Movie Captions

- Movie 1

Flow field evolution for the (J),Φ(J)-rheology in the well-posed, low packing fraction (₀=0.35), regime . Here the initial data is a small sinusoidal perturbation in vertical velocity away from steady homogeneous shearing. As expected, the disturbance decays smoothly in time.

- Movie 2

A demonstration of the ill-posed regime of the (J),Φ(J)-rheology corresponding to a high mean packing density of ₀=0.55. As in movie 1, the initial data consists of horizontal velocity u and solid volume fraction profiles which are precisely those of homogeneous steady shear. However, here the small amplitude perturbation in the vertical velocity w grows in time and comes to dominate the solution, as predicted by the ill-posedness analysis. The video ends when numerical convergence is no longer possible.

- Movie 3

Flow field evolution for the new vCIDR equations given the same initial data as employed in movie 1. As the equations are guaranteed to be well-posed, the initial sinusoidal profile of the vertical velocity w quickly decays towards the steady value of w=0.

- Movie 4

A demonstration that the vCIDR equations leads to long-time grid-resolved stability for high packing-fractions, unlike the (J),Φ(J)-rheology. Here the initial conditions are the same as those in movie 2: linear shearing in x, a small amplitude sinusoidal profile for velocity in z, and a homogeneous high uniform packing fraction ₀=0.55. This initial disturbance quickly decays towards the fully homogeneous steady state, unlike the blow-up observed in movie 2 for the (J),Φ(J)-rheology, because the equations are well-posed.

- Movie 5

The spatio-temporal evolution, using the vCIDR constitutive equations, of an initial particle concentration that is far from homogeneous. Here the non-uniform initial packing fraction consists of a single period of a sine wave of relatively large amplitude. This initial imbalance causes clear non-trivial coupling with the velocity fields which act to homogenise the sample in time. The video ends when decay to the steady homogeneous shearing state is complete.

- Movie 6

Here the same initially non-uniform packing fraction is employed as in movie 5. Because the (J),Φ(J)-rheology is employed here instead of the vCIDR equations, the region of the cell for which > _{crit} is in the ill-posed region of parameter space. Unlike in the simulation using the vCIDR equations, which are always well-posed, this region of ill-posedness prevents the smooth decay of the initial fields towards the steady homogeneous shearing state. Instead, a spontaneous oscillation emanates from the high- region and quickly dominates the simulation and leads to non-convergence.