Supplementary Material D. Internal wave beam and bolus transport

Internal waves generated by an oscillating tidal flow on a topographic slope were found to have highest amplitudes when the topographic slope matches the angle of propagation of the internal wave beam θ (with the horizontal), such that $\sin\theta = \omega/N$ (Zhang *et al.* 2008). If transport by boluses behaves similarly, it is reasonable to conjecture that internal waves shoaling on a constant slope topography will lead to maximum transport when the topographic slope s equals the internal wave beam slope $s_\theta = \tan(\theta)$.

Because N varies vertically, so does θ and we are primarily concerned with the beam angle at mid-depth, $\theta_{H/2}$, where the boluses are generated. This beam angle depends on the magnitude of $\mathrm{d}\rho_0/\mathrm{d}z(H/2)$. If the topographic slope remains constant and transport is maximum at the internal wave critical angle, then as we vary $\Delta\rho$, maximum transport will happen in thinner pycnoclines for small $\Delta\rho$ and broader pycnoclines for large $\Delta\rho$.

However, as presented in table S1, $\theta_{H/2}$ for the stratifications producing the largest bolus for each $\Delta\rho$ differ from the topographic slope $s = 0.176$, and the trends go against the conjecture that transport would maximize at the internal wave critical angle: boluses are larger for thinner pycnoclines when $\Delta\rho$ is larger and smaller for broader pycnoclines when $\Delta\rho$ is smaller.

$\Delta\rho$ (kg/m^3)	δ (m)	$\theta_{H/2}$	$s_\theta = \tan(\theta_{H/2})$
10	0.2	41.5°	0.885
20	0.15	24.0°	0.445
40	0.1	13.6°	0.242
80	0.1	9.5°	0.167

Table S1: Critical angle at mid-depth, $\theta_{H/2} = \theta(z = H/2)$, and corresponding critical slope s_θ for parameter combinations $(\Delta\rho, \delta)_{\max}$ that maximize the bolus size S_b.