0)}[x, t] \right) \right) + \\
& H^{(1,0)}[x, t]^2 \left(\frac{1}{12} (-3 + 2 c_1 + c_1^4) U_0[x, t] U_1^{(1,0)}[x, t] + \right. \\
& \quad U_1[x, t] \left(\frac{1}{4} (3 - 2 c_1 - c_1^4) U_0^{(1,0)}[x, t] + \frac{1}{40} (3 - 5 c_1^4 + 2 c_1^5) U_1^{(1,0)}[x, t] \right) \Big) + \\
& U_0^{(0,1)}[x, t] \left(-1 + c_1 + (-1 + c_1) h^{(1,0)}[x, t]^2 + (1 - c_1) h^{(1,0)}[x, t] H^{(1,0)}[x, t] + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} H[x, t] (-1 + c_1)^2 h^{(2,0)}[x, t] \Big) + U_1^{(0,1)}[x, t] \left(\frac{1}{2} (-1 + c_1^2) + \frac{1}{2} (-1 + c_1^2) h^{(1,0)}[x, t]^2 + \right. \\
& \frac{1}{6} (5 - 3 c_1 - 2 c_1^3) h^{(1,0)}[x, t] H^{(1,0)}[x, t] + \frac{1}{12} (-3 + 2 c_1 + c_1^4) H^{(1,0)}[x, t]^2 + \\
& H[x, t] \left(\frac{1}{6} (2 - 3 c_1 + c_1^3) h^{(2,0)}[x, t] + \frac{1}{24} (-3 + 4 c_1 - c_1^4) H^{(2,0)}[x, t] \right) \Big) + \\
& h^{(1,0)}[x, t] \left(g - g c_1 + (-1 + c_1) h^{(0,2)}[x, t] + \frac{(1 - 4 c_1^3 + 3 c_1^4) U_1[x, t]^2 H^{(1,0)}[x, t]^2}{12 H[x, t]} + \right. \\
& H^{(1,0)}[x, t] \left(U_0[x, t] \left((1 - c_1) U_0^{(1,0)}[x, t] + \frac{1}{6} (5 - 3 c_1 - 2 c_1^3) U_1^{(1,0)}[x, t] \right) + \right. \\
& U_1[x, t] \left(\frac{1}{6} (1 - 3 c_1 + 2 c_1^3) U_0^{(1,0)}[x, t] + \frac{1}{12} (7 - 7 c_1 + 2 c_1^3 - 2 c_1^4) U_1^{(1,0)}[x, t] \right) \Big) + \\
& 2 (-1 + c_1) U_0[x, t] h^{(1,1)}[x, t] + (-1 + c_1) U_0[x, t]^2 h^{(2,0)}[x, t] + \\
& U_1[x, t] \left((-1 + c_1^2) h^{(1,1)}[x, t] + (-1 + c_1^2) U_0[x, t] h^{(2,0)}[x, t] \right) + \\
& U_1[x, t]^2 \left(\frac{1}{3} (-1 + c_1^3) h^{(2,0)}[x, t] + \frac{1}{12} (1 - c_1 + 2 c_1^3 - 2 c_1^4) H^{(2,0)}[x, t] \right) + \\
& H[x, t] \left((1 - c_1) U_0^{(1,0)}[x, t]^2 + (1 - c_1) U_0^{(1,0)}[x, t] U_1^{(1,0)}[x, t] + \frac{1}{12} (4 - 5 c_1 + 2 c_1^3 - c_1^4) \right. \\
& U_1^{(1,0)}[x, t]^2 + (-1 + c_1) c_1 U_0^{(1,1)}[x, t] + \frac{1}{6} (1 - 3 c_1 + 2 c_1^3) U_1^{(1,1)}[x, t] + \\
& U_0[x, t] \left((-1 + c_1) c_1 U_0^{(2,0)}[x, t] + \frac{1}{6} (1 - 3 c_1 + 2 c_1^3) U_1^{(2,0)}[x, t] \right) + \\
& U_1[x, t] \left(\frac{1}{6} (-1 + c_1) (1 + 2 c_1)^2 U_0^{(2,0)}[x, t] + \frac{1}{12} (1 - 5 c_1 + 2 c_1^3 + 2 c_1^4) U_1^{(2,0)}[x, t] \right) \Big) \Big) + \\
& H^{(1,0)}[x, t] \left(g (-1 + c_1) - \frac{(-1 + c_1)^2 (1 + 2 c_1) U_1[x, t]^2}{12 H[x, t]} + (2 - 2 c_1) U_0[x, t] h^{(1,1)}[x, t] + \right. \\
& (1 - c_1) U_0[x, t]^2 h^{(2,0)}[x, t] + \\
& U_1[x, t] \left(\frac{1}{3} (4 - 3 c_1 - c_1^3) h^{(1,1)}[x, t] + \frac{1}{3} (4 - 3 c_1 - c_1^3) U_0[x, t] h^{(2,0)}[x, t] \right) + \\
& U_1[x, t]^2 \left(\frac{1}{24} (13 - 10 c_1 + 2 c_1^3 - 5 c_1^4) h^{(2,0)}[x, t] + \frac{1}{8} (-1 + c_1 - c_1^4 + c_1^5) H^{(2,0)}[x, t] \right) + \\
& H[x, t] \left((-1 + c_1) U_0^{(1,0)}[x, t]^2 + \frac{1}{6} (-3 + 2 c_1 + c_1^4) U_0^{(1,0)}[x, t] U_1^{(1,0)}[x, t] + \right. \\
& \frac{1}{20} (-1 + c_1^5) U_1^{(1,0)}[x, t]^2 + (1 - c_1) U_0^{(1,1)}[x, t] + \frac{1}{12} (3 - 2 c_1 - c_1^4) U_1^{(1,1)}[x, t] + \\
& U_0[x, t] \left((1 - c_1) U_0^{(2,0)}[x, t] + \frac{1}{12} (3 - 2 c_1 - c_1^4) U_1^{(2,0)}[x, t] \right) + \\
& U_1[x, t] \left(-\frac{3}{2} (-1 + c_1) U_0^{(2,0)}[x, t] + \frac{1}{20} (11 - 10 c_1 - c_1^5) U_1^{(2,0)}[x, t] \right) \Big) \Big) + \\
& H[x, t] \left((-1 + c_1)^2 U_0^{(1,0)}[x, t] h^{(1,1)}[x, t] + \frac{1}{3} (2 - 3 c_1 + c_1^3) U_1^{(1,0)}[x, t] h^{(1,1)}[x, t] + \right. \\
& \frac{1}{2} (-1 + c_1)^2 h^{(1,2)}[x, t] + U_0[x, t] \left(\frac{3}{2} (-1 + c_1)^2 U_0^{(1,0)}[x, t] h^{(2,0)}[x, t] + \right. \\
& U_1^{(1,0)}[x, t] \left(\frac{1}{2} (2 - 3 c_1 + c_1^3) h^{(2,0)}[x, t] + \frac{1}{24} (-3 + 4 c_1 - c_1^4) H^{(2,0)}[x, t] \right) + \\
& (-1 + c_1)^2 h^{(2,1)}[x, t] \Big) + \frac{1}{2} (-1 + c_1)^2 U_0[x, t]^2 h^{(3,0)}[x, t] + \\
& U_1[x, t] \left(U_0^{(1,0)}[x, t] \left(\frac{1}{2} (2 - 3 c_1 + c_1^3) h^{(2,0)}[x, t] + \frac{1}{8} (3 - 4 c_1 + c_1^4) H^{(2,0)}[x, t] \right) + \right. \\
& U_1^{(1,0)}[x, t] \left(\frac{1}{24} (19 - 26 c_1 + 2 c_1^3 + 5 c_1^4) h^{(2,0)}[x, t] + \frac{1}{80} (-1 + 5 c_1^4 - 4 c_1^5) H^{(2,0)}[x, t] \right) \Big) + \\
& \frac{1}{3} (2 - 3 c_1 + c_1^3) h^{(2,1)}[x, t] + \frac{1}{3} (2 - 3 c_1 + c_1^3) U_0[x, t] h^{(3,0)}[x, t] \Big) +
\end{aligned}$$

$$\begin{aligned}
& U_1[x, t]^2 \left(\frac{1}{12} (3 - 4 c_1 + c_1^4) h^{(3,0)}[x, t] + \frac{1}{240} (-9 + 10 c_1 + 5 c_1^4 - 6 c_1^5) H^{(3,0)}[x, t] \right) + \\
& H[x, t]^2 \left(U_0^{(1,0)}[x, t] \left(-\frac{1}{6} (-1 + c_1)^2 (2 + c_1) U_0^{(2,0)}[x, t] + \frac{1}{24} (-3 + 4 c_1 - c_1^4) U_1^{(2,0)}[x, t] \right) + \right. \\
& U_1^{(1,0)}[x, t] \left(\frac{1}{24} (3 - 4 c_1 + c_1^4) U_0^{(2,0)}[x, t] + \frac{1}{80} (7 - 10 c_1 + 5 c_1^4 - 2 c_1^5) U_1^{(2,0)}[x, t] \right) + \\
& \frac{1}{6} (2 - 3 c_1 + c_1^3) U_0^{(2,1)}[x, t] + \frac{1}{24} (3 - 4 c_1 + c_1^4) U_1^{(2,1)}[x, t] + \\
& U_0[x, t] \left(\frac{1}{6} (2 - 3 c_1 + c_1^3) U_0^{(3,0)}[x, t] + \frac{1}{24} (3 - 4 c_1 + c_1^4) U_1^{(3,0)}[x, t] \right) + \\
& \left. U_1[x, t] \left(\frac{1}{8} (3 - 4 c_1 + c_1^4) U_0^{(3,0)}[x, t] + \frac{1}{240} (39 - 50 c_1 + 5 c_1^4 + 6 c_1^5) U_1^{(3,0)}[x, t] \right) \right)
\end{aligned}$$