

# Swinging and tumbling of multicomponent vesicles in flow – Supplementary materials

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## I. MOVIE CAPTIONS

- Movie M1: Vesicle dynamics with bending stiffness variation in spatial mode  $M=2$ . Beyond a critical magnitude of variation the elongated axis transitions from swinging to tumbling.
- Movie M2: As in Movie M1, but with a smaller enclosed area.
- Movie M3: As in Movies M1-M2, with yet smaller enclosed area.
- Movie M4: Vesicles in a shear flow with different spatial modes of material property variation. The  $M=2$  mode most strongly interacts with the deformation imposed by the background shear flow. The magnitude of the variation is just large enough for the case with two domains ( $M=2$ ) to tumble.
- Movie M5: As in Movie M3, but with increased variation in the bending stiffnesses. The swinging amplitudes of vesicles with even numbers of domains ( $M$  even) are increasing.
- Movie M6: As in Movies M1-M2, with yet smaller enclosed area.

## II. MOVIE PARAMETERS

Movie	Mode(s)	$\varepsilon$	$\bar{\kappa}C^{-1}$
M1	2	0.05	{0.3, 0.87, 0.871, 2}
M2	2	0.1	{0.3, 0.698, 0.699, 2}
M3	2	0.15	{0.3, 0.7, 1, 2}
M4	{1,2,...,8}	0.1	0.75
M5	{1,2,...,8}	0.1	2
M6	{1,2,...,8}	0.1	4

## III. DESCRIPTION OF MOVIES M1–M3

Visualizations of the different types of observed dynamics. Let  $(\varepsilon, \bar{\kappa}C^{-1})$  denote the amount of deformation and the variation in the bending rigidity. The results can be grouped thusly:

- Swinging: (0.05, 0.3), (0.05, 0.87), (0.1, 0.3), (0.1, 0.698)
- Phase-Lagging: (0.05, 0.871), (0.1, 0.699), (0.15, 0.7), (0.15, 1)

- Tumbling: (0.05, 2.0), (0.1, 2.0), (0.15, 2)

The systems (0.05, 0.87) and (0.1, 0.698) are just below the critical  $\bar{\kappa}\mathcal{C}^{-1}$  value required for tumbling, while (0.05, 0.871) and (0.1, 0.699) are just above it. Just below the critical  $\bar{\kappa}\mathcal{C}^{-1}$  value the tip of the vesicle dips below horizontal but does not continue due to the phase-lagging of the high-bending rigidity region (in red). Just above the critical value the vesicle begins to tumble, with the surface domains sliding between the tip and the edge, never making a full rotation around the perimeter. As  $\bar{\kappa}\mathcal{C}^{-1}$  further increases the high-bending rigidity regions become locked away from the tips due to the large bending energy.

#### IV. DESCRIPTION OF MOVIES M4–M6

Visualization of how the mode (the number of domains) influences the dynamics of a vesicle with  $\varepsilon = 0.1$  for Movie M4:  $\bar{\kappa}\mathcal{C}^{-1} = 0.7$ , Movie M5:  $\bar{\kappa}\mathcal{C}^{-1} = 2$ , and Movie M6:  $\bar{\kappa}\mathcal{C}^{-1} = 4$ . A value of  $\bar{\kappa}\mathcal{C}^{-1} = 0.7$  is just larger than the critical value necessary for the mode-2 vesicle to tumble, see Movie M2. In general only even modes are observed to tumble, with higher modes requiring a higher bending rigidity difference. Due to the non-symmetric nature of the odd modes, the membrane deformation is no longer symmetric.