

SUPPLEMENTARY MATERIAL FOR
**Designing vortices in pipe flow with
topography-driven Langmuir circulation**

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Names of equations and figures refer to the main article, except when preceded by a capital letter:
(A.1), (B.2), etc.

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A. Plots and graphs for the full set of simulation cases

The simulated cases are given in Table A.1. In the following we shall refer mainly to the ID of each case. In some cases the same figure might be present in the main paper, but note that scaling can then be different. We list in table A.1 also in which figures of the main paper the different cases appear.

ID#	Re_τ	a	m	$\varphi[^\circ]$	Re	Figure(s)
1	20	0.05	1	18.56	195.8	
2	40	0.05	1	18.56	774.3	3(a), 4(a,l)
3	60	0.05	1	18.56	1618	3(f)
4	80	0.05	1	18.56	2328	3(k)
5	40	0.01	1	18.56	799.0	4(a)
6	40	0.025	1	18.56	793.6	4(a)
7	40	0.09	1	18.56	726.8	4(a)
8	40	0.125	1	18.56	680.0	4(a,m)
9	40	0.20	1	18.56	521.5	4(a,n)
10	40	0.05	1	45	791.3	3(b), 4(e)
11	60	0.05	1	45	1718	3(g)
12	80	0.05	1	45	2719	3(l)
13	40	0.01	1	45	799.7	
14	40	0.025	1	45	798.1	
15	40	0.20	1	45	597.7	4(g)
16	40	0.05	1	60	796.2	3(c), 4(f)
17	60	0.05	1	60	1734	3(h)
18	80	0.05	1	60	2751	3(m)
19	40	0.20	1	60	669.5	4(h)
20	40	0.05	3	18.56	757.8	4(b)
21	40	0.05	3	45	782.6	4(c)
22	40	0.05	3	60	790.3	4(d)

TABLE A.1. Simulation cases

A.1. *Streamwise-averaged pressure and streamlines*

Plot of the same kind as Figure 4.

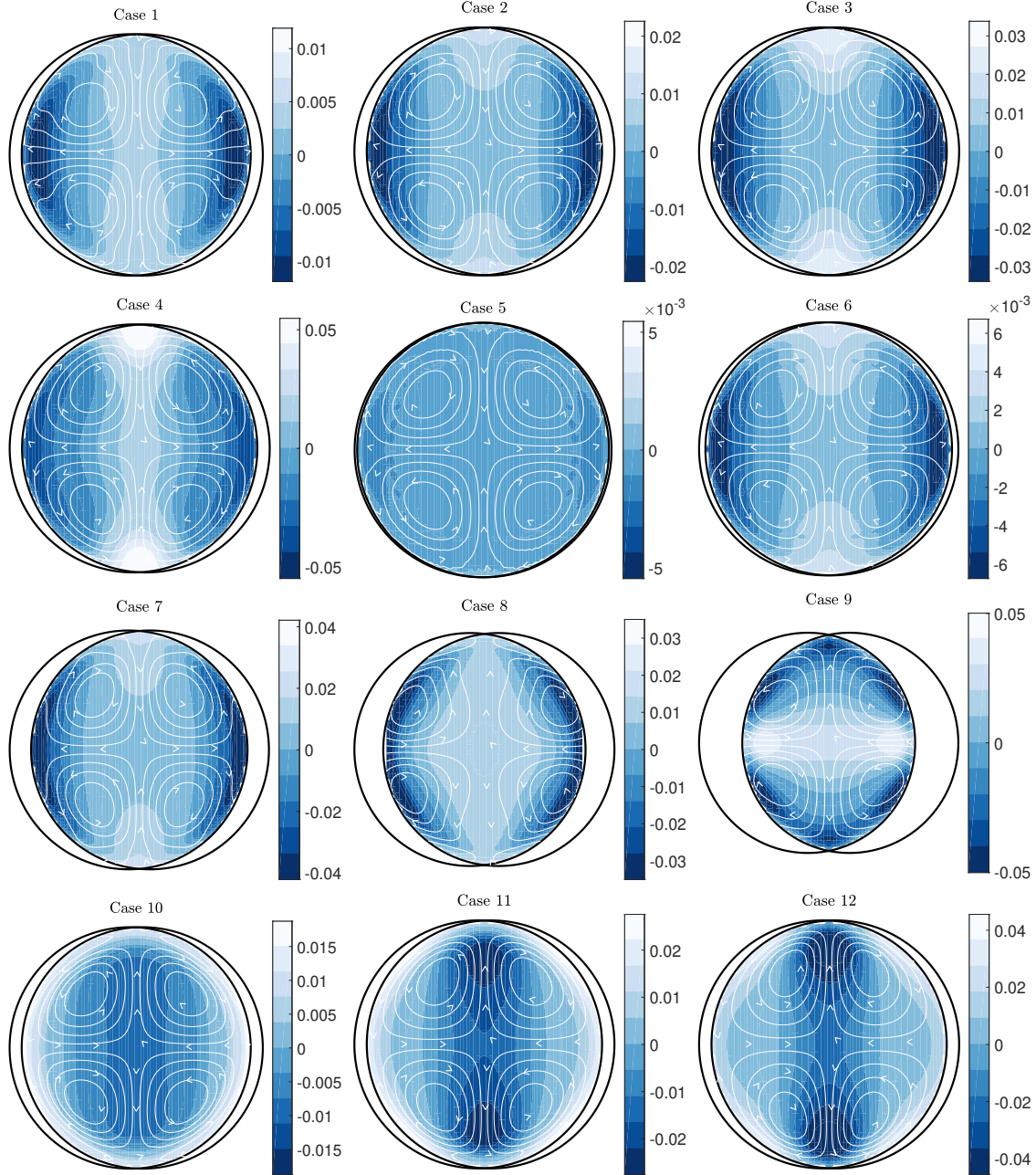


FIGURE A.1. Streamwise-averaged excess pressure (contours) and streamlines.

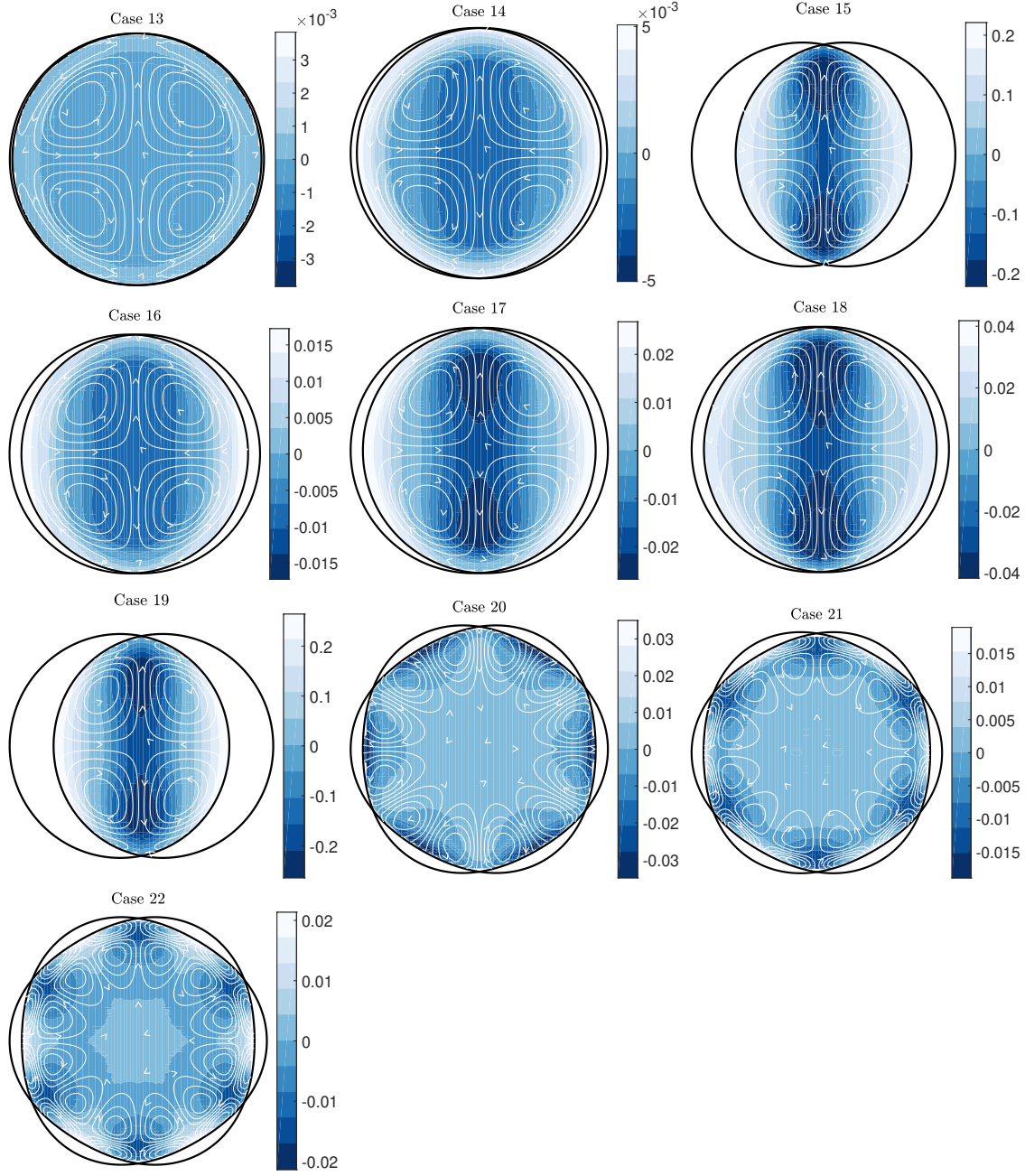


FIGURE A.1. (cont'd). Streamwise-averaged excess pressure (contours) and streamlines.

A.2. Circulation strength as function of radius

Here follow plots of the approximate circulation strength $\tilde{\psi}/Re a^2$ along a line at polar angle $\pi/4m$ running approximately through the centre of the ‘first’ vortex. Some of these appear in figure 3(d,h,l,m-o).

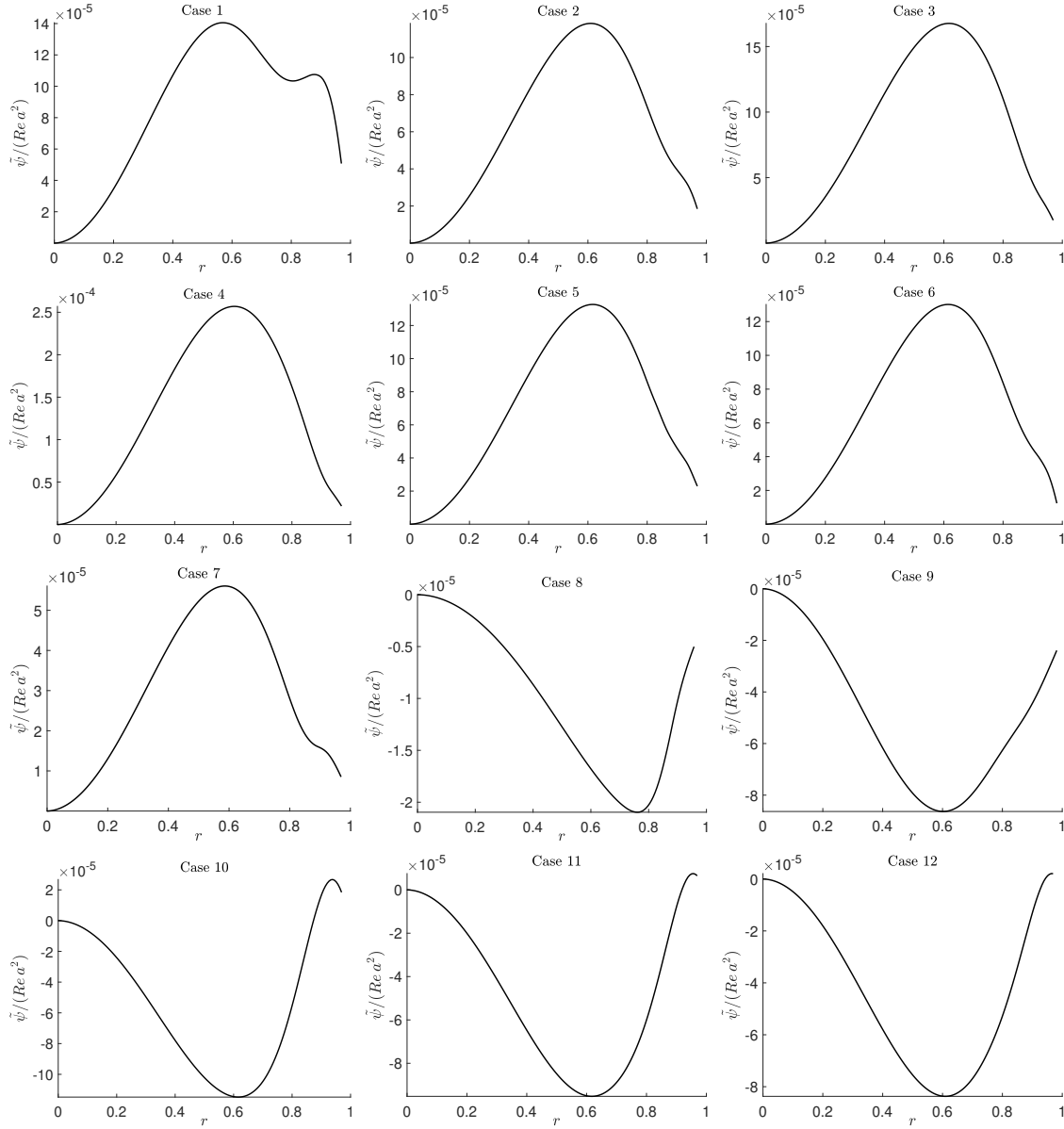


FIGURE A.2. Circulation strength as function of radius.

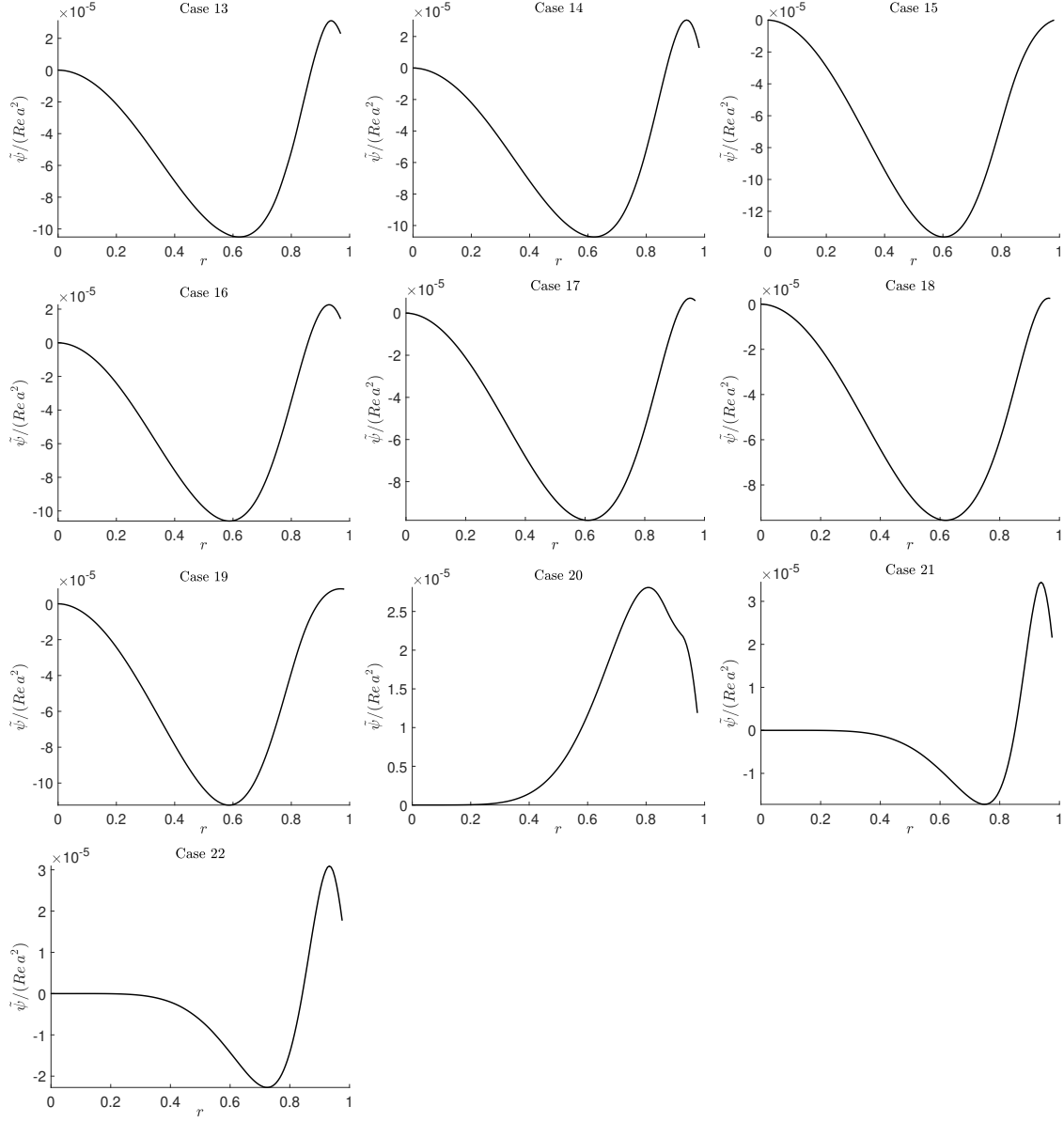


FIGURE A.2. (cont'd) Circulation strength as function of radius.

A.3. Streamlines of mean cross-plane flow

Figures of the type seen in Figures 2b–d and 3a–c, e–g and i–k.

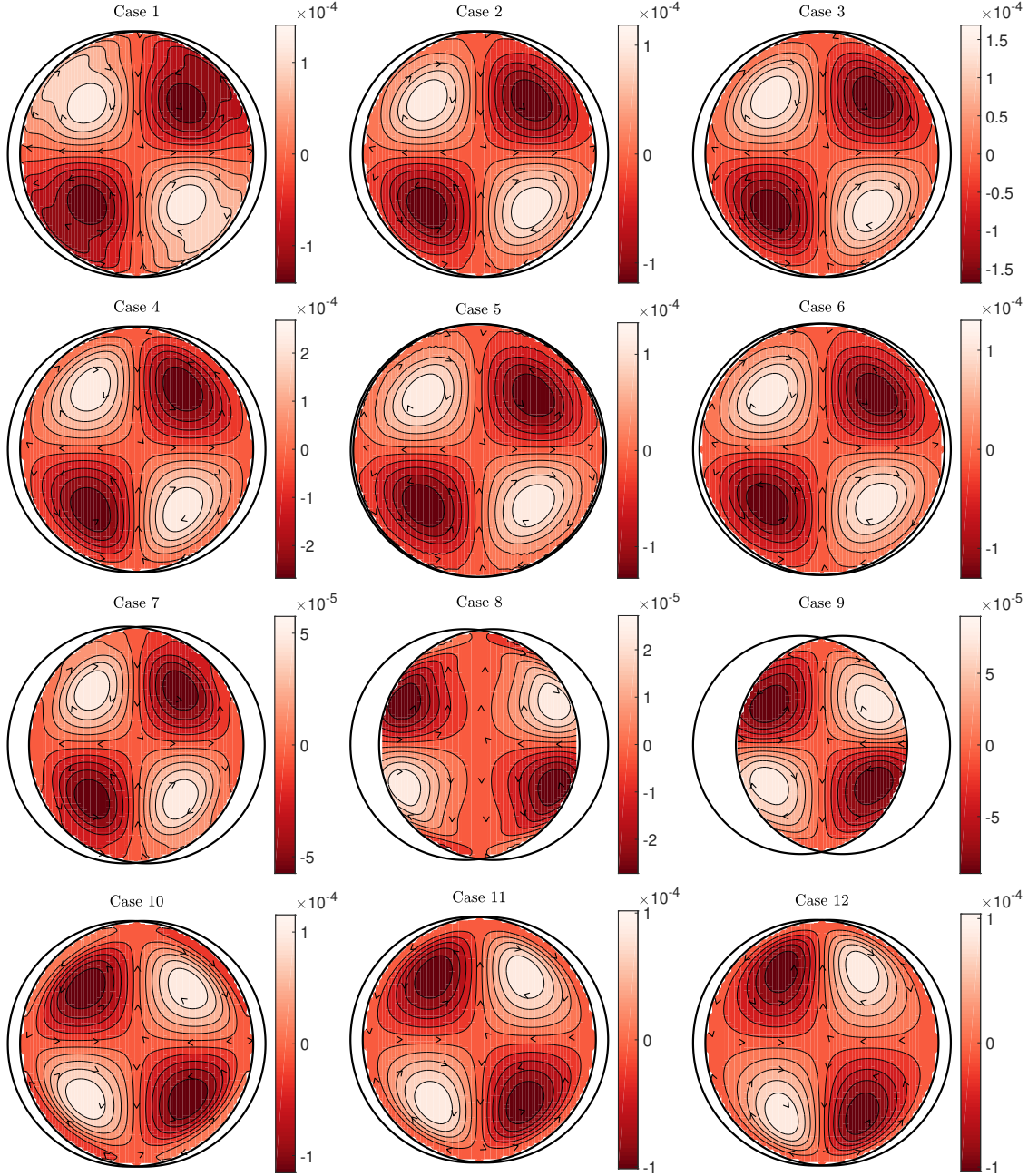


FIGURE A.3. Streamwise-averaged cross-plane velocity. Contours: values of $\psi/(Re a^2)$.

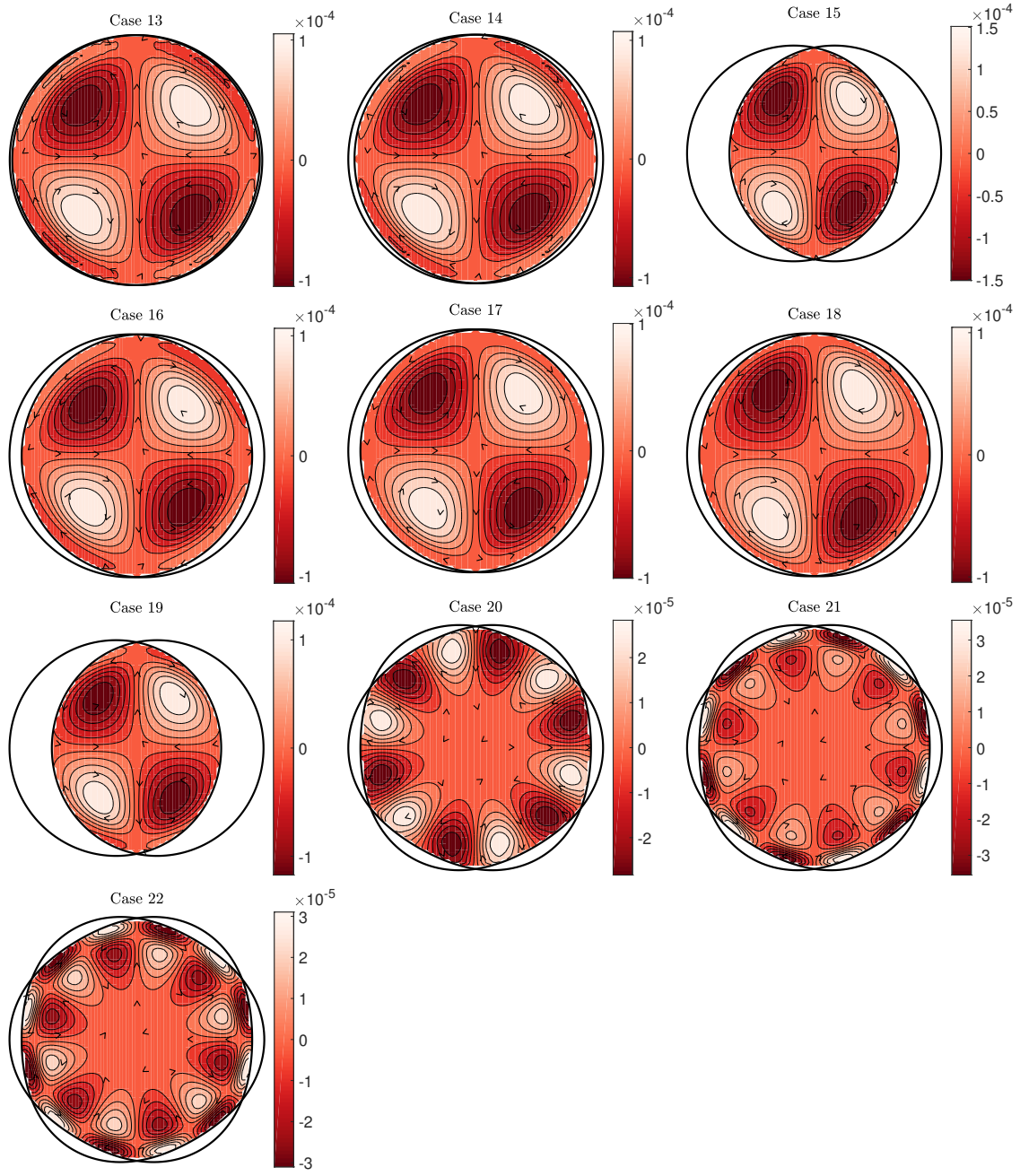


FIGURE A.3. (Cont'd).

A.4. Deviation of mean flow from Poiseuille flow

Contour plots as seen in Fig. 3 of $[\overline{u_{z,\text{tot}}}(r, \theta) - U_P(r)]/U_0$ (dimensional quantities) where $\overline{u_{z,\text{tot}}}$ is the streamwise-averaged axial velocity, U_{avg} is the mean velocity (flow rate divided by πR^2), U_0 is the mean centreline velocity and $U_P(r) = 2U_{\text{avg}}(1 - r^2/R^2)$ is the Poiseuille flow with the same volume flow rate as the simulated flow.

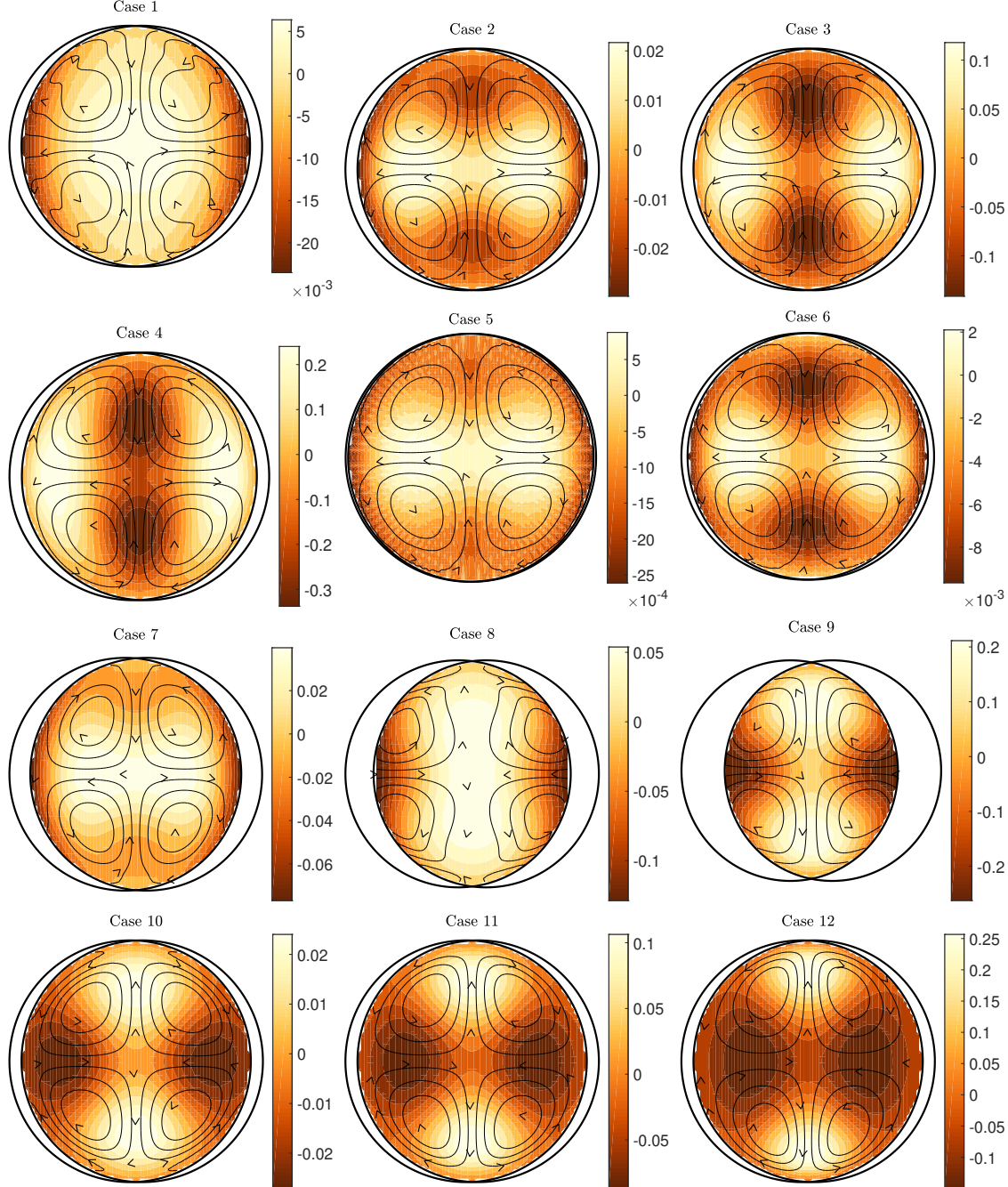


FIGURE A.4. Deviation of streamwise-average axial velocity from Poiseuille flow with same flow rate. Streamlines of streamwise-averaged cross-plane flow superposed.

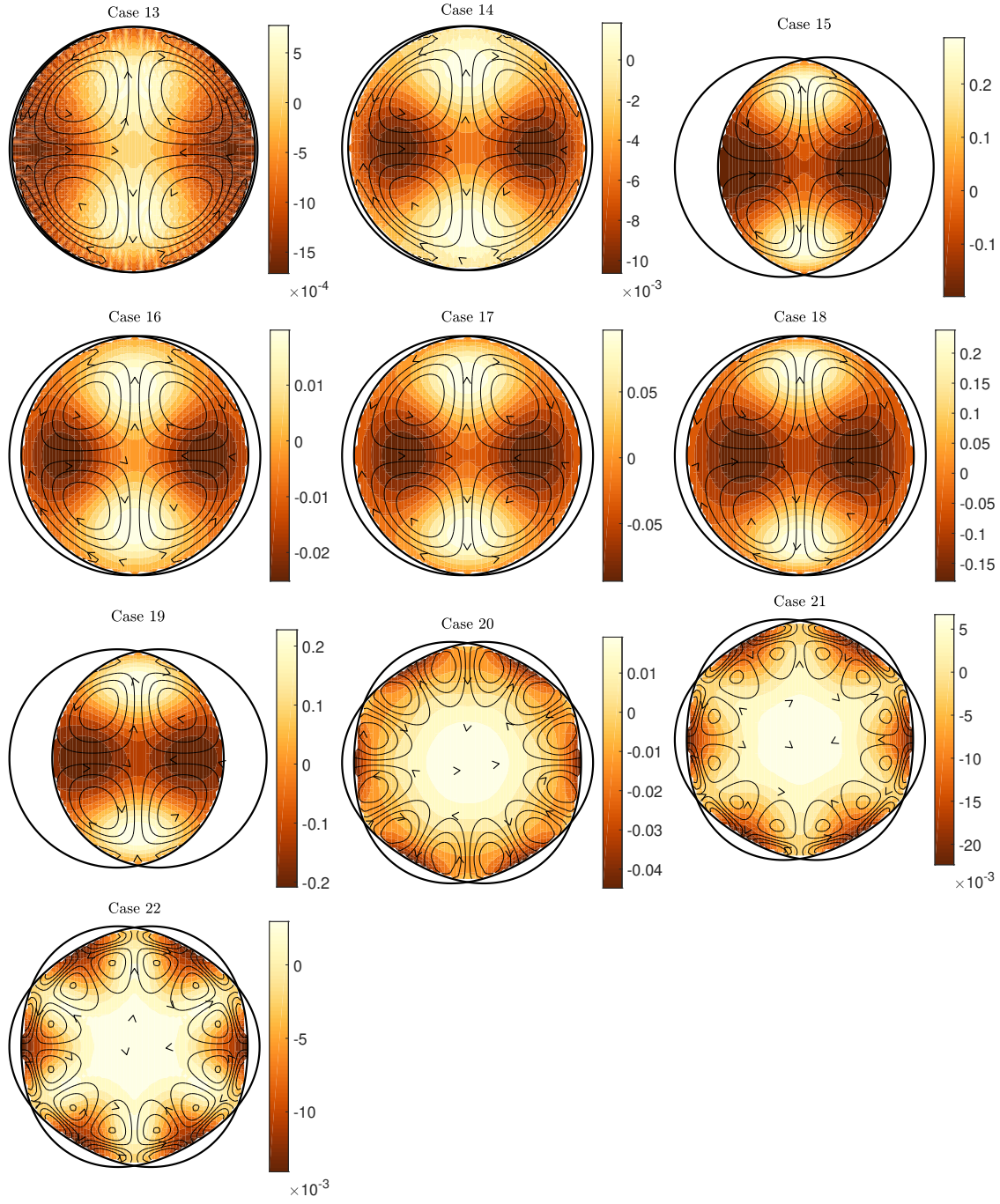


FIGURE A.4. (cont'd.)

A.5. Motion in the cross-plane

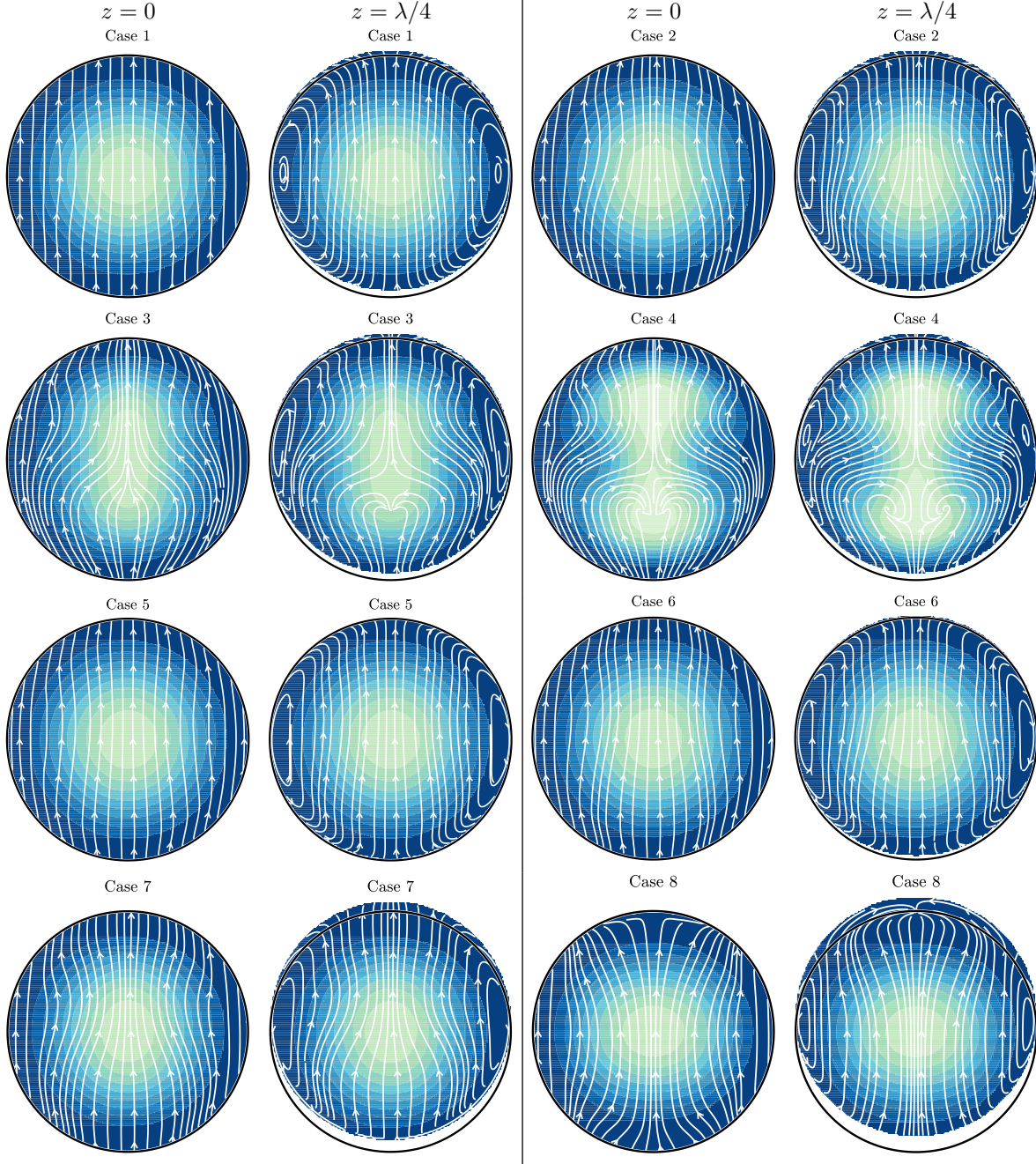


FIGURE A.5. Cross-flow (u, v) at two different cross-sections, at $z = 0$ where the pipe has the average, circular cross-section, and at $z = \lambda/4$ where the wall-wave is at its highest. Colour contours indicate kinetic energy $(\hat{u}_r^2 + \hat{u}_\theta^2 + \hat{u}_z^2)/2$.

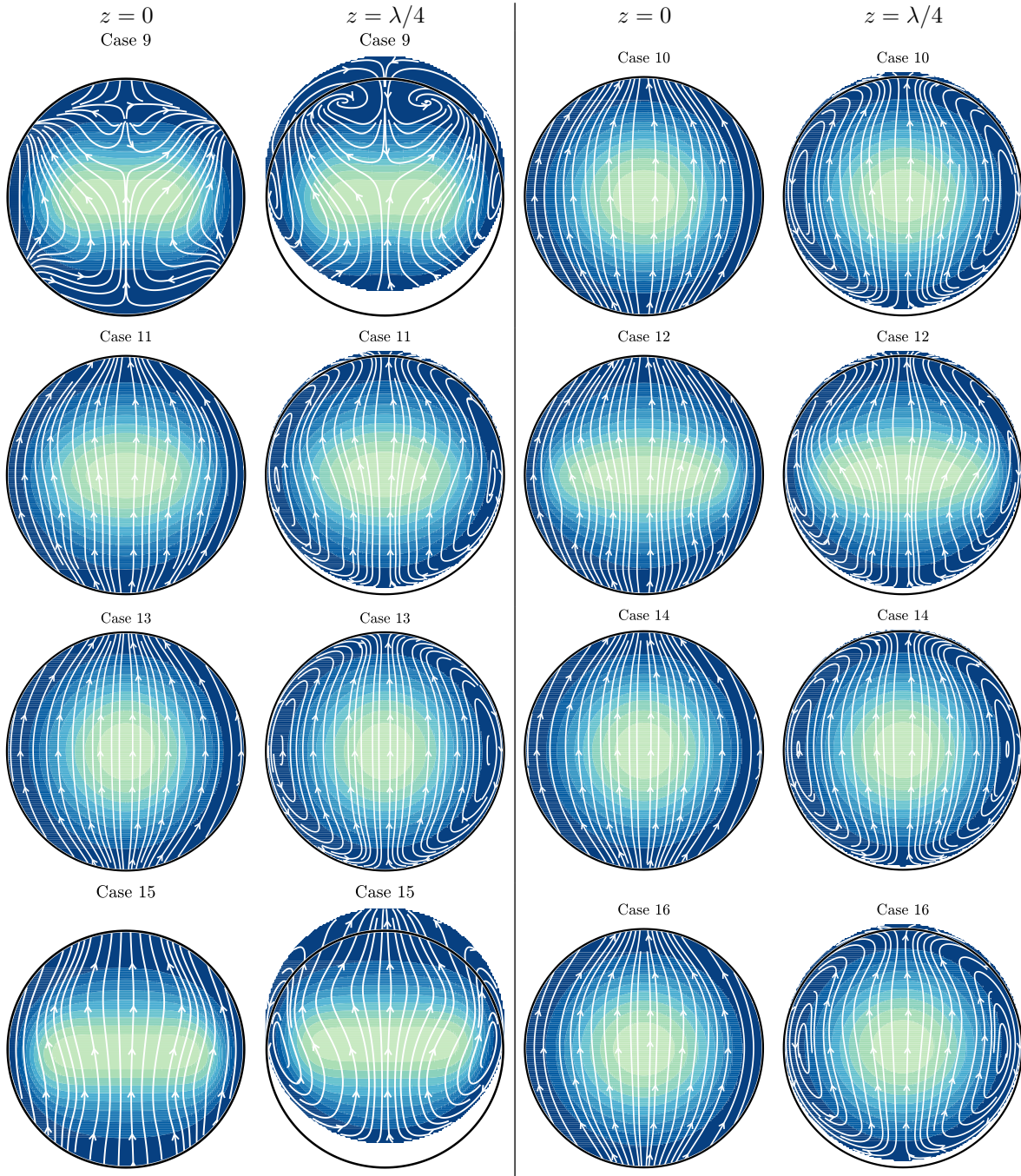


FIGURE A.5. (cont'd.)

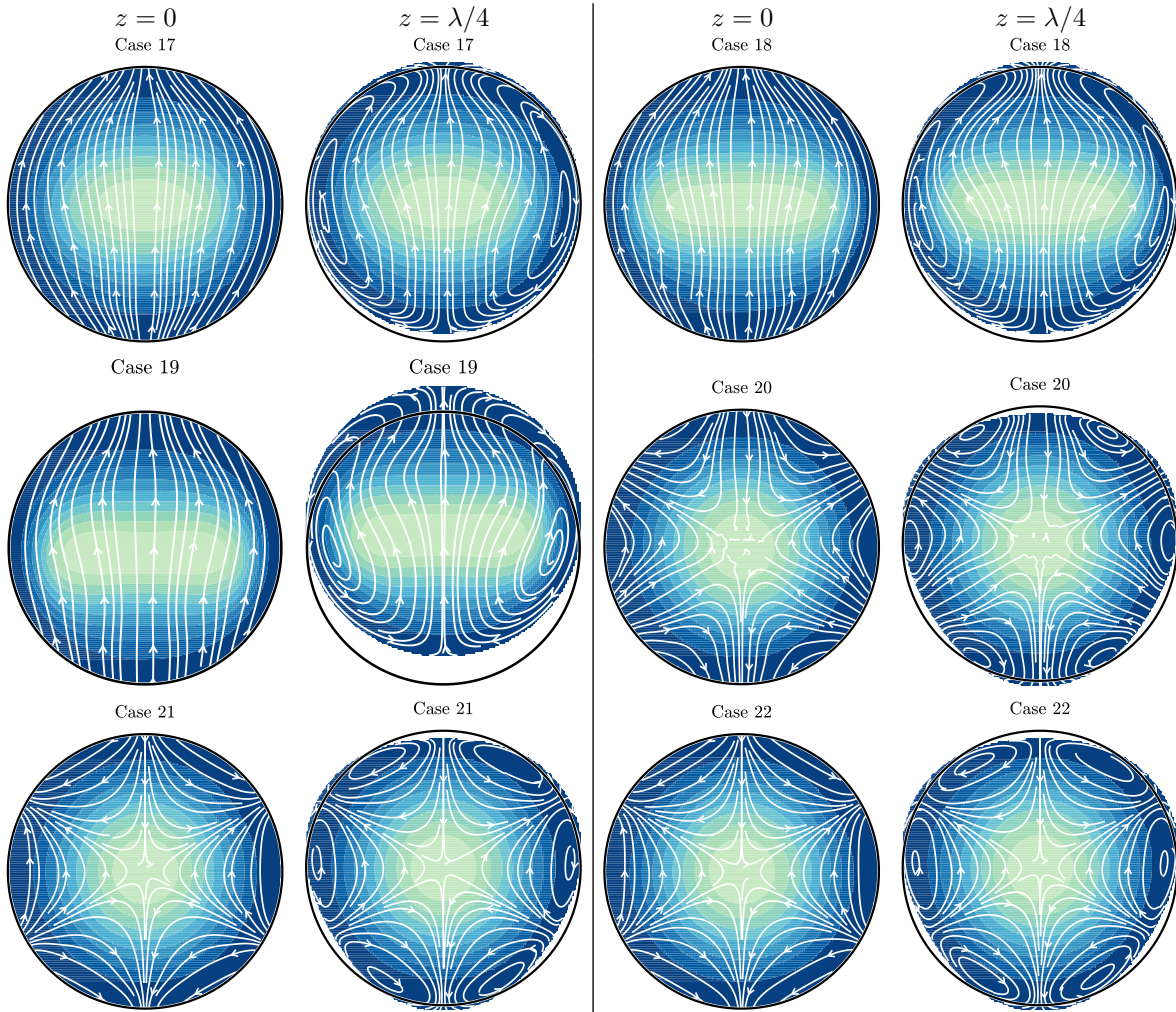


FIGURE A.5. (cont'd.)