Supplementary Information

S.1.1. The coefficients for the concentration fields of both fluids when the Janus particle and the encapsulating droplet are concentrically located as defined in section 2.1:

$$X_n = (x_n)\left(\frac{(k-D)(n+1)}{(2n+1)\left(\frac{1}{\chi}\right)^{2n+1}}\right)$$

$$Y_n = (x_n)\left(\frac{nk+(n+1)D}{2n+1}\right)$$

$$x_n = \frac{(2n+1)^2}{(2n+2)}\left(\frac{\left(\frac{1}{\chi}\right)^{2n+1}}{n(k-D)-\left(\frac{1}{\chi}\right)^{2n+1}(nk+(n+1)D)}\right)\int_1^{\cos\alpha} p_n(\vartheta)\,d\vartheta$$

S.1.2. The coefficients of the stream function for the velocity fields of both fluids when the Janus particle and the encapsulating droplet are concentrically located as defined in section 2.2:

$$A_n = \begin{cases} \dfrac{\left(\begin{array}{l} 3\mu\chi^2\left(2-3\left(1+\left(\frac{V}{U}-1\right)\right)\chi+\left(1+3\left(\frac{V}{U}-1\right)\right)\chi^3\right) \\ -\chi^4\left(6\left(\frac{V}{U}-1\right)\chi+\left(2-3\chi+\chi^3\right)H_1\right) \end{array}\right)}{3(-1+\chi)^3\left(\begin{array}{l} -2\left(2+3\chi+3\chi^2+2\chi^3\right) \\ +\mu\left(-4-3\chi+3\chi^2+4\chi^3\right) \end{array}\right)} & n=1 \\ \dfrac{\chi^2\left(-1+2\left(\frac{1}{\chi}\right)^{(-1+2n)}-\chi^2+2n\left(-1+\chi^2\right)\right)H_n}{(1+2n)\left(\begin{array}{l} 2\left(\frac{1}{\chi}\right)^{2n}-2\left(\frac{1}{\chi}\right)^{(-4+2n)}-4\left(\frac{1}{\chi}\right)^{(-1+4n)} \\ +4\chi^3-4\mu\chi^3+4n^2\mu\left(\frac{1}{\chi}\right)^{2n}\left(-1+\chi^2\right)^2 \\ +\mu\left(\frac{1}{\chi}\right)^{2n}\left(1-4\left(\frac{1}{\chi}\right)^{(-1+2n)}+6\chi^2+\chi^4\right) \\ +4n\left(\frac{1}{\chi}\right)^{2n}\left(-1+\chi^2\right)\left(-1-\chi^2+\mu\left(-1+\chi^2\right)\right) \end{array}\right)} & n>1 \end{cases}$$

$$B_n = \begin{cases} \dfrac{\left(\dfrac{V}{U}-1\right)\left(-1+3\left(\dfrac{1}{\chi}\right)^2\right)}{\left(-1+\dfrac{1}{\chi}\right)^2\left(1+\dfrac{2}{\chi}\right)} - \dfrac{A_1\left(3+\dfrac{6}{\chi}+4\left(\dfrac{1}{\chi}\right)^2+2\left(\dfrac{1}{\chi}\right)^3\right)}{1+\dfrac{2}{\chi}} & n=1 \\ -\dfrac{A_n\left(-1-2n+3\left(\dfrac{1}{\chi}\right)^2+2n\left(\dfrac{1}{\chi}\right)^2-2\left(\dfrac{1}{\chi}\right)^{(3+2n)}\right)}{1-2n+\left(\dfrac{1}{\chi}\right)^2+2n\left(\dfrac{1}{\chi}\right)^2-2\left(\dfrac{1}{\chi}\right)^{(1+2n)}} & n>1 \end{cases}$$

$$C_n = \begin{cases} -\dfrac{3\left(\dfrac{V}{U}-1\right)\left(\dfrac{1}{\chi}\right)^3}{\left(-1+\dfrac{1}{\chi}\right)^2\left(1+\dfrac{2}{\chi}\right)} - \dfrac{A_1\left(-2-\dfrac{4}{\chi}-6\left(\dfrac{1}{\chi}\right)^2-3\left(\dfrac{1}{\chi}\right)^3\right)}{1+\dfrac{2}{\chi}} & n=1 \\ -\dfrac{A_n\left(2-3\left(\dfrac{1}{\chi}\right)^{(1+2n)}-2n\left(\dfrac{1}{\chi}\right)^{(1+2n)}+\left(\dfrac{1}{\chi}\right)^{(3+2n)}+2n\left(\dfrac{1}{\chi}\right)^{(3+2n)}\right)}{1-2n+\left(\dfrac{1}{\chi}\right)^2+2n\left(\dfrac{1}{\chi}\right)^2-2\left(\dfrac{1}{\chi}\right)^{(1+2n)}} & n>1 \end{cases}$$

$$D_n = \begin{cases} \dfrac{\left(\dfrac{V}{U}-1\right)\left(\dfrac{1}{\chi}\right)^3}{\left(-1+\dfrac{1}{\chi}\right)^2\left(1+\dfrac{2}{\chi}\right)} - \dfrac{A_1\left(2\left(\dfrac{1}{\chi}\right)^2+\left(\dfrac{1}{\chi}\right)^3\right)}{1+\dfrac{2}{\chi}} & n=1 \\ \dfrac{A_n\left(\dfrac{2}{\chi}-\left(\dfrac{1}{\chi}\right)^{2n}-2n\left(\dfrac{1}{\chi}\right)^{2n}-\left(\dfrac{1}{\chi}\right)^{(2+2n)}+2n\left(\dfrac{1}{\chi}\right)^{(2+2n)}\right)}{\chi\left(1-2n+\left(\dfrac{1}{\chi}\right)^2+2n\left(\dfrac{1}{\chi}\right)^2-2\left(\dfrac{1}{\chi}\right)^{(1+2n)}\right)} & n>1 \end{cases}$$

$$a_n = \begin{cases} \dfrac{3\left(1+\left(\frac{V}{U}-1\right)+\frac{1}{\chi}+\dfrac{\left(\frac{V}{U}-1\right)}{\chi}-2\left(\frac{1}{\chi}\right)^2\right)}{\chi\left(2\left(-1+\frac{1}{\chi}\right)\left(1+\frac{2}{\chi}\right)\right)} - \dfrac{A_n\left(-4+\frac{1}{\chi}+6\left(\frac{1}{\chi}\right)^2+\left(\frac{1}{\chi}\right)^3-4\left(\frac{1}{\chi}\right)^4\right)}{2\left(1+\frac{2}{\chi}\right)} & n=1 \\ -\dfrac{A_n\left(\begin{array}{l} -4\chi^3+4n\left(\frac{1}{\chi}\right)^{2n}\left(-1+\chi^2\right)^2+4n^2\left(\frac{1}{\chi}\right)^{2n}\left(-1+\chi^2\right)^2 \\ +\left(\frac{1}{\chi}\right)^{2n}\left(1-4\left(\frac{1}{\chi}\right)^{(-1+2n)}+6\chi^2+\chi^4\right) \end{array}\right)}{2\chi\left(-1+2\left(\frac{1}{\chi}\right)^{(-1+2n)}-\chi^2+2n\left(-1+\chi^2\right)\right)} & n>1 \end{cases}$$

$$b_n = \begin{cases} \dfrac{\left(\begin{array}{l} 12+\left(6-18\left(\frac{V}{U}-1\right)\right)\chi-\chi^5 A_1+4\chi^6 A_1+4\chi^2\left(-9\left(\frac{V}{U}-1\right)+A_1\right) \\ -6\chi^4\left(2+2\left(\frac{V}{U}-1\right)+A_1\right)-\chi^3\left(6+24\left(\frac{V}{U}-1\right)+A_1\right) \end{array}\right)}{6\left(-1+\chi\right)\chi^3\left(\begin{array}{l} -2\left(2+3\chi+3\chi^2+2\chi^3\right) \\ +\mu\left(-4-3\chi+3\chi^2+4\chi^3\right) \end{array}\right)} & n=1 \\ \dfrac{\left(\begin{array}{l} -4\chi^3+4n\left(\frac{1}{\chi}\right)^{2n}\left(-1+\chi^2\right)^2+4n^2\left(\frac{1}{\chi}\right)^{2n}\left(-1+\chi^2\right)^2 \\ +\left(\frac{1}{\chi}\right)^{2n}\left(1-4\left(\frac{1}{\chi}\right)^{(-1+2n)}+6\chi^2+\chi^4\right) \end{array}\right)A_n}{2\left(1+2n\right)\chi\left(\begin{array}{l} 2\left(\frac{1}{\chi}\right)^{2n}-2\left(\frac{1}{\chi}\right)^{(-4+2n)}-4\left(\frac{1}{\chi}\right)^{(-1+4n)} \\ +4\chi^3-4\mu\chi^3+4n^2\mu\left(\frac{1}{\chi}\right)^{2n}\left(-1+\chi^2\right)^2 \\ +\mu\left(\frac{1}{\chi}\right)^{2n}\left(1-4\left(\frac{1}{\chi}\right)^{(-1+2n)}+6\chi^2+\chi^4\right) \\ +4n\left(\frac{1}{\chi}\right)^{2n}\left(-1+\chi^2\right)\left(-1-\chi^2+\mu\left(-1+\chi^2\right)\right) \end{array}\right)} & n>1 \end{cases}$$

Where $H_n = -\frac{2k_2 J R_1 \beta \gamma_0 x_n}{D_1 \mu_1 U}$.

Multiplying both sides by $\frac{g_n(\zeta)}{1-\zeta^2}$ and integrating the boundary conditions from $\zeta = (-1 \leftrightarrow 1)$ we can reduce the above equations to a system of linear equations (using the orthogonality property of the Modified Gegenbauer polynomials).

S.2.1. The system of equations corresponding to the boundary conditions for the concentration fields of both fluids as defined for the eccentric configuration in section 2.2.1

$$(X_n + Y_n) = (k)(x_n)$$

$$\left(\begin{array}{l} \sum_{n=0}^{\infty} \left[\begin{array}{l} X_n(\sinh\xi_2 + (2n+1)\cosh\xi_2) \\ +Y_n(\sinh\xi_2 - (2n+1)\cosh\xi_2) \\ -(D)x_n(\sinh\xi_2 \pm (2n+1)\cosh\xi_2) \end{array} \right] p_n(\zeta) \\ -\sum_{m=0}^{\infty} \left[X_m - Y_m \mp (D)x_m \right]((2m+1)\zeta)\, p_m(\zeta) \end{array} \right) = 0$$

$$\left(\begin{array}{l} \sum_{n=0}^{\infty} \left[\begin{array}{l} X_n(\sinh\xi_1 + (2n+1)\cosh\xi_1)e^{(n+1/2)(\xi_1-\xi_2)} \\ +Y_n(\sinh\xi_1 - (2n+1)\cosh\xi_1)e^{-(n+1/2)(\xi_1-\xi_2)} \end{array} \right] p_n(\zeta) \\ -\sum_{m=0}^{\infty} \left[X_m e^{(n+1/2)(\xi_1-\xi_2)} - Y_m e^{-(n+1/2)(\xi_1-\xi_2)} \right]((2m+1)\zeta)\, p_m(\zeta) \end{array} \right) = \frac{\pm 2F}{\sqrt{(\cosh\xi_1 - \zeta)}}$$

Multiplying both sides by $p_n(\zeta)$ and integrating the boundary conditions from $\zeta = (-1 \leftrightarrow 1)$ we can reduce the above equations to a system of coupled linear equations (using the orthogonality property of Legendre polynomials).

S.2.2. The system of equations corresponding to the boundary conditions for the stream functions of the velocity fields of both fluids as defined for the eccentric configuration in section 2.2.2

$$\sum_{n=1}^{\infty} \left[(A_n + B_n + C_n + D_n)\left((\cosh\xi - \zeta) g_n'(\zeta) + \left(\frac{3}{2}\right) g_n(\zeta) \right) \right] = \frac{-(1-\zeta\cosh\xi)}{\sqrt{(\cosh\xi - \zeta)}}$$

$$\sum_{n=1}^{\infty} \left[(a_n + b_n)\left((\cosh\xi - \zeta) g_n'(\zeta) + \frac{3}{2} g_n(\zeta) \right) \right] = \frac{-(1-\zeta\cosh\xi)}{\sqrt{(\cosh\xi - \zeta)}}$$

$$\left(\sum_{n=1}^{\infty}\left[\begin{array}{l}\left(\left(n-\frac{1}{2}\right)\left(A_n-B_n\mp a_n\right)+\left(n+\frac{3}{2}\right)\left(C_n-D_n\mp b_n\right)\right)\left(\cosh\xi-\zeta\right)\\-\left(\frac{3}{2}\right)\sinh\xi\left(A_n+B_n+C_n+D_n-\left(a_n+b_n\right)\right)\end{array}\right]g_n\left(\zeta\right)\right)=0$$

$$\left(\begin{array}{l}\sum_{n=1}^{\infty}\left[\left(A_n+B_n+C_n+D_n-\left(\mu\right)\left(a_n+b_n\right)\right)\left(\left(\frac{3}{4}-n\left(n+1\right)\right)+\left(\frac{\sinh\xi_2}{\cosh\xi_2-\zeta}\right)^2\right)g_n\left(\zeta\right)\right]\\-\sum_{m=1}^{\infty}\left[\left(\left(m-\frac{1}{2}\right)^2\left(A_m+B_m\right)+\left(m+\frac{3}{2}\right)^2\left(C_m+D_m\right)-\left(\mu\right)\left(\left(m-\frac{1}{2}\right)^2a_m+\left(m+\frac{3}{2}\right)^2b_m\right)\right)g_m\left(\zeta\right)\right]\end{array}\right)$$

$$=\pm\left(\frac{\beta\gamma_0k_2}{U\mu_1}\right)\left(\frac{Jc}{D_1}\right)\sum_{l=1}^{\infty}\left[\left(x_l\right)\left(\frac{\left(1-\zeta^2\right)}{2\left(\cosh\xi-\zeta\right)}p_l\left(\zeta\right)+\left(l\left(l+1\right)\right)g_l\left(\zeta\right)\right)\right]$$

$$\sum_{n=1}^{\infty}\left[\left(\begin{array}{l}A_ne^{(n-1/2)(\xi_1-\xi_2)}+B_ne^{-(n-1/2)(\xi_1-\xi_2)}\\+C_ne^{(n+3/2)(\xi_1-\xi_2)}+D_ne^{-(n+3/2)(\xi_1-\xi_2)}\end{array}\right)\left(\left(\cosh\xi_1-\zeta\right)\frac{dg_n\left(\zeta\right)}{d\zeta}+\frac{3}{2}g_n\left(\zeta\right)\right)\right]=\frac{-\left(V/U\right)\left(1-\zeta\cosh\xi_1\right)}{\sqrt{\left(\cosh\xi_1-\zeta\right)}}$$

$$\sum_{n=1}^{\infty}\left[\left(\begin{array}{l}\left(\begin{array}{l}\left(n-\frac{1}{2}\right)\left(A_ne^{(n-1/2)(\xi_1-\xi_2)}-B_ne^{-(n-1/2)(\xi_1-\xi_2)}\right)\\+\left(n+\frac{3}{2}\right)\left(C_ne^{(n+3/2)(\xi_1-\xi_2)}-D_ne^{-(n+3/2)(\xi_1-\xi_2)}\right)\end{array}\right)\left(\cosh\xi_1-\zeta\right)\\-\left(\frac{3}{2}\right)\sinh\xi_1\left(\begin{array}{l}A_ne^{(n-1/2)(\xi_1-\xi_2)}+B_ne^{-(n-1/2)(\xi_1-\xi_2)}\\+C_ne^{(n+3/2)(\xi_1-\xi_2)}+D_ne^{-(n+3/2)(\xi_1-\xi_2)}\end{array}\right)\end{array}\right)g_n\left(\zeta\right)\right]=\frac{\left(V/U\right)\sinh\xi_1\left(1-\zeta^2\right)}{\sqrt{\left(\cosh\xi_1-\zeta\right)}}$$

In deriving the above equations, the following recurrence relationship is used

$$(1-\zeta^2)\frac{d^2g_n\left(\zeta\right)}{d\zeta^2}=-n\left(n+1\right)g_n\left(\zeta\right)$$

Using the orthogonality properties of the modified Gegenbauer polynomials (similar to the arguments for the concentric case stream functions), the above system of equations can be reduced to a linear coupled matrix that can be suitably truncated and inverted as required.

Note that the dimensional scaling for the terminal velocity is still $\dfrac{U}{\left(\dfrac{\beta\gamma_0k_2}{\mu_1}\dfrac{JR_1}{D_1}\right)}$ since $p=R_1\sinh|\xi_1|$.

Force free relations for the drop and the Janus particle

$$\left\{ \sum_{n=1}^{\infty} \left(a_n e^{\mp(n-1/2)\xi_2} + b_n e^{\mp(n+3/2)\xi_2} \right) = 0 \right\},$$

$$\left\{ \begin{array}{ll} \sum_{n=1}^{\infty} \left(A_n e^{-(n-1/2)\xi_2} + C_n e^{-(n+3/2)\xi_2} \right) = 0 & \forall \{ \xi_2 > 0 \} \\ \sum_{n=1}^{\infty} \left(B_n e^{(n-1/2)\xi_2} + D_n e^{(n+3/2)\xi_2} \right) = 0 & \forall \{ \xi_2 < 0 \} \end{array} \right\}$$

In the case of the bispherical solutions, the truncation errors in computing the first 20 coefficients for this analysis are reduced by taking up to 25 terms in the matrix of the coupled system of equations. The convergence of the solutions are estimated by comparing the terminal velocities computed with 15 and 20 terms wherein the variation is well within 1%