**Appendix A. Selected properties for water-ice and water-liquid**

Saturation vapor pressure of water in mbar with in is (Pruppacher & Klett 1980):

For water-ice

For water-liquid

Saturated liquid density in () is (Pruppacher & Klett 1980):

Surface tension () is (Schnerr & Dohrmann 1990):

,

Condensation coefficient is (Schnerr & Dohrmann 1990):

For water-ice:

For water-liquid:

For : for :

and for :

Specific latent heat in ( is (Pruppacher & Klett 1980):

where .

**Appendix B. Derivation of reduced-order model**

To eliminate the pressure term, we cross differentiate the second-order radial (40) and axial (42) momentum equations, divide (40) by and subtract to obtain a relationship for and ,

. (B1)

Equation (B1) represents one relationship between and The second-order energy equation (46) gives:

. (B2)

In addition, from the second-order axial momentum equation,

. (B3)

By taking the -derivative of the second order equation of state (36) and substituting expressions (B2) and (B3), we obtain the expression,

. (B4)

We substitute (B4) into (B1) to find an equation for the second-order streamfunction ,

, (B5)

where

, (B6)

(B7)

(B8)

(B9)

The functionals in (B6) - (B9) are determined by the critical perturbations (59) and (62) - (67). These functionals are given in the form

Equation (B5) must satisfy the following boundary conditions for resulting from equations (54)-(58)

(B10)

(B11)

Integrating (B5) with respect to yields

Multiplying this expression by the adjoint function given by equations (68)-(70) and integrating over the flow domain, yields

where

. (B12)

From partial differential theory, it can be shown that the first term on the left hand side of this equation vanishes.

**Appendix C. Physical insight into vortex flow and condensation processes**

To shed more light on the flow dynamics, we use the steady azimuthal vorticity equation developed in Rusak, Kapila & Choi (2002, equation (75)):

Here is Gibbs free energy and the partial derivative is taken at fixed and. This equation shows that convection of the reduced azimuthal vorticity is balanced by several terms on the right-hand side of the equation. The first term represents azimuthal vorticity changes due to the stretching of axial vorticity, resulting from the swirl axial gradient. The second term represents a baroclinic effect resulting from interaction between swirl and axial temperature gradient. Note that these terms are functions of the square of the swirl level . The third term represents azimuthal vorticity changes resulting from interaction between the azimuthal vorticity and flow dilatation, due to density changes within the flow. The fourth and fifth terms also represent baroclinic effects which result from the interaction between radial and axial gradients of the velocity components and radial and axial temperature gradients. The last term represents vorticity changes resulting from condensation within the flow.

It can be shown that at orders and considered in the present analysis, the fourth, fifth, and sixth terms on the right-hand side of (C1) are of order or , which are much smaller than the other terms in (C1) and may be neglected. The third term related with dilatation effects is of order and is not directly dependent on swirl level. Then, to the orders considered, only the first two terms on the right-hand side of (C1) depend on the swirl level . The present analysis shows that a flow perturbation with a positive radial speed ) gives from the steady version of (6) a negative axial gradient of swirl ; this is the most dominant mode of disturbance at near-critical swirl levels and with the assumed inlet and outlet conditions. Such a disturbance produces from the first term on the right-hand side of (C1) a negative gradient of and reduces the azimuthal vorticity along the pipe axis. Thereby, from flow kinematics, it acts to decelerate the flow around the pipe centerline and causes from continuity the divergence of streamlines. As the swirl level increases, the flow deceleration also increases and reaches a critical balance at a certain critical swirl, independent of flow compressibility or condensation. The compressible moist air flow problem shows that temperature rises with compressibility and condensation effects and, therefore, the second term provides an additional production of negative , which also depends on swirl. Therefore, a lower level of the swirl ratio is needed to create a critical balance once condensation occurs. This means that the critical swirl decreases with increase of , see Figures 3 and 4*.*