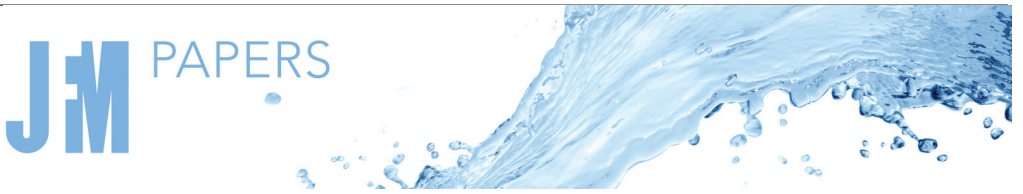

Supplementary Material

Linking turbulent waves and bubble diffusion in self-aerated open-channel flows: Two-state air concentration

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1. Presentation

This supplementary material presents the application of the two-state model to more than 500 concentration profiles from different data sets, comprising *smooth chute* data from Straub and Anderson (1958, 74 profiles), Killen (1968, 17 profiles), Bung (2009, 28 profiles), Severi (2018, 261 profiles), and *stepped chute* data from Bung (2009, 151 profiles), Zhang (2017, 6 profiles), and Kramer and Chanson (2018, 34 profiles). A summary of key parameters is given in Table 1.

In the following, the chute angle (θ), specific discharge (q), and streamwise location (x) of the respective profile are indicated in the top left corner of each subfigure, while the background of the first profile of each series of measurements is shaded in gray. The parameter x is presented as distance (in m) from the upstream crest for smooth chutes or as step edge (SE) number for stepped chutes.

Laboratory flows with depth-averaged air concentrations $\langle\overline{c}\rangle \lessapprox 0.25$ are dominated by free-surface instabilities and turbulent waves (air transported between wave crests and troughs), and an analytical solution for the air concentration ($\overline{c}$) involves the Gaussian error function [Eq. (2.2); blue lines]. In general, Eq. (2.2) is applicable to flows where the bubbly flow layer does not protrude to the channel bottom, which holds true for small chute angles ($\theta < 15°$) or for locations upstream or immediately downstream of the inception point of air entrainment. For $\langle\overline{c}\rangle \gtrapprox 0.25$, air bubbles are diffused deeper into the water column, and the time-averaged air concentration is described by the two-state principle, involving a convolution of the Rouse profile and the Gaussian error function with the interface probability [Eq. (2.8); red lines].

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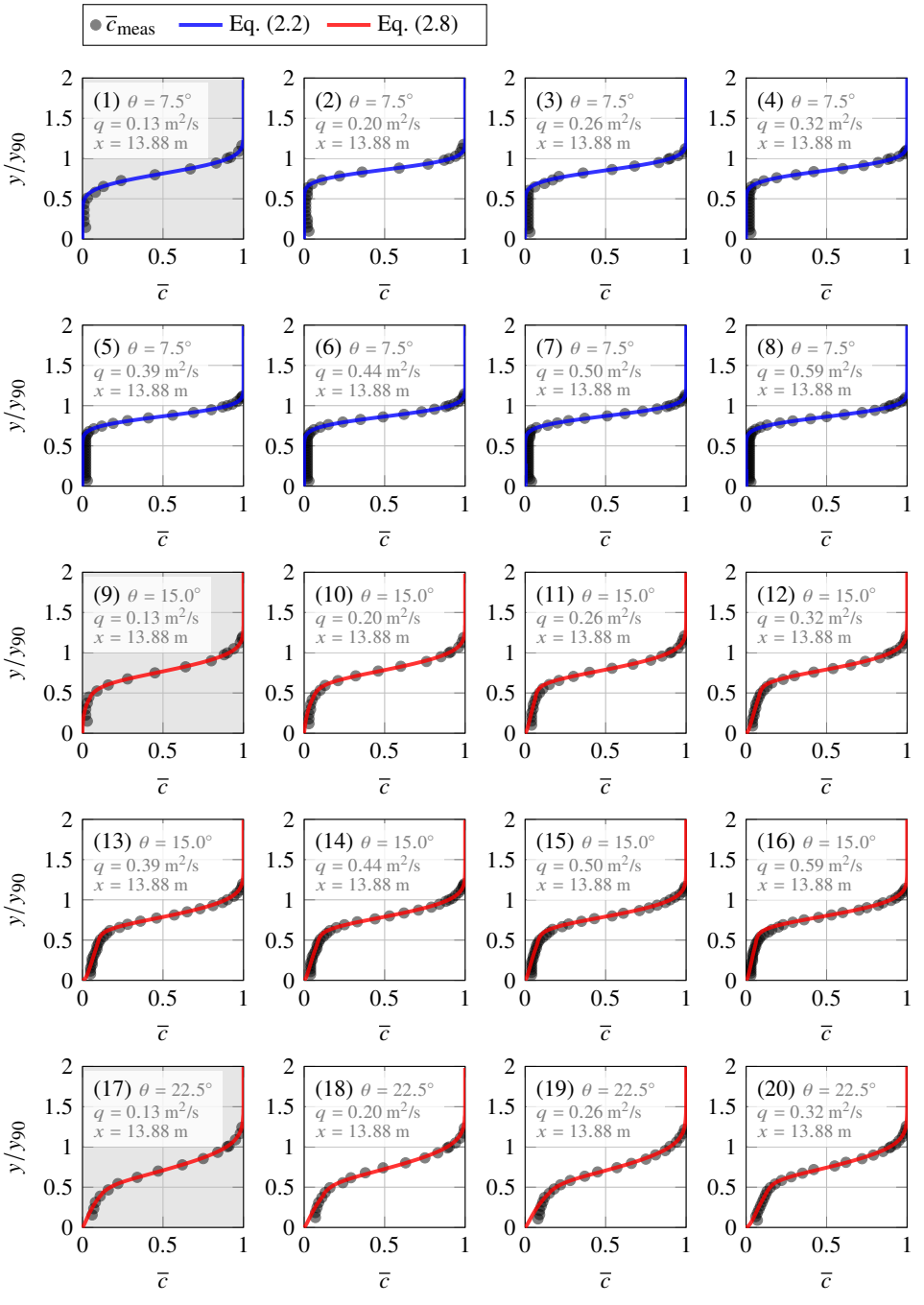
Table 1: Key parameters of the re-analysed data sets - smooth chutes (upper part) and stepped chutes (lower part)

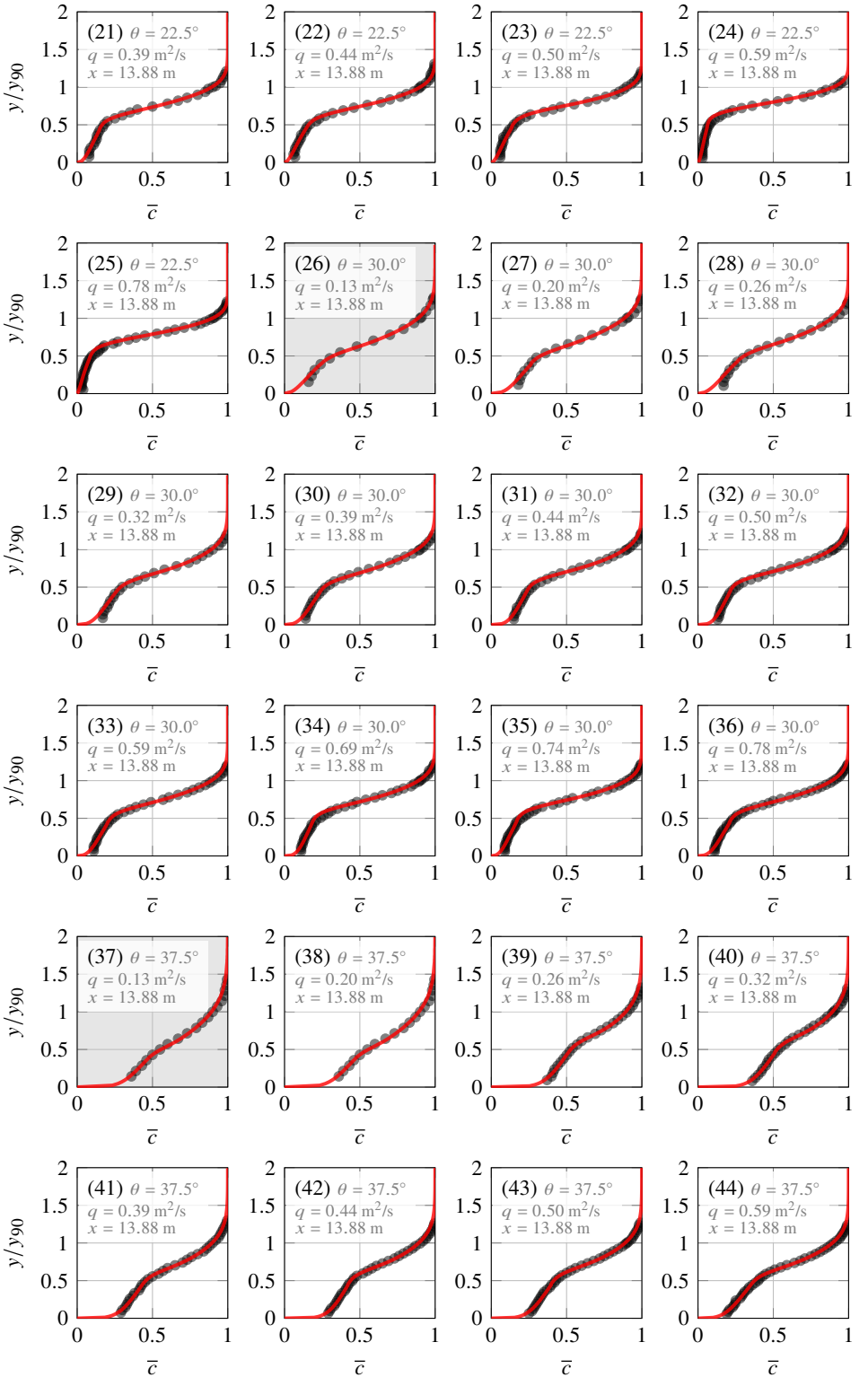
Reference	Straub and Anderson (1958)	Killen (1968)	Bung (2009)	Severi (2018)
chute type	smooth	smooth	smooth	smooth
q (m^2/s)	0.13 to 0.92	0.39 and 0.78	0.07 to 0.11	0.03 to 0.38
θ (°)	7.5 to 75	30 and 52.5	18.4	10.8
k_s (mm)	0.71	0.71	8	1.6 to 9.5
L_{chute} (m)	15.24	15.24	7.6	8
B_{chute} (m)	0.46	0.46	0.3	0.8
comment	granular roughness	granular roughness	micro-roughness	micro-roughness
Reference	Bung (2009)	Zhang (2017)	Kramer and Chanson (2018)	
chute type	stepped	stepped	stepped	
q (m^2/s)	0.07 to 0.11	0.14	0.032 to 0.11	
θ (°)	18.4 and 26.6	45.0	45.0	
h (m)	0.03 to 0.06	0.1	0.1	
k_s (mm)	26.8 to 56.9	70.7	70.7	
L_{chute} (m)	5.4 to 7.6	1.7	1.7	
B_{chute} (m)	0.3	0.99	0.99	
comment	skimming flow	skimming flow	transition + skimming flow	

q = specific discharge; θ = chute angle; k_s = roughness height; h = step height (stepped spillways)
L_{chute} = chute length; B_{chute} = chute width

2. Smooth chute data sets

2.1. *Straub and Anderson (1958, 74 profiles)*

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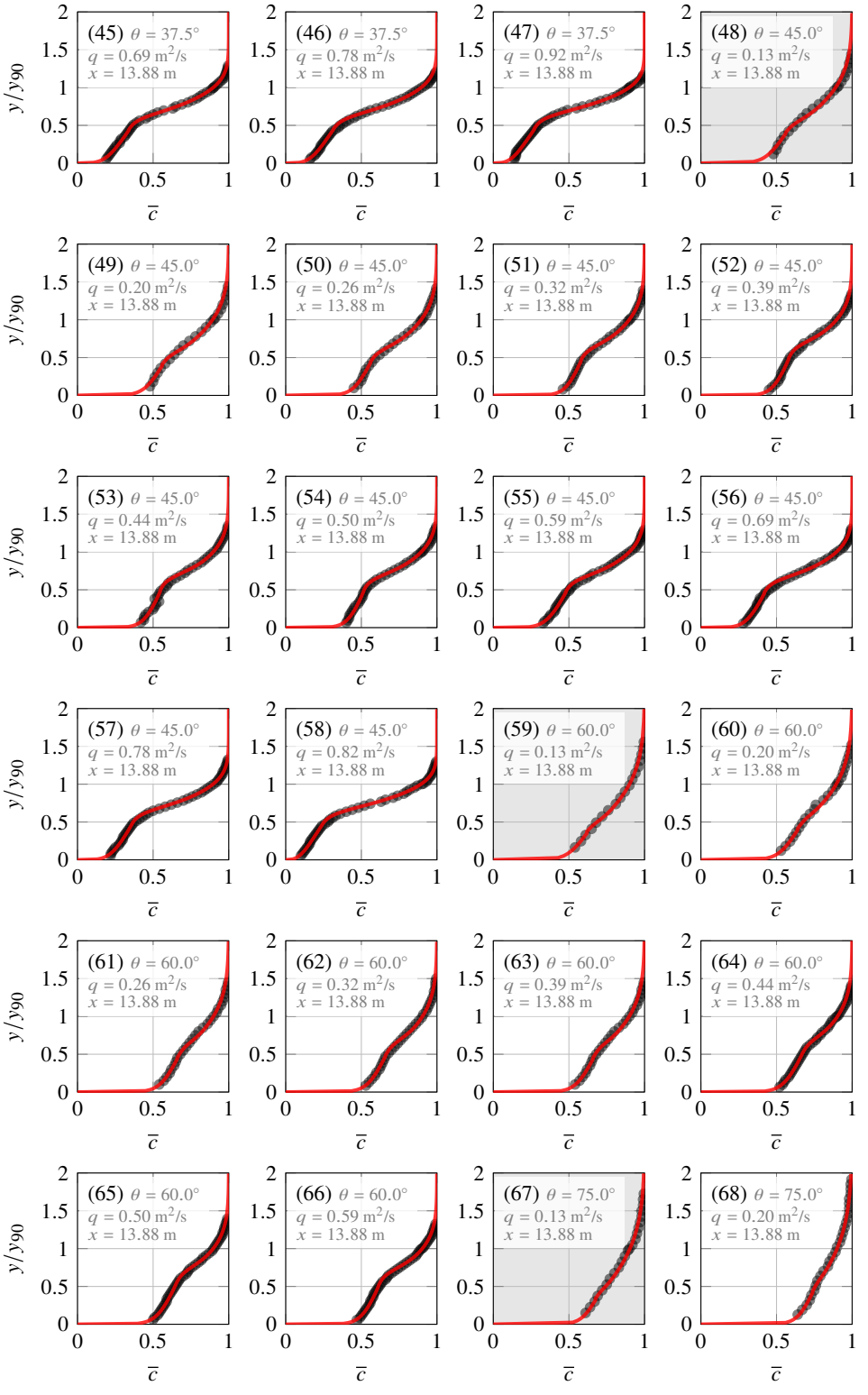
c̄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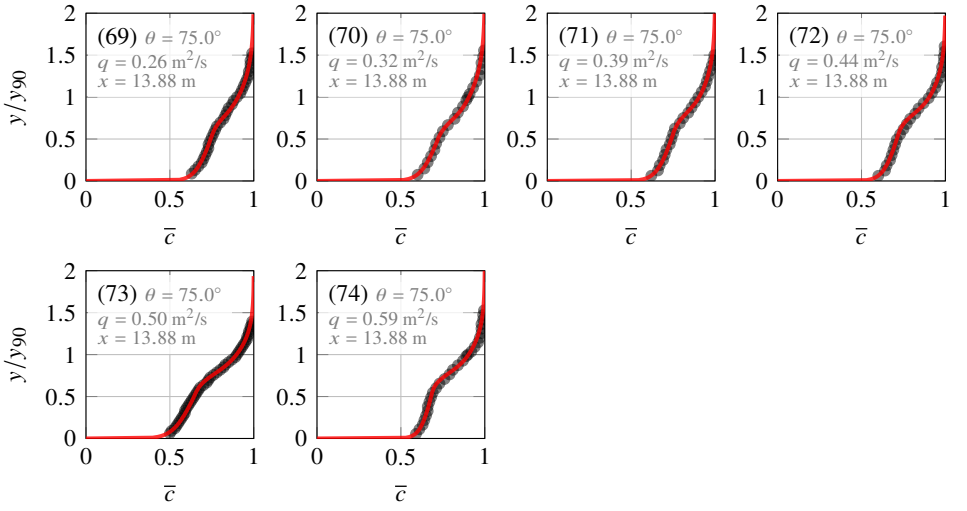
y/y90
c̄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x = 13.88 m
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q = 0.59 m²/s
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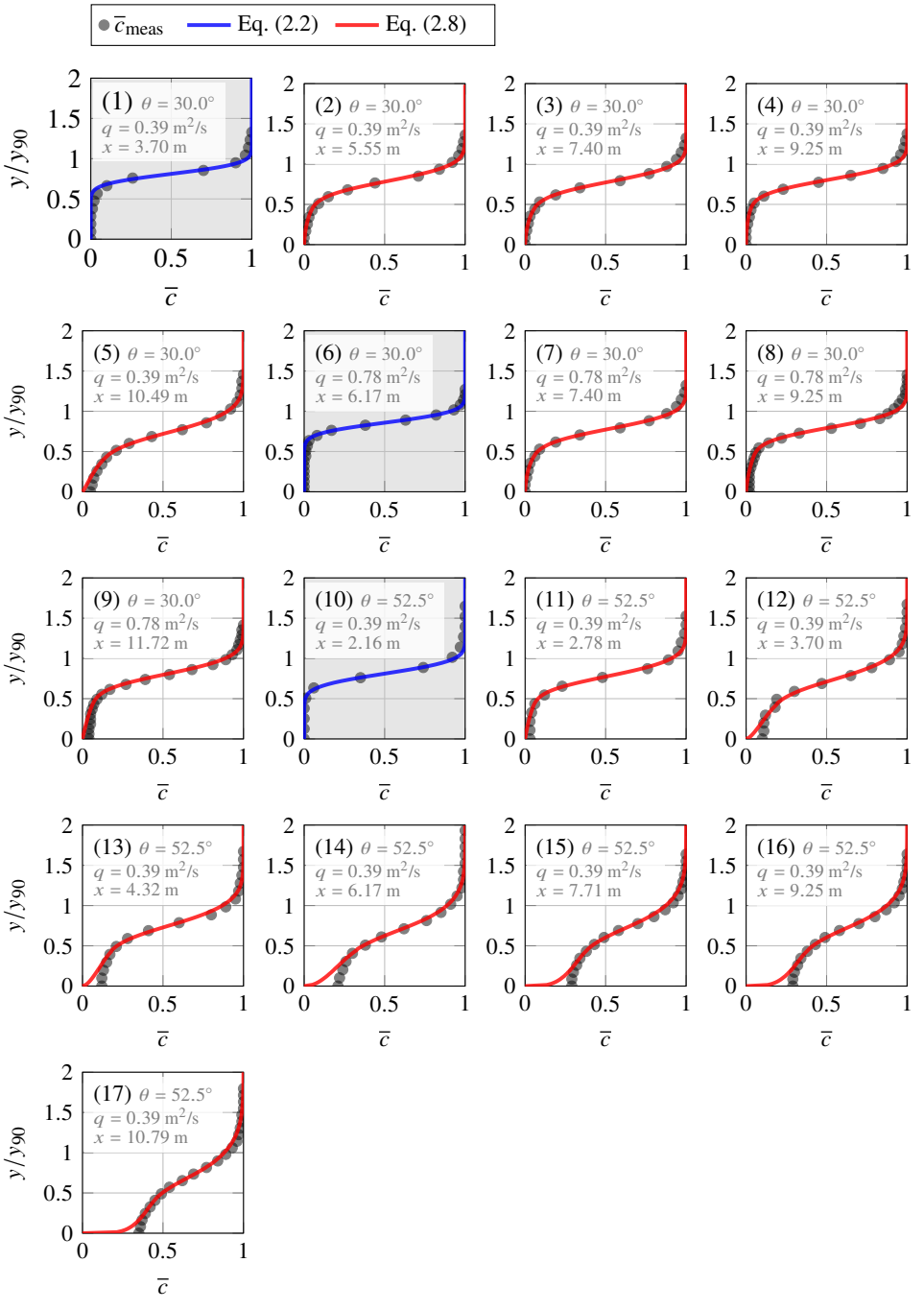
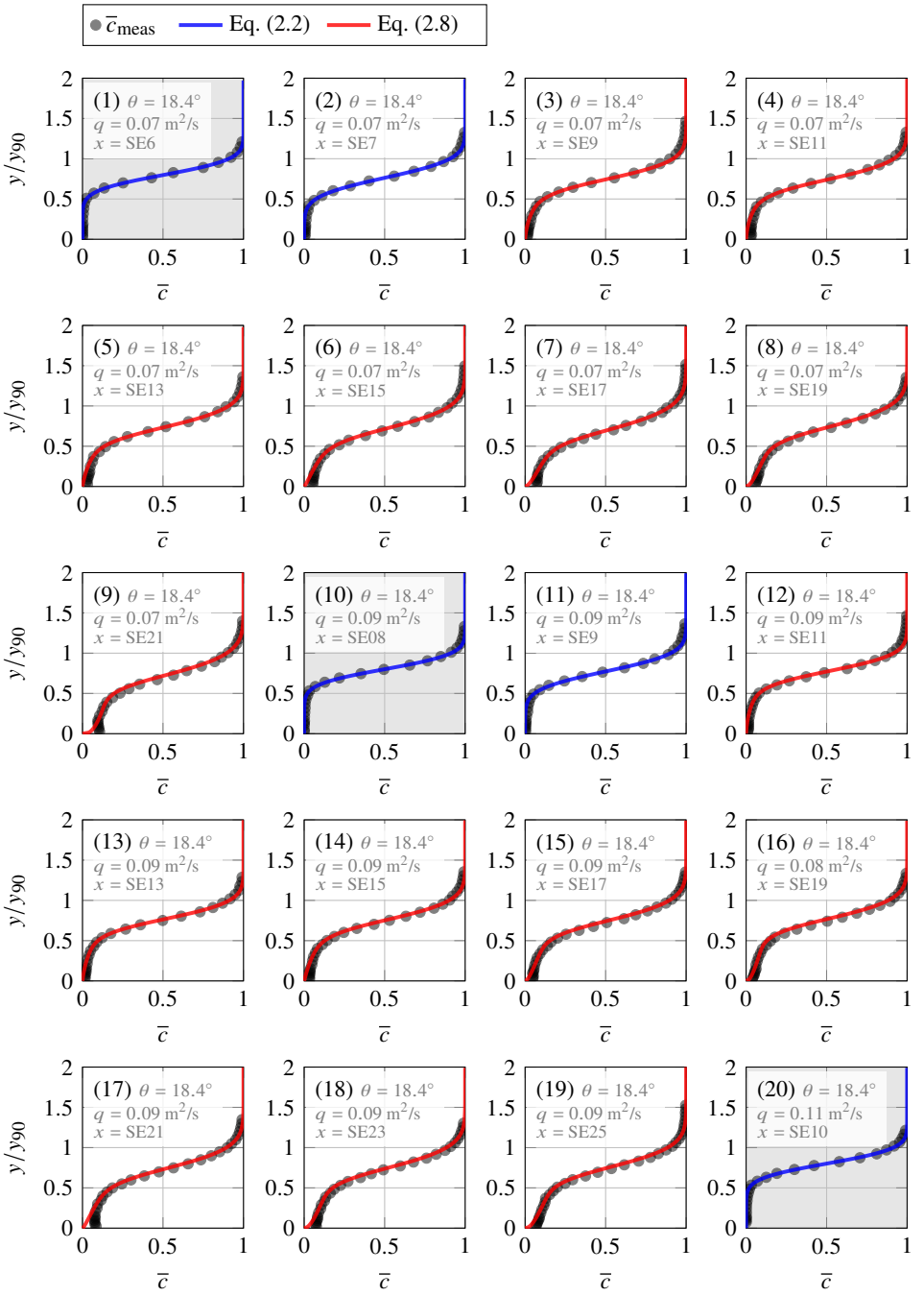
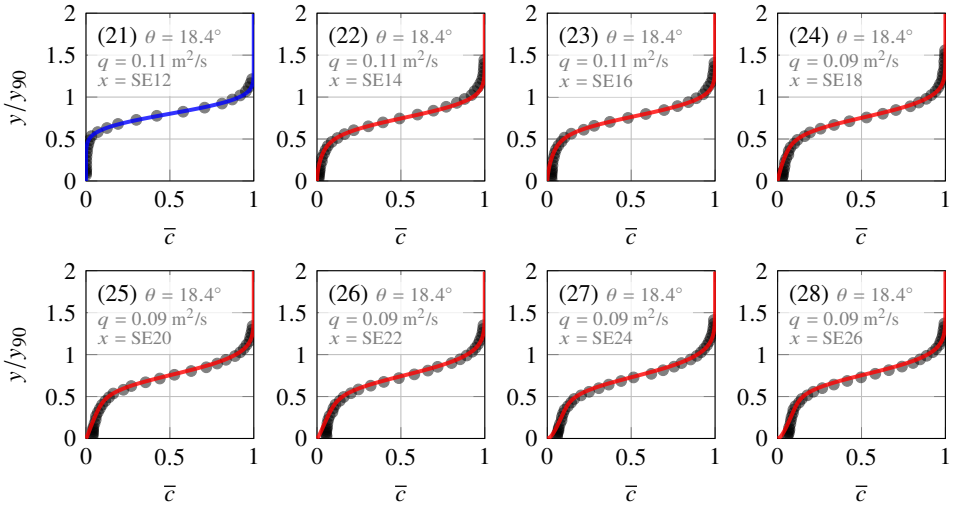
y/y90
c̄

2.2. *Killen (1968, 17 profiles)*



2.3. *Bung (2009, 28 profiles)*

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y/y90
c̄
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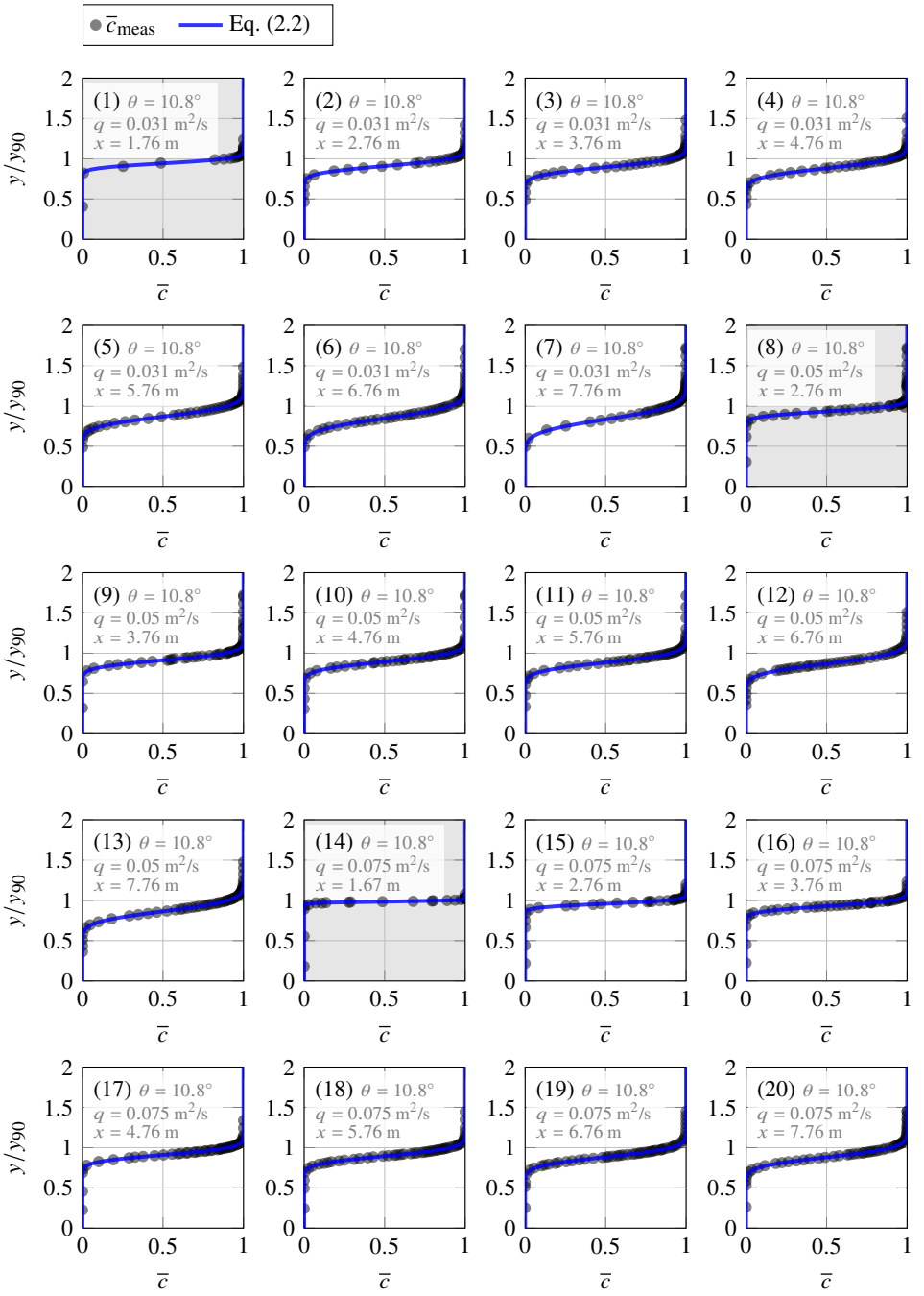
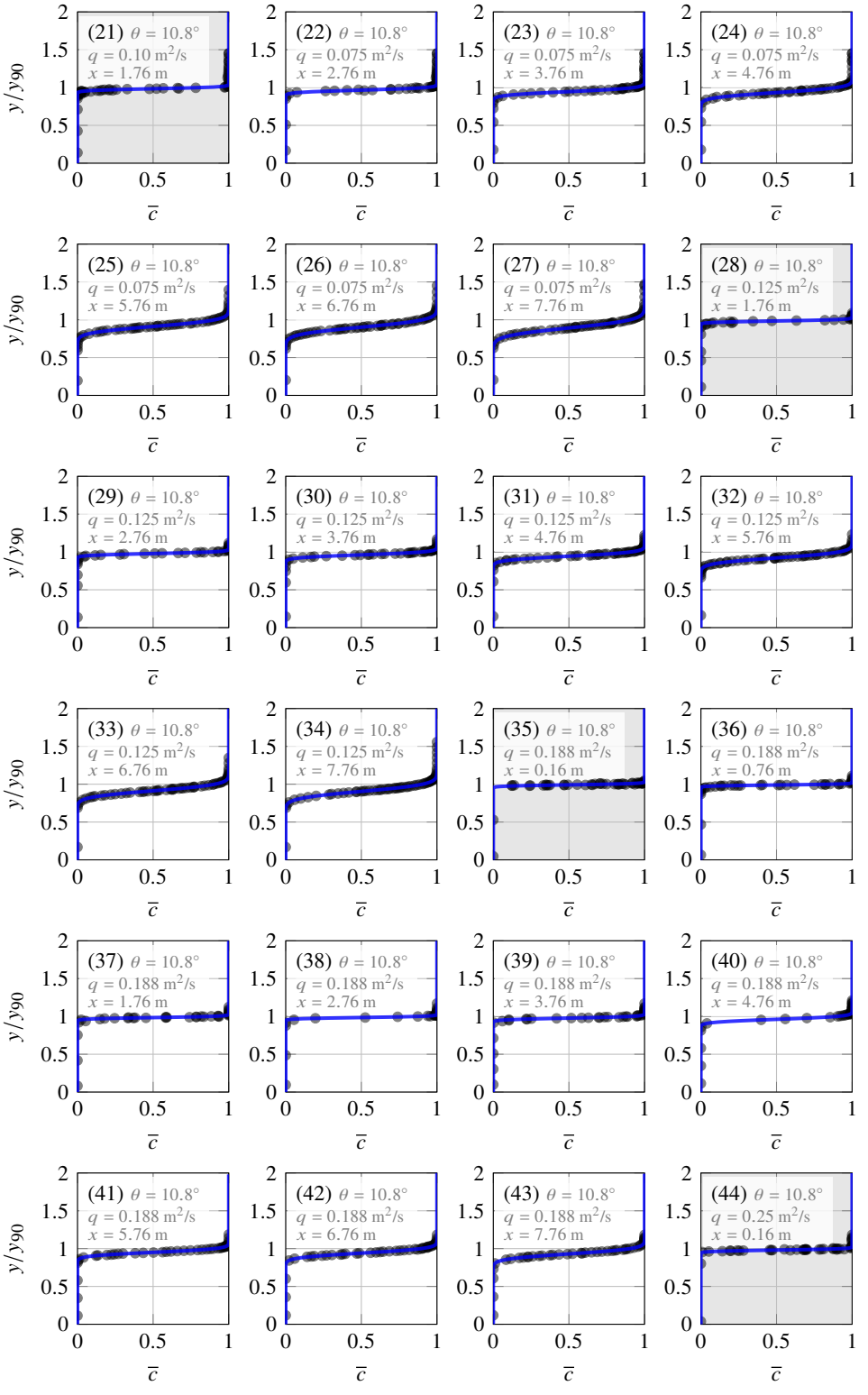
0.5
1
1.5
2

2.4. *Severi (2018, 261 profiles)*

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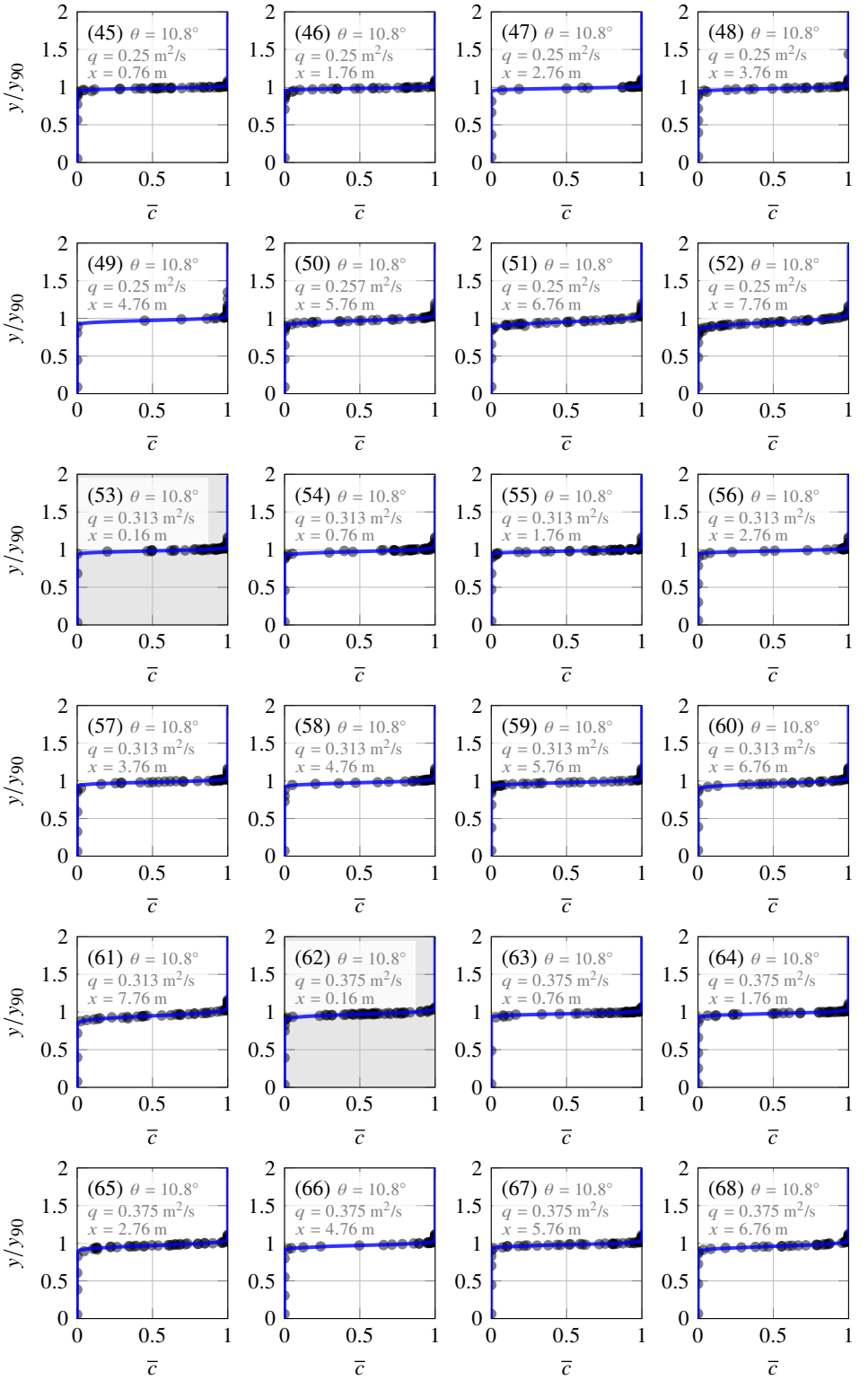
y/y90
c̄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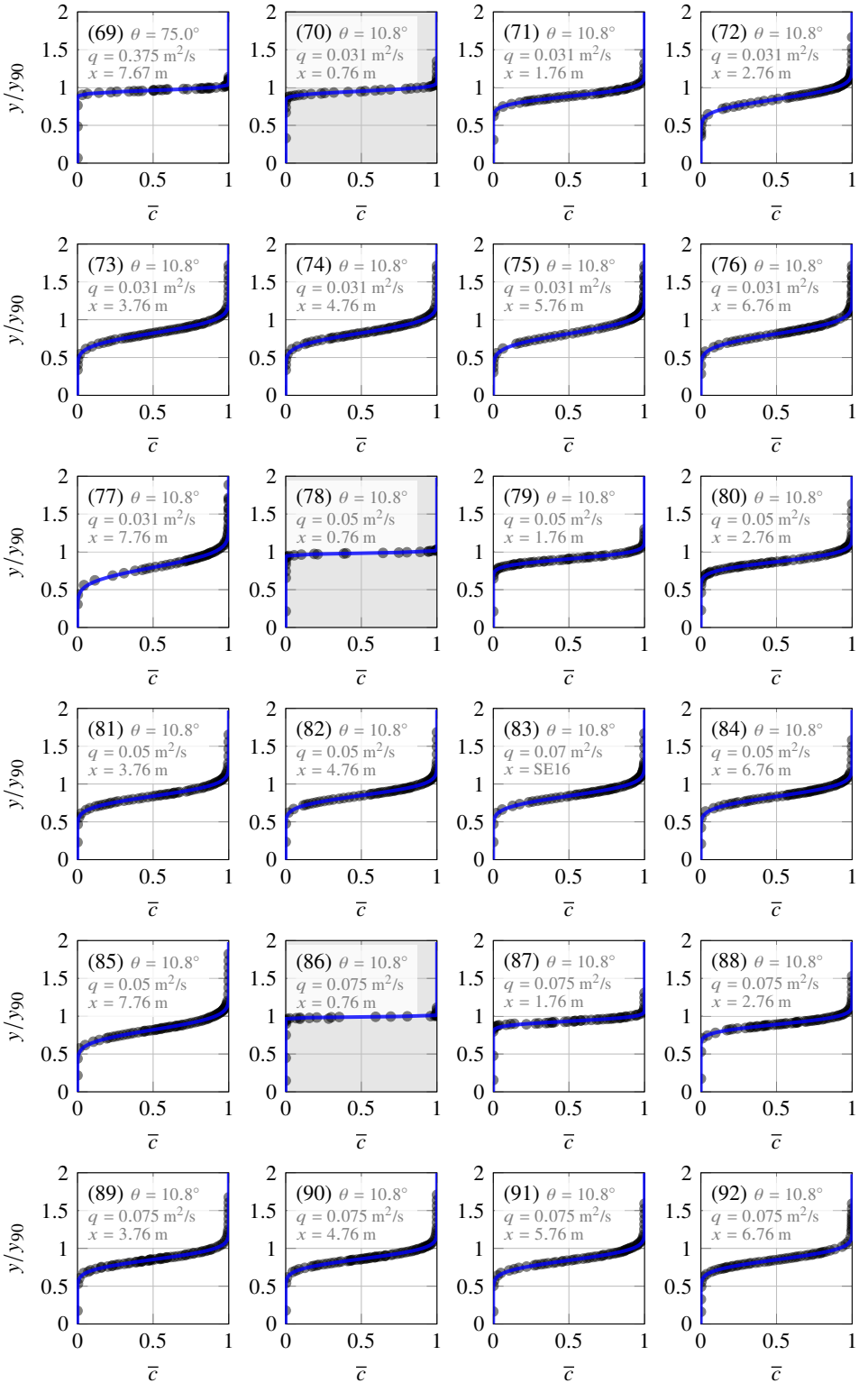
y/y90
c̄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y/y90
c̄
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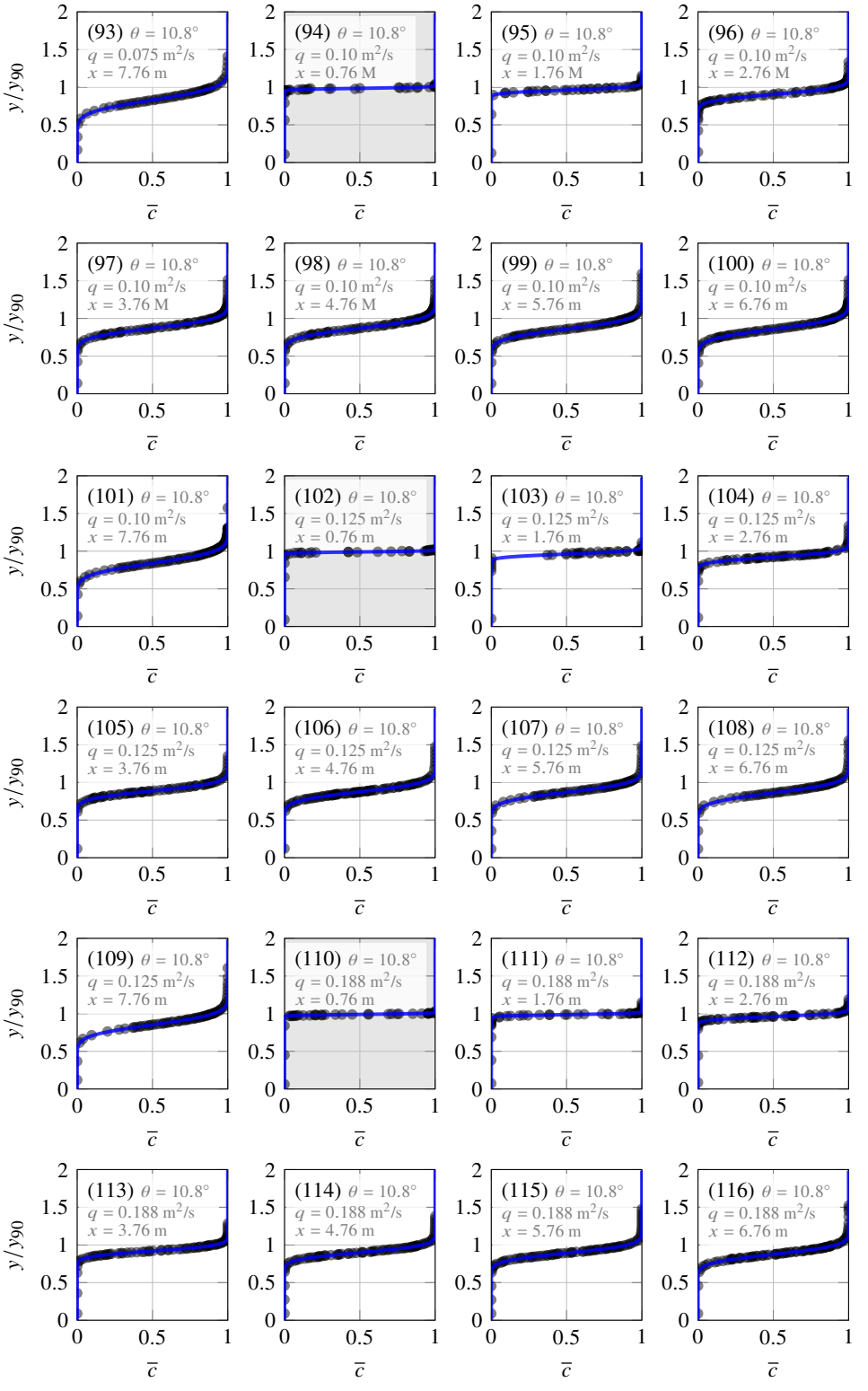
2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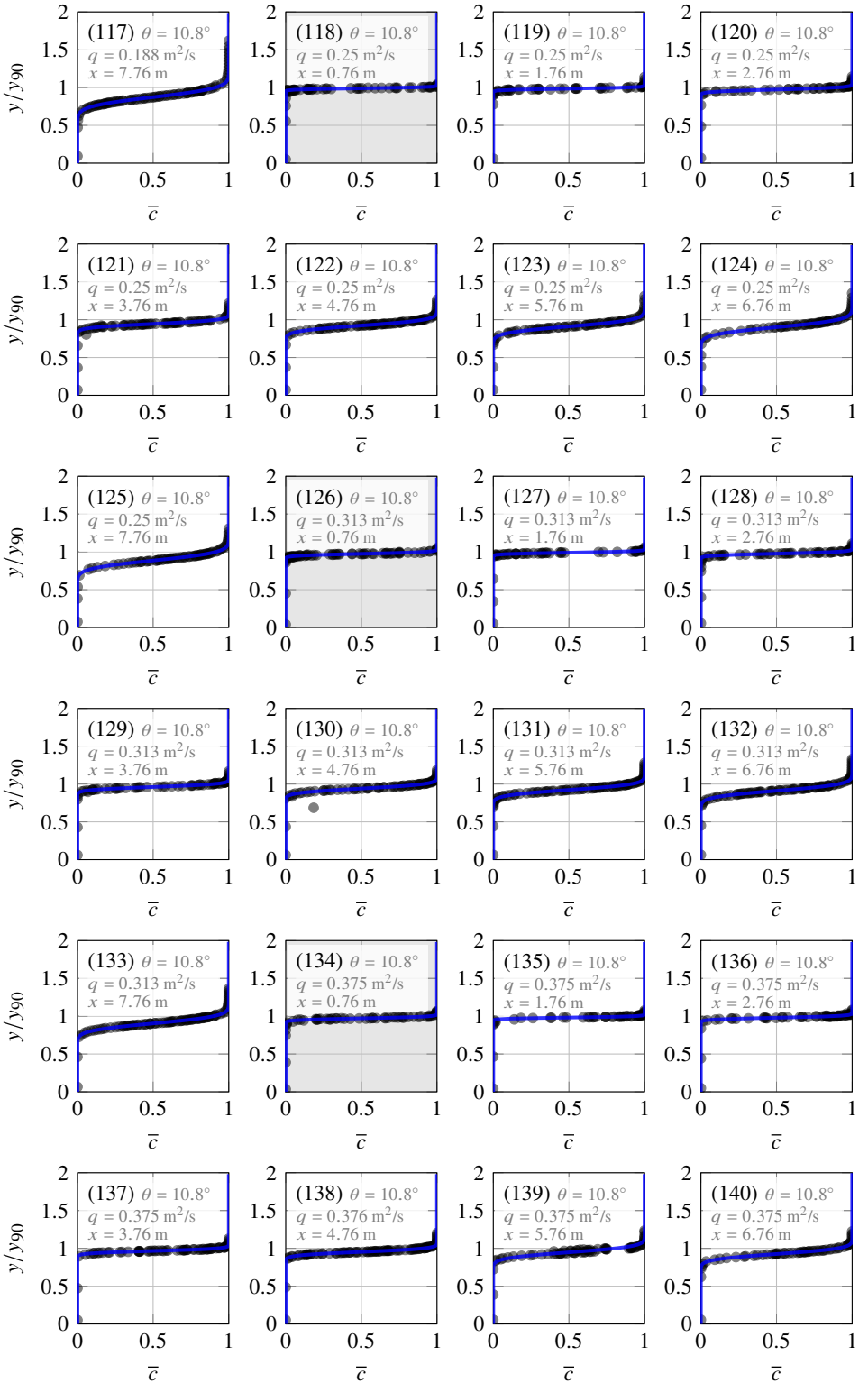
y/y90
c̄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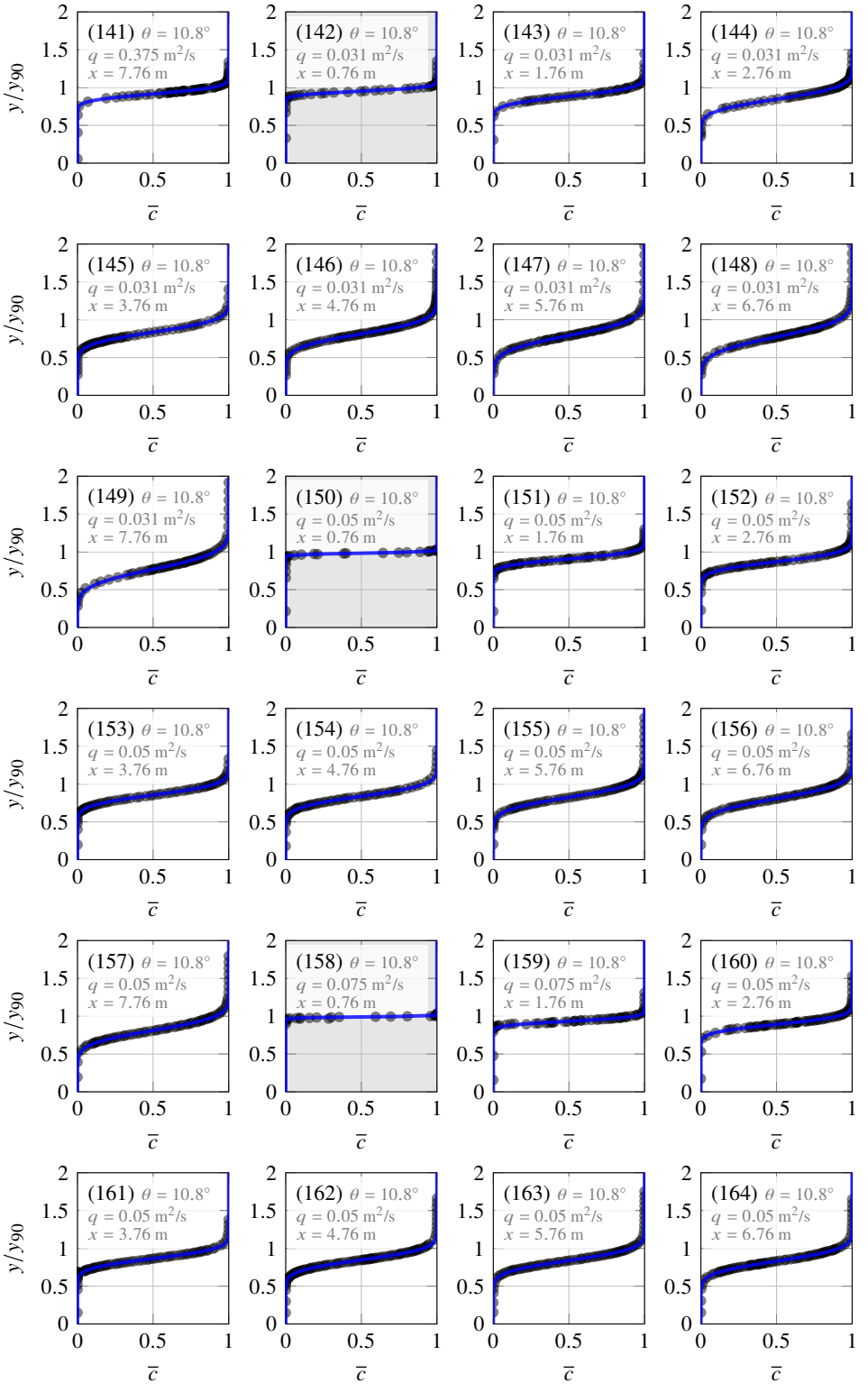
y/y90
c̄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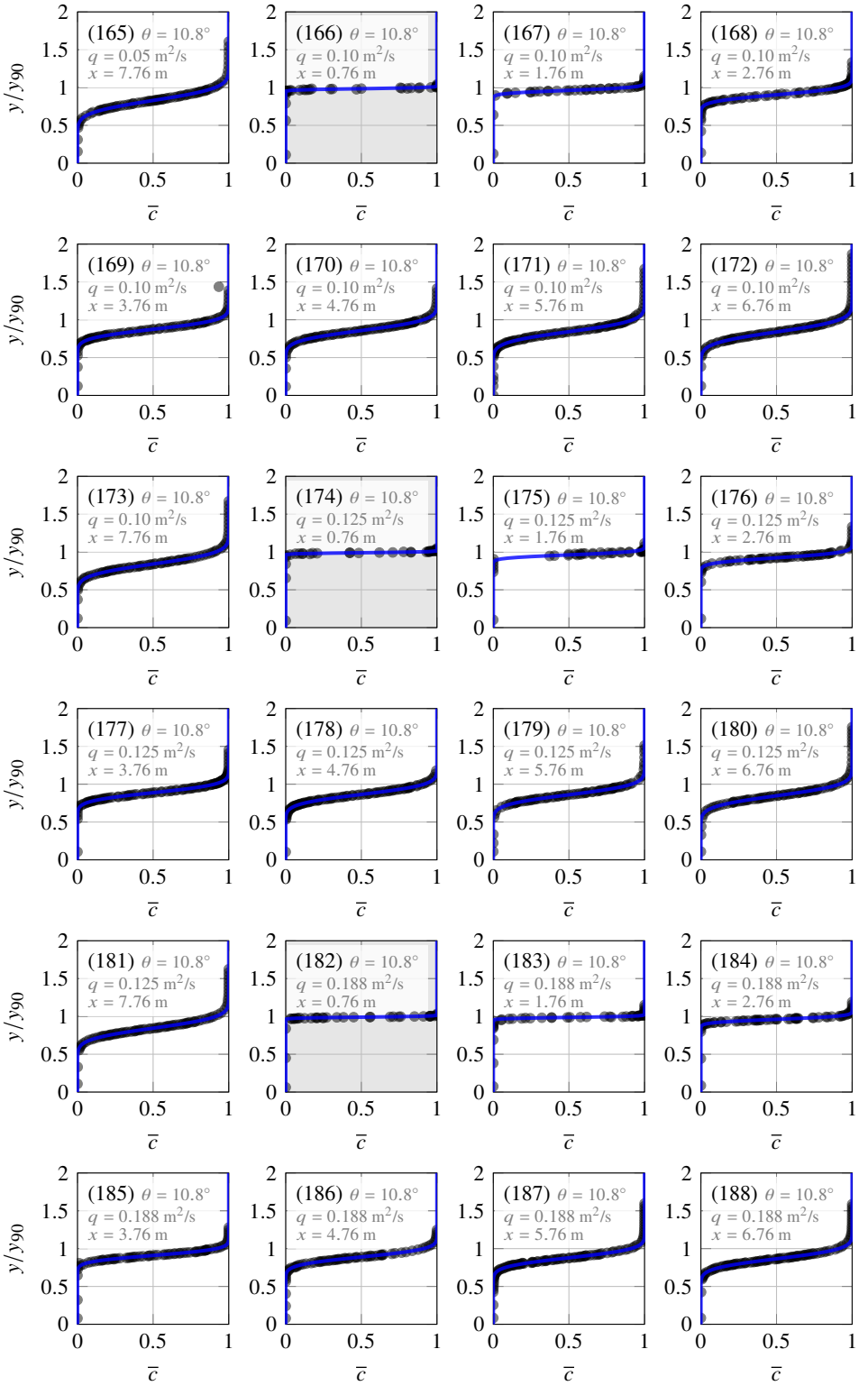
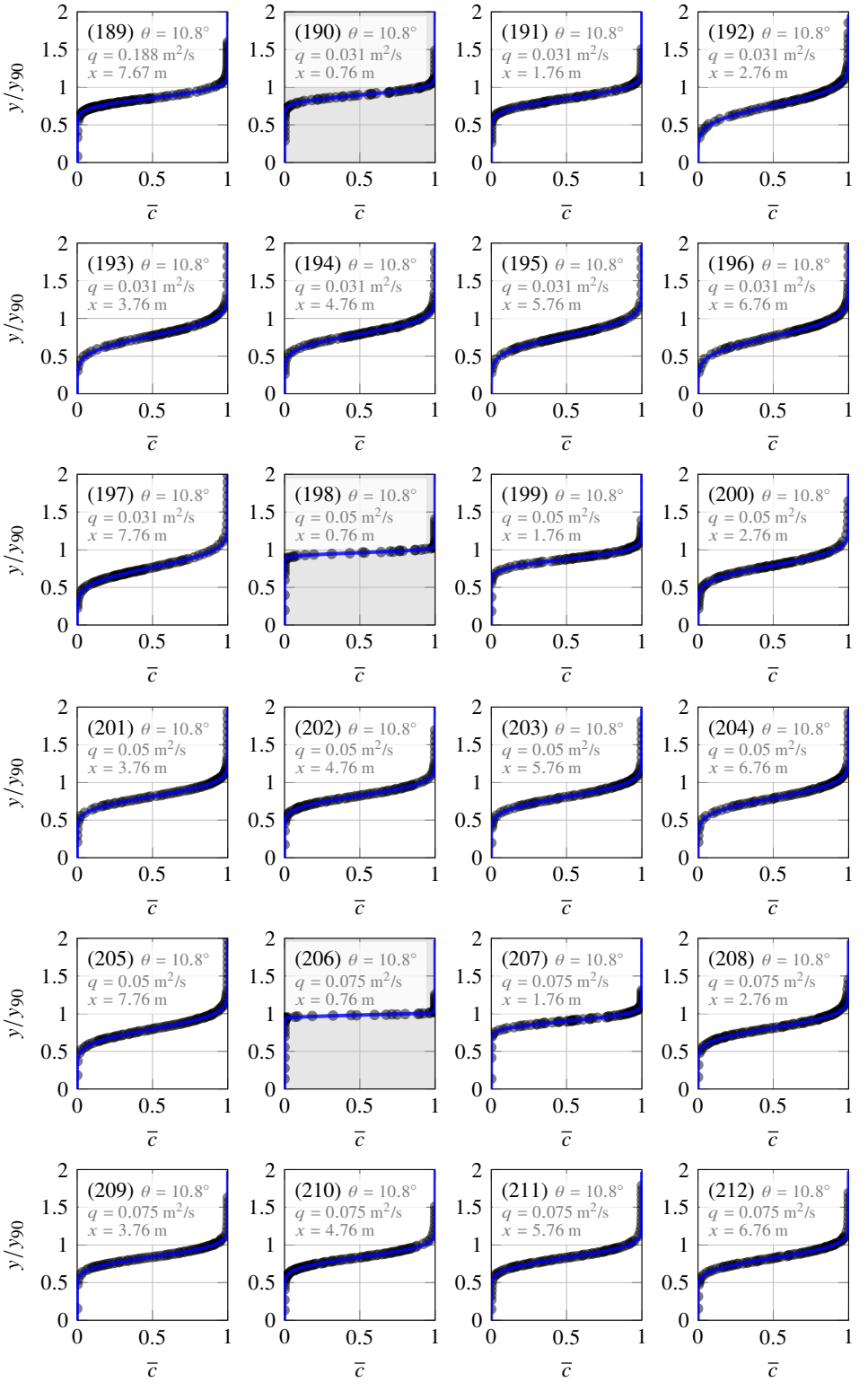
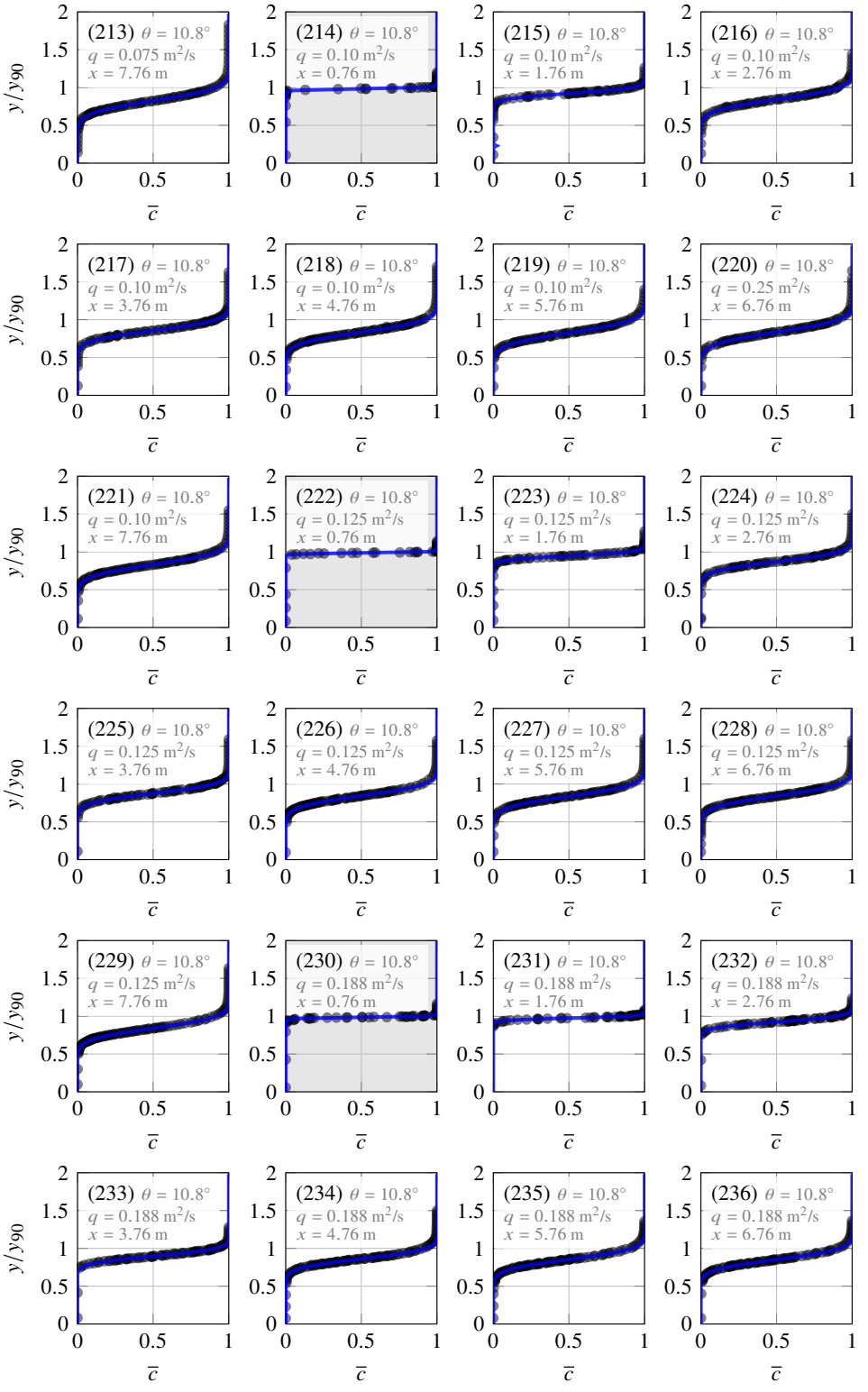
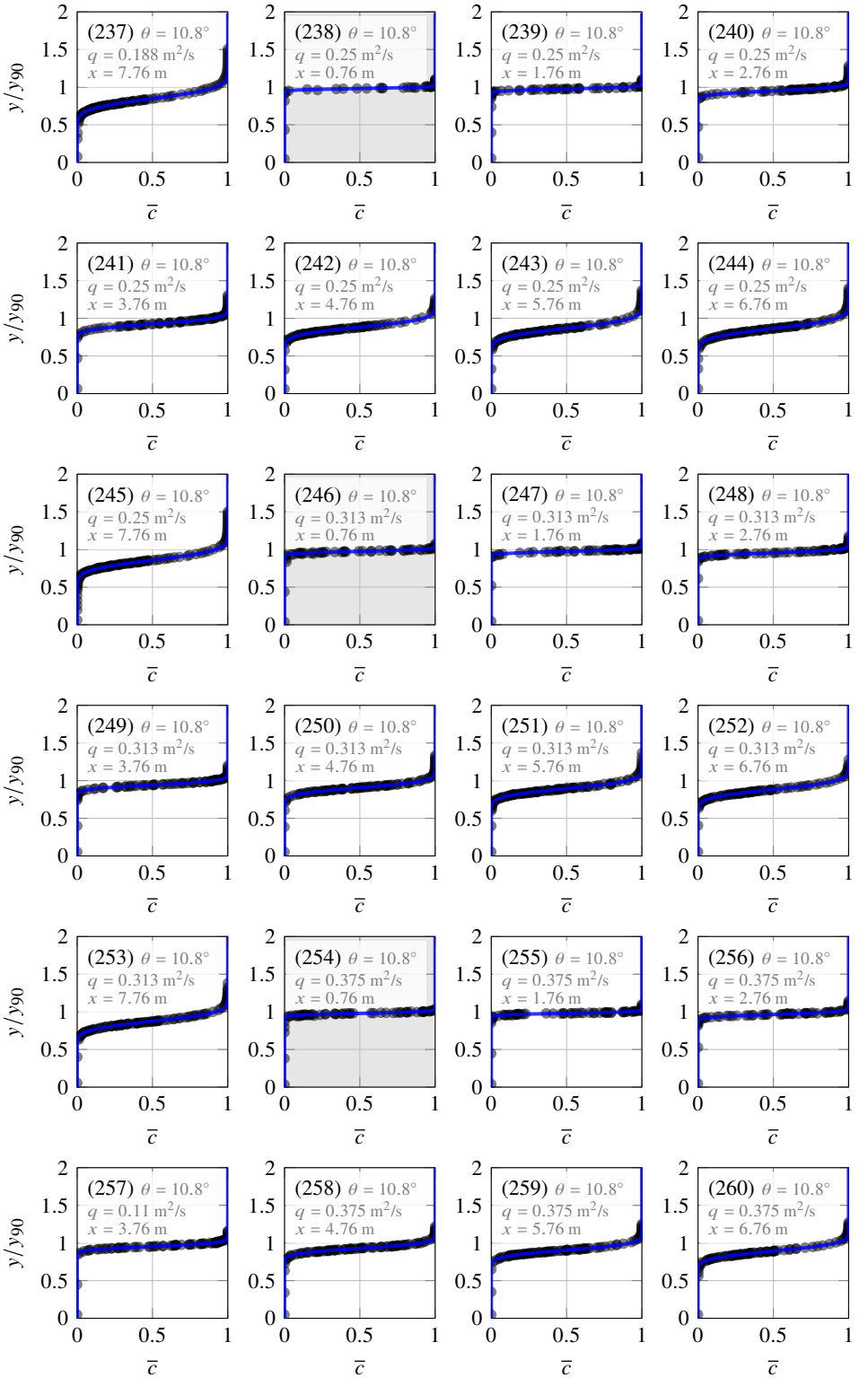
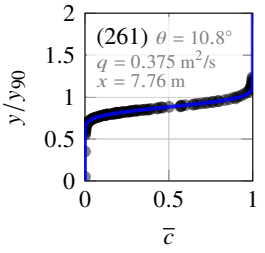
(144) θ = 10.8° q = 0.031 m²/s x = 2.76 m
(145) θ = 10.8° q = 0.031 m²/s x = 3.76 m
(146) θ = 10.8° q = 0.031 m²/s x = 4.76 m
(147) θ = 10.8° q = 0.031 m²/s x = 5.76 m
(148) θ = 10.8° q = 0.031 m²/s x = 6.76 m
(149) θ = 10.8° q = 0.031 m²/s x = 7.76 m
(150) θ = 10.8° q = 0.05 m²/s x = 0.76 m
(151) θ = 10.8° q = 0.05 m²/s x = 1.76 m
(152) θ = 10.8° q = 0.05 m²/s x = 2.76 m
(153) θ = 10.8° q = 0.05 m²/s x = 3.76 m
(154) θ = 10.8° q = 0.05 m²/s x = 4.76 m
(155) θ = 10.8° q = 0.05 m²/s x = 5.76 m
(156) θ = 10.8° q = 0.05 m²/s x = 6.76 m
(157) θ = 10.8° q = 0.05 m²/s x = 7.76 m
(158) θ = 10.8° q = 0.075 m²/s x = 0.76 m
(159) θ = 10.8° q = 0.075 m²/s x = 1.76 m
(160) θ = 10.8° q = 0.05 m²/s x = 2.76 m
(161) θ = 10.8° q = 0.05 m²/s x = 3.76 m
(162) θ = 10.8° q = 0.05 m²/s x = 4.76 m
(163) θ = 10.8° q = 0.05 m²/s x = 5.76 m
(164) θ = 10.8° q = 0.05 m²/s x = 6.76 m
y/y90
c̄


(165) θ = 10.8° q = 0.05 m²/s x = 7.76 m
(166) θ = 10.8° q = 0.10 m²/s x = 0.76 m
(167) θ = 10.8° q = 0.10 m²/s x = 1.76 m
(168) θ = 10.8° q = 0.10 m²/s x = 2.76 m
(169) θ = 10.8° q = 0.10 m²/s x = 3.76 m
(170) θ = 10.8° q = 0.10 m²/s x = 4.76 m
(171) θ = 10.8° q = 0.10 m²/s x = 5.76 m
(172) θ = 10.8° q = 0.10 m²/s x = 6.76 m
(173) θ = 10.8° q = 0.10 m²/s x = 7.76 m
(174) θ = 10.8° q = 0.125 m²/s x = 0.76 m
(175) θ = 10.8° q = 0.125 m²/s x = 1.76 m
(176) θ = 10.8° q = 0.125 m²/s x = 2.76 m
(177) θ = 10.8° q = 0.125 m²/s x = 3.76 m
(178) θ = 10.8° q = 0.125 m²/s x = 4.76 m
(179) θ = 10.8° q = 0.125 m²/s x = 5.76 m
(180) θ = 10.8° q = 0.125 m²/s x = 6.76 m
(181) θ = 10.8° q = 0.125 m²/s x = 7.76 m
(182) θ = 10.8° q = 0.188 m²/s x = 0.76 m
(183) θ = 10.8° q = 0.188 m²/s x = 1.76 m
(184) θ = 10.8° q = 0.188 m²/s x = 2.76 m
(185) θ = 10.8° q = 0.188 m²/s x = 3.76 m
(186) θ = 10.8° q = 0.188 m²/s x = 4.76 m
(187) θ = 10.8° q = 0.188 m²/s x = 5.76 m
(188) θ = 10.8° q = 0.188 m²/s x = 6.76 m
y/y90
c̄


(189) θ = 10.8° q = 0.188 m²/s x = 7.67 m
(190) θ = 10.8° q = 0.031 m²/s x = 0.76 m
(191) θ = 10.8° q = 0.031 m²/s x = 1.76 m
(192) θ = 10.8° q = 0.031 m²/s x = 2.76 m
(193) θ = 10.8° q = 0.031 m²/s x = 3.76 m
(194) θ = 10.8° q = 0.031 m²/s x = 4.76 m
(195) θ = 10.8° q = 0.031 m²/s x = 5.76 m
(196) θ = 10.8° q = 0.031 m²/s x = 6.76 m
(197) θ = 10.8° q = 0.031 m²/s x = 7.76 m
(198) θ = 10.8° q = 0.05 m²/s x = 0.76 m
(199) θ = 10.8° q = 0.05 m²/s x = 1.76 m
(200) θ = 10.8° q = 0.05 m²/s x = 2.76 m
(201) θ = 10.8° q = 0.05 m²/s x = 3.76 m
(202) θ = 10.8° q = 0.05 m²/s x = 4.76 m
(203) θ = 10.8° q = 0.05 m²/s x = 5.76 m
(204) θ = 10.8° q = 0.05 m²/s x = 6.76 m
(205) θ = 10.8° q = 0.05 m²/s x = 7.76 m
(206) θ = 10.8° q = 0.075 m²/s x = 0.76 m
(207) θ = 10.8° q = 0.075 m²/s x = 1.76 m
(208) θ = 10.8° q = 0.075 m²/s x = 2.76 m
(209) θ = 10.8° q = 0.075 m²/s x = 3.76 m
(210) θ = 10.8° q = 0.075 m²/s x = 4.76 m
(211) θ = 10.8° q = 0.075 m²/s x = 5.76 m
(212) θ = 10.8° q = 0.075 m²/s x = 6.76 m
y/y90
c̄

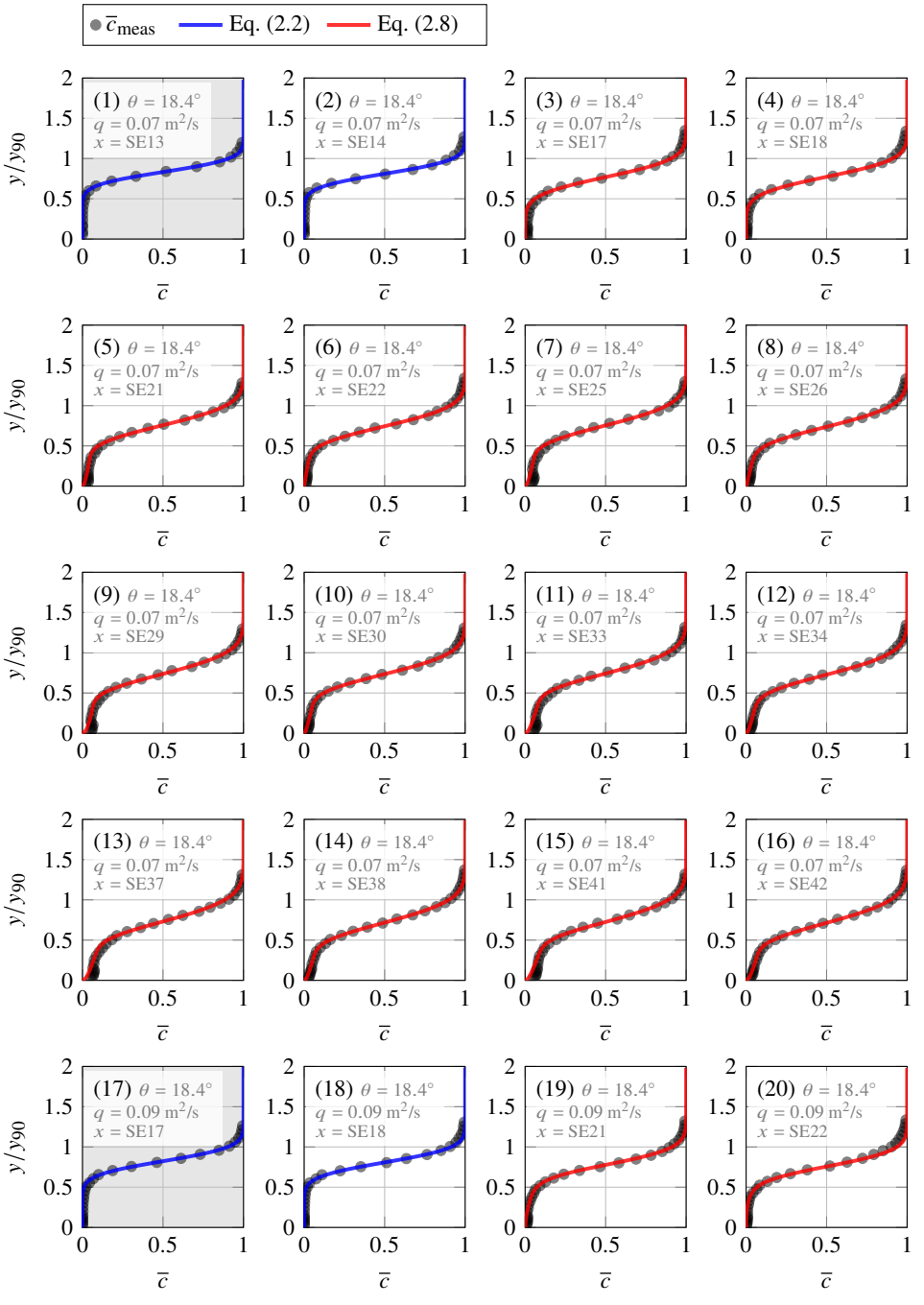

(213) θ = 10.8° q = 0.075 m²/s x = 7.76 m
(214) θ = 10.8° q = 0.10 m²/s x = 0.76 m
(215) θ = 10.8° q = 0.10 m²/s x = 1.76 m
(216) θ = 10.8° q = 0.10 m²/s x = 2.76 m
(217) θ = 10.8° q = 0.10 m²/s x = 3.76 m
(218) θ = 10.8° q = 0.10 m²/s x = 4.76 m
(219) θ = 10.8° q = 0.10 m²/s x = 5.76 m
(220) θ = 10.8° q = 0.25 m²/s x = 6.76 m
(221) θ = 10.8° q = 0.10 m²/s x = 7.76 m
(222) θ = 10.8° q = 0.125 m²/s x = 0.76 m
(223) θ = 10.8° q = 0.125 m²/s x = 1.76 m
(224) θ = 10.8° q = 0.125 m²/s x = 2.76 m
(225) θ = 10.8° q = 0.125 m²/s x = 3.76 m
(226) θ = 10.8° q = 0.125 m²/s x = 4.76 m
(227) θ = 10.8° q = 0.125 m²/s x = 5.76 m
(228) θ = 10.8° q = 0.125 m²/s x = 6.76 m
(229) θ = 10.8° q = 0.125 m²/s x = 7.76 m
(230) θ = 10.8° q = 0.188 m²/s x = 0.76 m
(231) θ = 10.8° q = 0.188 m²/s x = 1.76 m
(232) θ = 10.8° q = 0.188 m²/s x = 2.76 m
(233) θ = 10.8° q = 0.188 m²/s x = 3.76 m
(234) θ = 10.8° q = 0.188 m²/s x = 4.76 m
(235) θ = 10.8° q = 0.188 m²/s x = 5.76 m
(236) θ = 10.8° q = 0.188 m²/s x = 6.76 m
y/y90
c̄

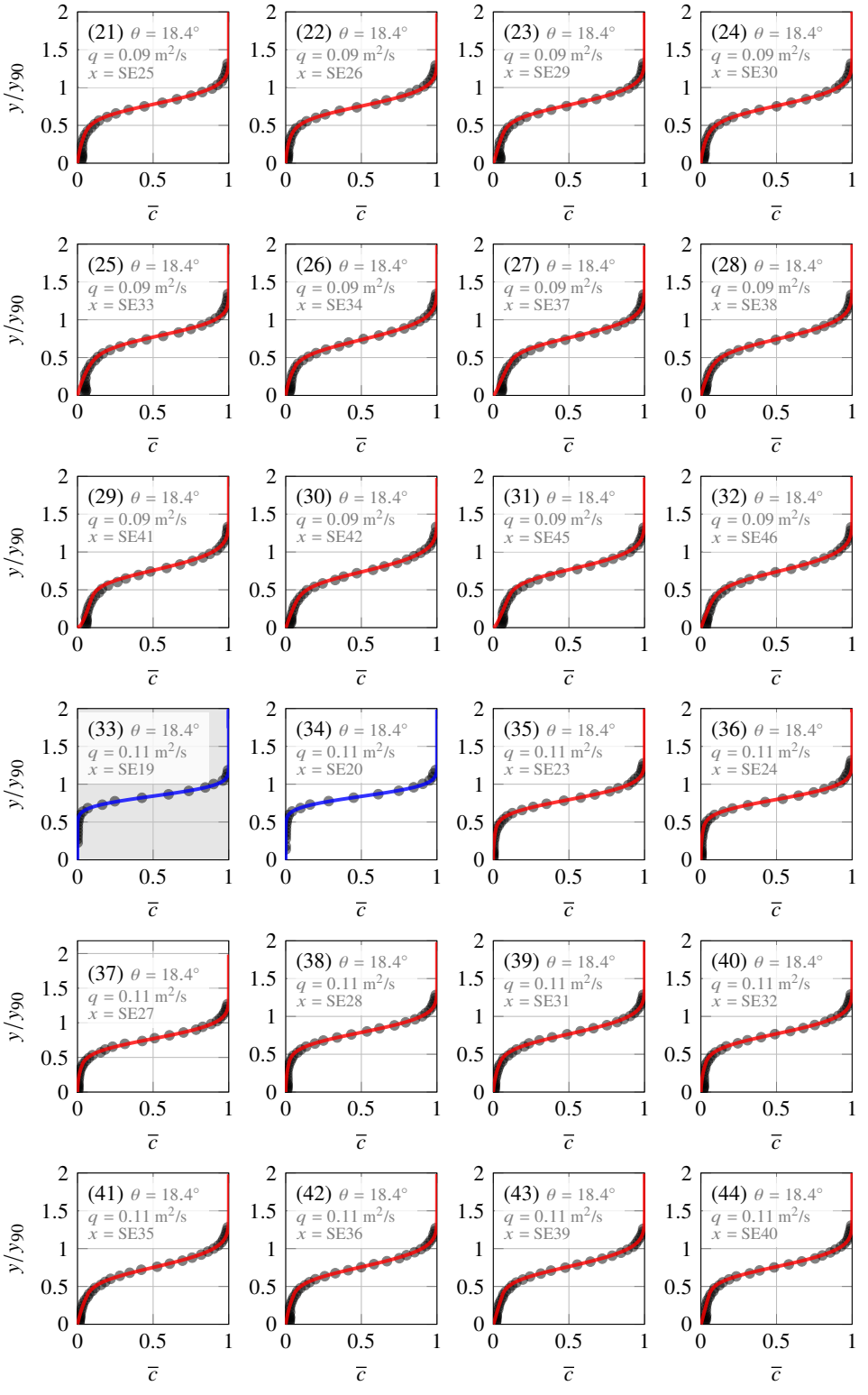

(237) θ = 10.8°
q = 0.188 m²/s
x = 7.76 m
(238) θ = 10.8°
q = 0.25 m²/s
x = 0.76 m
(239) θ = 10.8°
q = 0.25 m²/s
x = 1.76 m
(240) θ = 10.8°
q = 0.25 m²/s
x = 2.76 m
(241) θ = 10.8°
q = 0.25 m²/s
x = 3.76 m
(242) θ = 10.8°
q = 0.25 m²/s
x = 4.76 m
(243) θ = 10.8°
q = 0.25 m²/s
x = 5.76 m
(244) θ = 10.8°
q = 0.25 m²/s
x = 6.76 m
(245) θ = 10.8°
q = 0.25 m²/s
x = 7.76 m
(246) θ = 10.8°
q = 0.313 m²/s
x = 0.76 m
(247) θ = 10.8°
q = 0.313 m²/s
x = 1.76 m
(248) θ = 10.8°
q = 0.313 m²/s
x = 2.76 m
(249) θ = 10.8°
q = 0.313 m²/s
x = 3.76 m
(250) θ = 10.8°
q = 0.313 m²/s
x = 4.76 m
(251) θ = 10.8°
q = 0.313 m²/s
x = 5.76 m
(252) θ = 10.8°
q = 0.313 m²/s
x = 6.76 m
(253) θ = 10.8°
q = 0.313 m²/s
x = 7.76 m
(254) θ = 10.8°
q = 0.375 m²/s
x = 0.76 m
(255) θ = 10.8°
q = 0.375 m²/s
x = 1.76 m
(256) θ = 10.8°
q = 0.375 m²/s
x = 2.76 m
(257) θ = 10.8°
q = 0.11 m²/s
x = 3.76 m
(258) θ = 10.8°
q = 0.375 m²/s
x = 4.76 m
(259) θ = 10.8°
q = 0.375 m²/s
x = 5.76 m
(260) θ = 10.8°
q = 0.375 m²/s
x = 6.76 m
y/y90
c̄

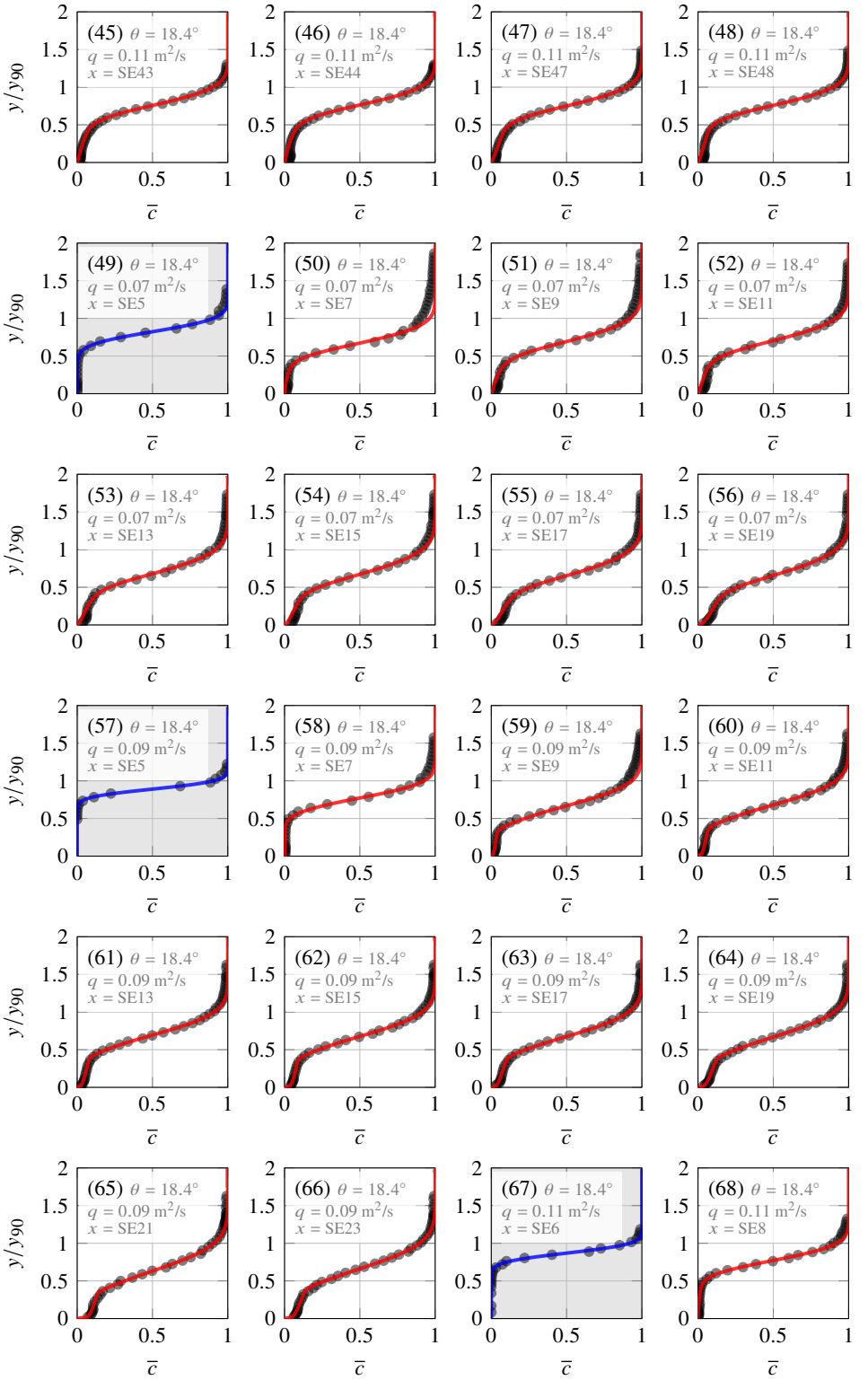

(261) θ = 10.8°
q = 0.375 m²/s
x = 7.76 m
y/y90
c̄
0
0.5
1
1.5
2

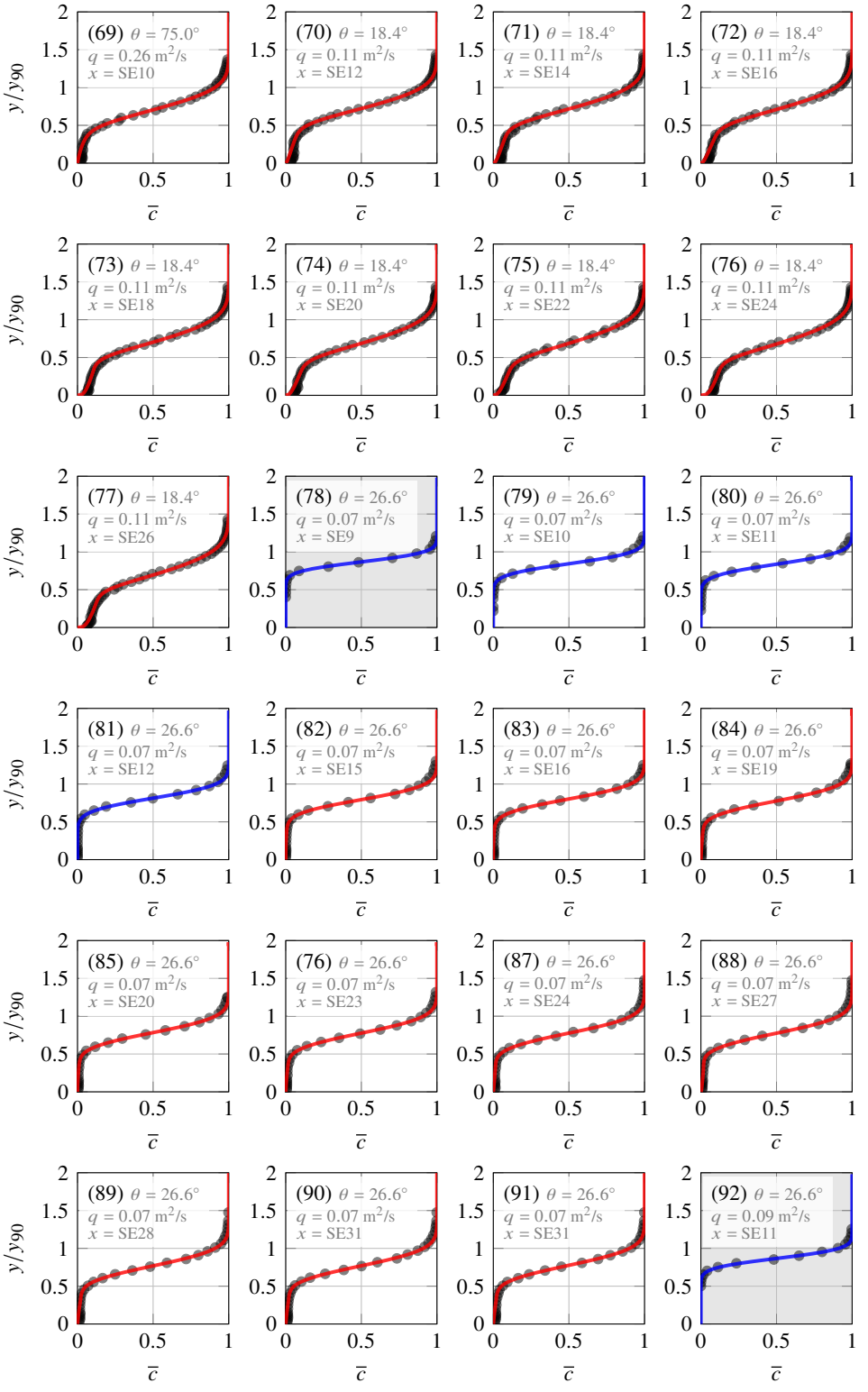
3. Stepped chute data sets

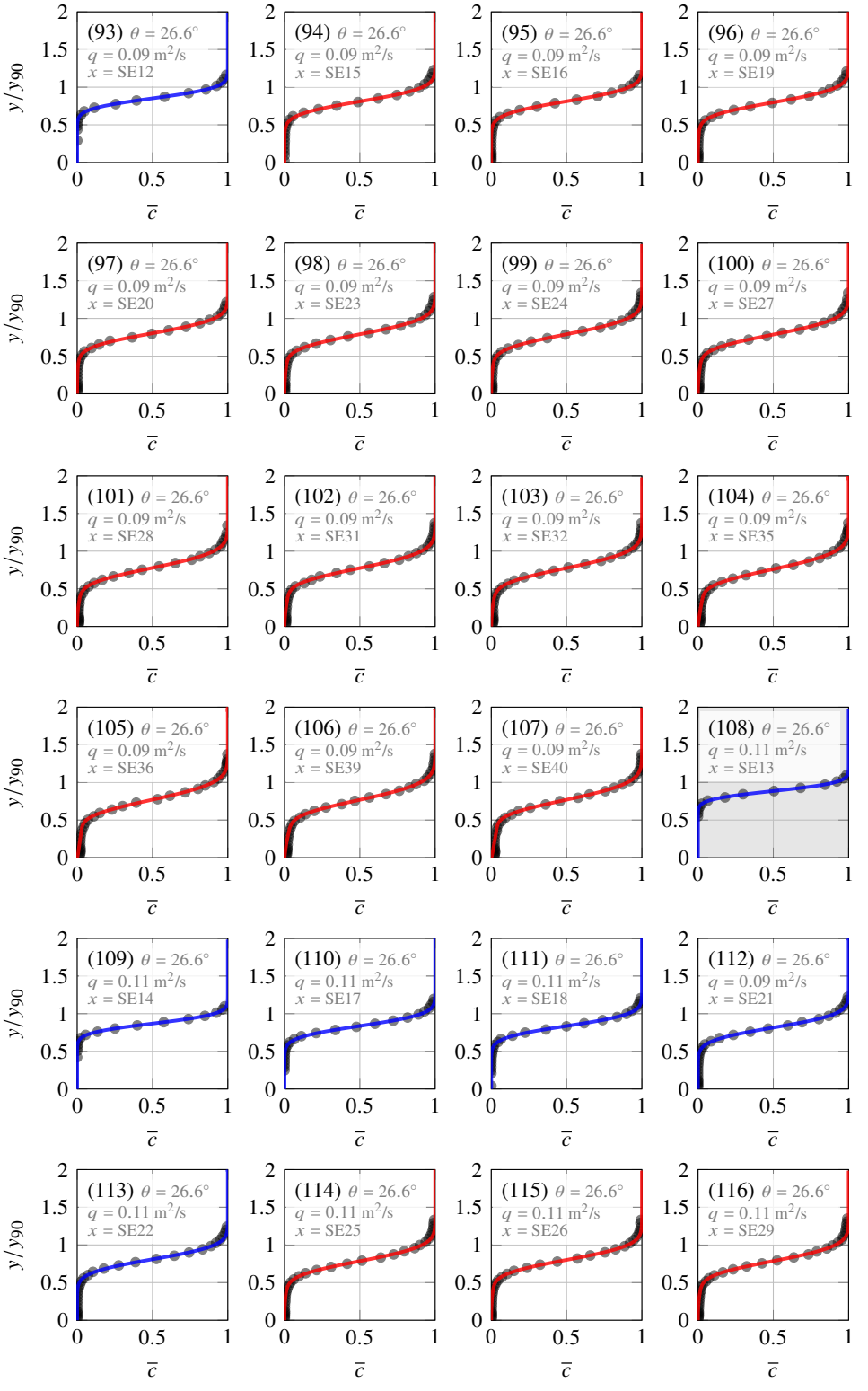
3.1. *Bung (2009, 151 profiles)*

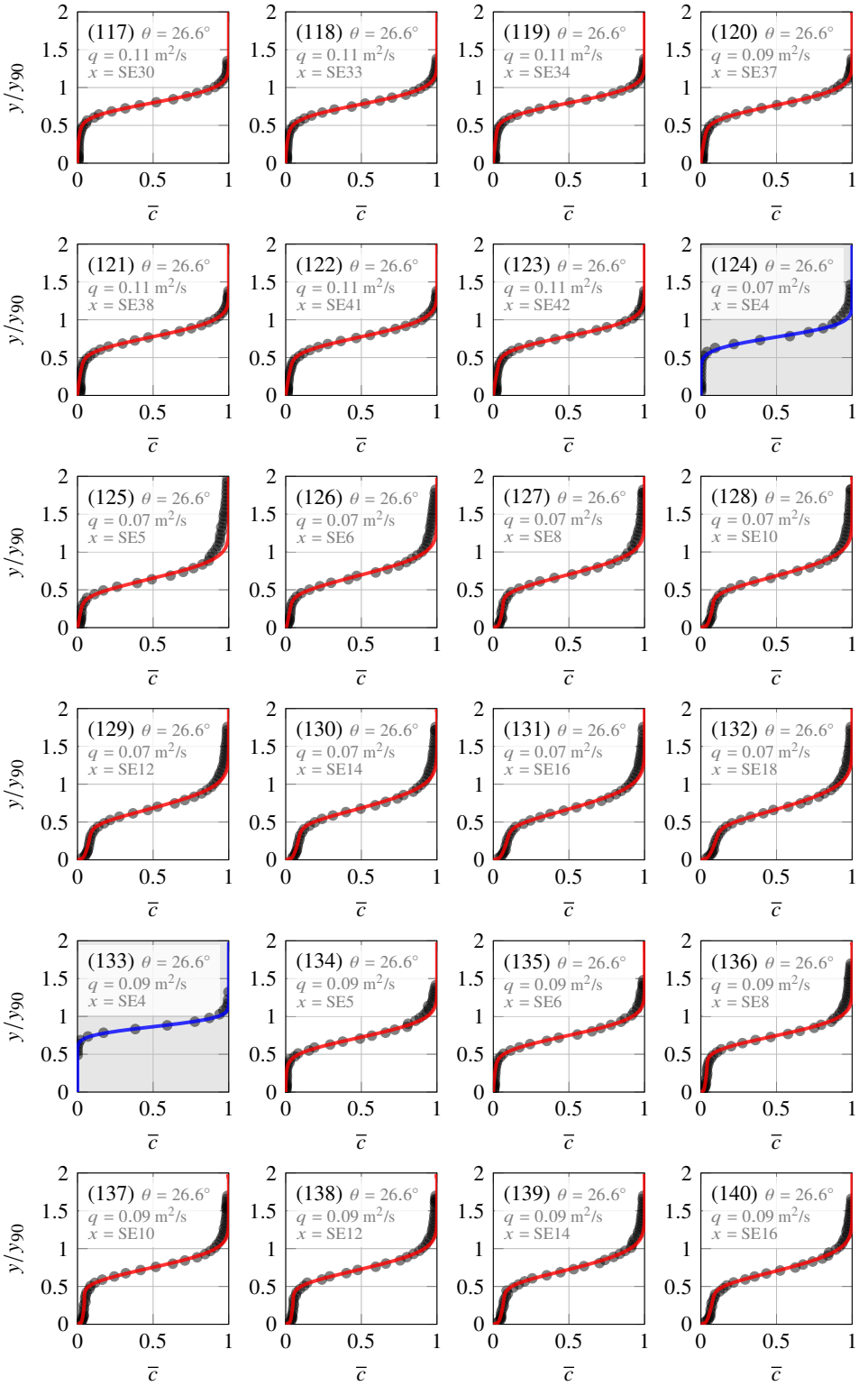


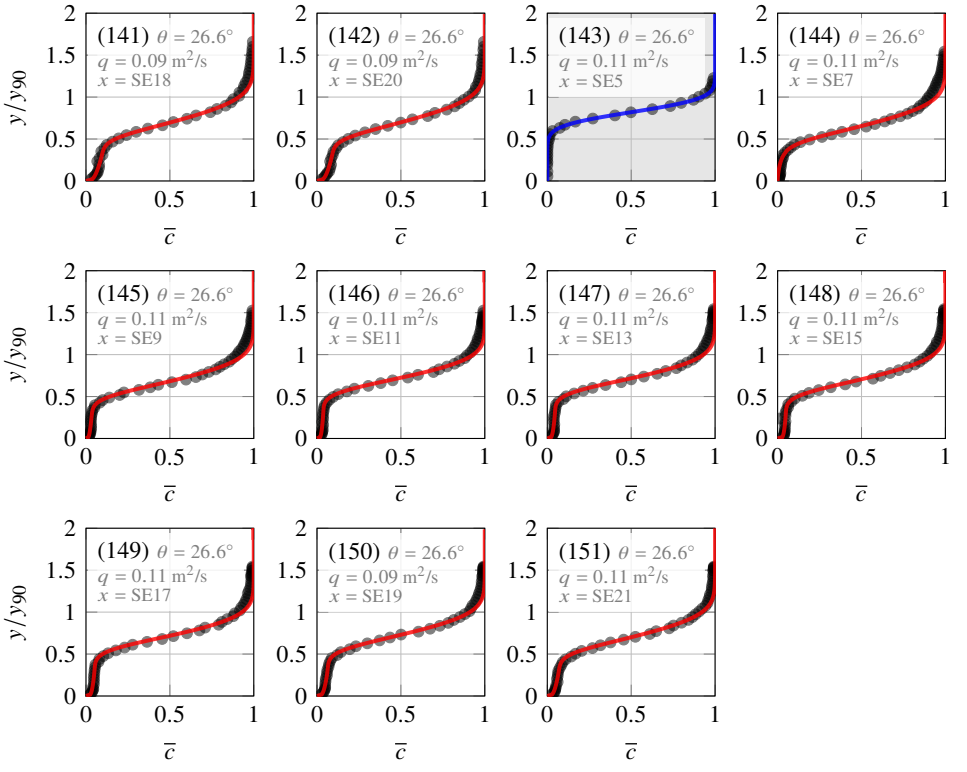

(21) θ = 18.4° q = 0.09 m²/s x = SE25
(22) θ = 18.4° q = 0.09 m²/s x = SE26
(23) θ = 18.4° q = 0.09 m²/s x = SE29
(24) θ = 18.4° q = 0.09 m²/s x = SE30
(25) θ = 18.4° q = 0.09 m²/s x = SE33
(26) θ = 18.4° q = 0.09 m²/s x = SE34
(27) θ = 18.4° q = 0.09 m²/s x = SE37
(28) θ = 18.4° q = 0.09 m²/s x = SE38
(29) θ = 18.4° q = 0.09 m²/s x = SE41
(30) θ = 18.4° q = 0.09 m²/s x = SE42
(31) θ = 18.4° q = 0.09 m²/s x = SE45
(32) θ = 18.4° q = 0.09 m²/s x = SE46
(33) θ = 18.4° q = 0.11 m²/s x = SE19
(34) θ = 18.4° q = 0.11 m²/s x = SE20
(35) θ = 18.4° q = 0.11 m²/s x = SE23
(36) θ = 18.4° q = 0.11 m²/s x = SE24
(37) θ = 18.4° q = 0.11 m²/s x = SE27
(38) θ = 18.4° q = 0.11 m²/s x = SE28
(39) θ = 18.4° q = 0.11 m²/s x = SE31
(40) θ = 18.4° q = 0.11 m²/s x = SE32
(41) θ = 18.4° q = 0.11 m²/s x = SE35
(42) θ = 18.4° q = 0.11 m²/s x = SE36
(43) θ = 18.4° q = 0.11 m²/s x = SE39
(44) θ = 18.4° q = 0.11 m²/s x = SE40
y/y90
c̄


(45) θ = 18.4° q = 0.11 m²/s x = SE43
(46) θ = 18.4° q = 0.11 m²/s x = SE44
(47) θ = 18.4° q = 0.11 m²/s x = SE47
(48) θ = 18.4° q = 0.11 m²/s x = SE48
(49) θ = 18.4° q = 0.07 m²/s x = SE5
(50) θ = 18.4° q = 0.07 m²/s x = SE7
(51) θ = 18.4° q = 0.07 m²/s x = SE9
(52) θ = 18.4° q = 0.07 m²/s x = SE11
(53) θ = 18.4° q = 0.07 m²/s x = SE13
(54) θ = 18.4° q = 0.07 m²/s x = SE15
(55) θ = 18.4° q = 0.07 m²/s x = SE17
(56) θ = 18.4° q = 0.07 m²/s x = SE19
(57) θ = 18.4° q = 0.09 m²/s x = SE5
(58) θ = 18.4° q = 0.09 m²/s x = SE7
(59) θ = 18.4° q = 0.09 m²/s x = SE9
(60) θ = 18.4° q = 0.09 m²/s x = SE11
(61) θ = 18.4° q = 0.09 m²/s x = SE13
(62) θ = 18.4° q = 0.09 m²/s x = SE15
(63) θ = 18.4° q = 0.09 m²/s x = SE17
(64) θ = 18.4° q = 0.09 m²/s x = SE19
(65) θ = 18.4° q = 0.09 m²/s x = SE21
(66) θ = 18.4° q = 0.09 m²/s x = SE23
(67) θ = 18.4° q = 0.11 m²/s x = SE6
(68) θ = 18.4° q = 0.11 m²/s x = SE8
y/y90
c̄
0
0.5
1
1.5
2

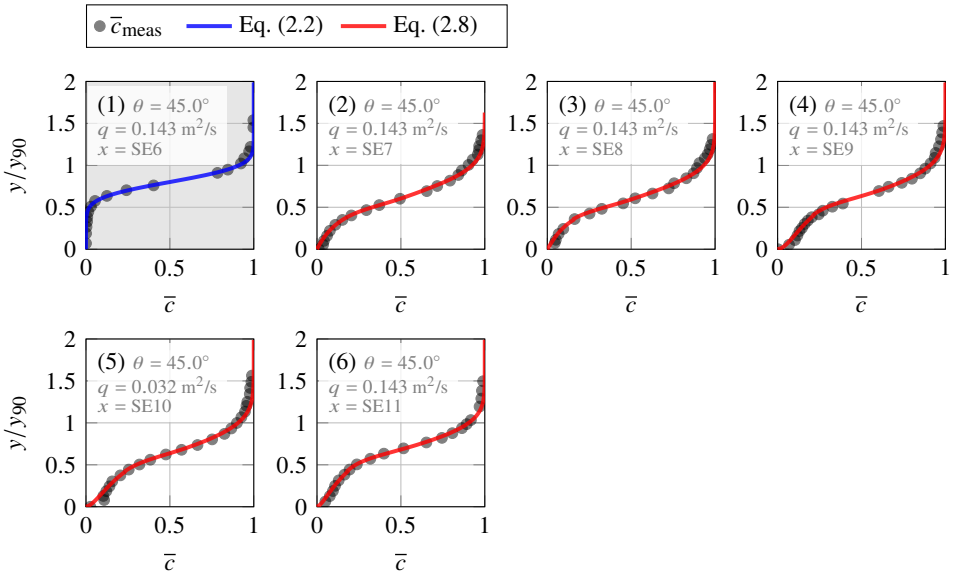

(69) θ = 75.0°
q = 0.26 m²/s
x = SE10
(70) θ = 18.4°
q = 0.11 m²/s
x = SE12
(71) θ = 18.4°
q = 0.11 m²/s
x = SE14
(72) θ = 18.4°
q = 0.11 m²/s
x = SE16
(73) θ = 18.4°
q = 0.11 m²/s
x = SE18
(74) θ = 18.4°
q = 0.11 m²/s
x = SE20
(75) θ = 18.4°
q = 0.11 m²/s
x = SE22
(76) θ = 18.4°
q = 0.11 m²/s
x = SE24
(77) θ = 18.4°
q = 0.11 m²/s
x = SE26
(78) θ = 26.6°
q = 0.07 m²/s
x = SE9
(79) θ = 26.6°
q = 0.07 m²/s
x = SE10
(80) θ = 26.6°
q = 0.07 m²/s
x = SE11
(81) θ = 26.6°
q = 0.07 m²/s
x = SE12
(82) θ = 26.6°
q = 0.07 m²/s
x = SE15
(83) θ = 26.6°
q = 0.07 m²/s
x = SE16
(84) θ = 26.6°
q = 0.07 m²/s
x = SE19
(85) θ = 26.6°
q = 0.07 m²/s
x = SE20
(76) θ = 26.6°
q = 0.07 m²/s
x = SE23
(87) θ = 26.6°
q = 0.07 m²/s
x = SE24
(88) θ = 26.6°
q = 0.07 m²/s
x = SE27
(89) θ = 26.6°
q = 0.07 m²/s
x = SE28
(90) θ = 26.6°
q = 0.07 m²/s
x = SE31
(91) θ = 26.6°
q = 0.07 m²/s
x = SE31
(92) θ = 26.6°
q = 0.09 m²/s
x = SE11
y/y90
c̄


(93) θ = 26.6°
q = 0.09 m²/s
x = SE12
(94) θ = 26.6°
q = 0.09 m²/s
x = SE15
(95) θ = 26.6°
q = 0.09 m²/s
x = SE16
(96) θ = 26.6°
q = 0.09 m²/s
x = SE19
(97) θ = 26.6°
q = 0.09 m²/s
x = SE20
(98) θ = 26.6°
q = 0.09 m²/s
x = SE23
(99) θ = 26.6°
q = 0.09 m²/s
x = SE24
(100) θ = 26.6°
q = 0.09 m²/s
x = SE27
(101) θ = 26.6°
q = 0.09 m²/s
x = SE28
(102) θ = 26.6°
q = 0.09 m²/s
x = SE31
(103) θ = 26.6°
q = 0.09 m²/s
x = SE32
(104) θ = 26.6°
q = 0.09 m²/s
x = SE35
(105) θ = 26.6°
q = 0.09 m²/s
x = SE36
(106) θ = 26.6°
q = 0.09 m²/s
x = SE39
(107) θ = 26.6°
q = 0.09 m²/s
x = SE40
(108) θ = 26.6°
q = 0.11 m²/s
x = SE13
(109) θ = 26.6°
q = 0.11 m²/s
x = SE14
(110) θ = 26.6°
q = 0.11 m²/s
x = SE17
(111) θ = 26.6°
q = 0.11 m²/s
x = SE18
(112) θ = 26.6°
q = 0.09 m²/s
x = SE21
(113) θ = 26.6°
q = 0.11 m²/s
x = SE22
(114) θ = 26.6°
q = 0.11 m²/s
x = SE25
(115) θ = 26.6°
q = 0.11 m²/s
x = SE26
(116) θ = 26.6°
q = 0.11 m²/s
x = SE29
y/y90
c̄
0
0.5
1
1.5
2

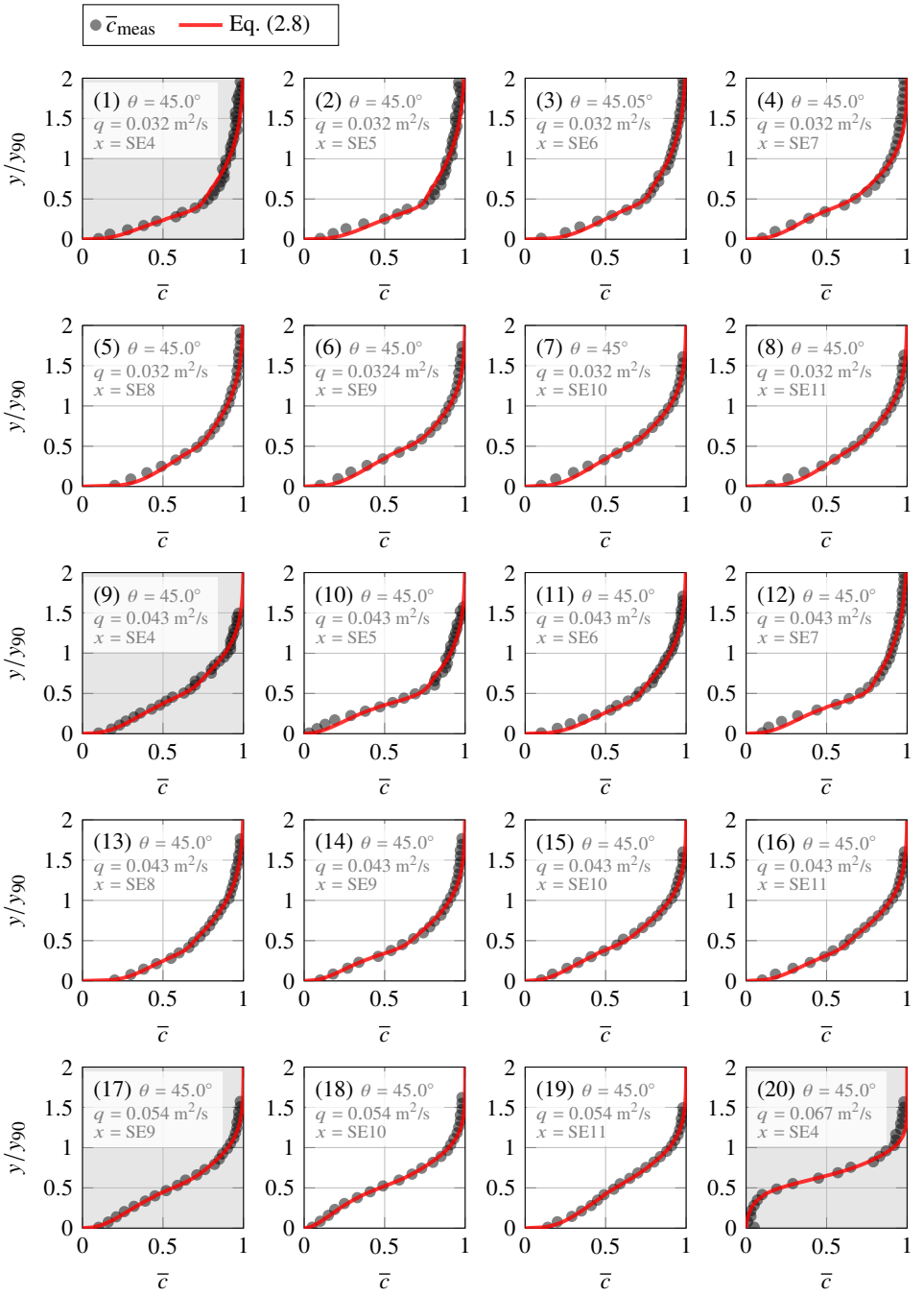

(117) θ = 26.6°
q = 0.11 m²/s
x = SE30
(118) θ = 26.6°
q = 0.11 m²/s
x = SE33
(119) θ = 26.6°
q = 0.11 m²/s
x = SE34
(120) θ = 26.6°
q = 0.09 m²/s
x = SE37
(121) θ = 26.6°
q = 0.11 m²/s
x = SE38
(122) θ = 26.6°
q = 0.11 m²/s
x = SE41
(123) θ = 26.6°
q = 0.11 m²/s
x = SE42
(124) θ = 26.6°
q = 0.07 m²/s
x = SE4
(125) θ = 26.6°
q = 0.07 m²/s
x = SE5
(126) θ = 26.6°
q = 0.07 m²/s
x = SE6
(127) θ = 26.6°
q = 0.07 m²/s
x = SE8
(128) θ = 26.6°
q = 0.07 m²/s
x = SE10
(129) θ = 26.6°
q = 0.07 m²/s
x = SE12
(130) θ = 26.6°
q = 0.07 m²/s
x = SE14
(131) θ = 26.6°
q = 0.07 m²/s
x = SE16
(132) θ = 26.6°
q = 0.07 m²/s
x = SE18
(133) θ = 26.6°
q = 0.09 m²/s
x = SE4
(134) θ = 26.6°
q = 0.09 m²/s
x = SE5
(135) θ = 26.6°
q = 0.09 m²/s
x = SE6
(136) θ = 26.6°
q = 0.09 m²/s
x = SE8
(137) θ = 26.6°
q = 0.09 m²/s
x = SE10
(138) θ = 26.6°
q = 0.09 m²/s
x = SE12
(139) θ = 26.6°
q = 0.09 m²/s
x = SE14
(140) θ = 26.6°
q = 0.09 m²/s
x = SE16
y/y90
c̄
0
0.5
1
1.5
2

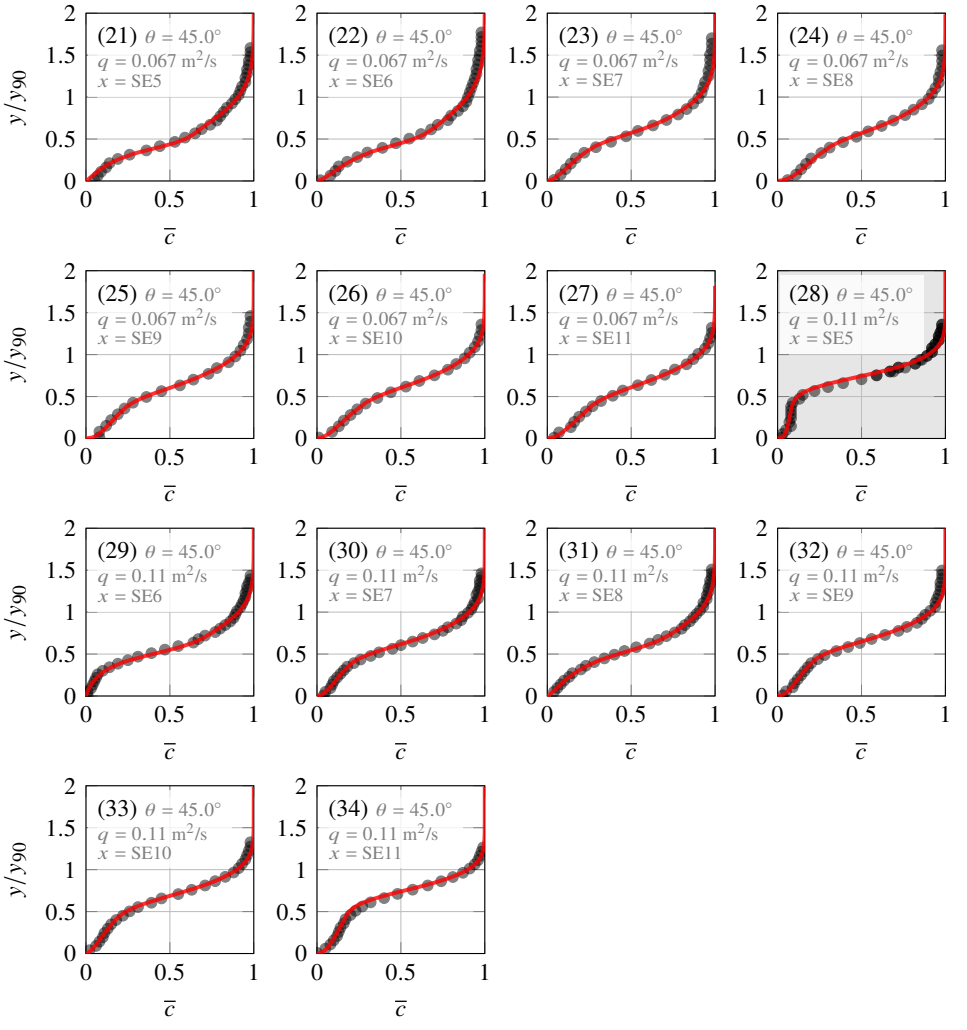

(141) θ = 26.6°
q = 0.09 m²/s
x = SE18
(142) θ = 26.6°
q = 0.09 m²/s
x = SE20
(143) θ = 26.6°
q = 0.11 m²/s
x = SE5
(144) θ = 26.6°
q = 0.11 m²/s
x = SE7
(145) θ = 26.6°
q = 0.11 m²/s
x = SE9
(146) θ = 26.6°
q = 0.11 m²/s
x = SE11
(147) θ = 26.6°
q = 0.11 m²/s
x = SE13
(148) θ = 26.6°
q = 0.11 m²/s
x = SE15
(149) θ = 26.6°
q = 0.11 m²/s
x = SE17
(150) θ = 26.6°
q = 0.09 m²/s
x = SE19
(151) θ = 26.6°
q = 0.11 m²/s
x = SE21
y/y90
c̄

3.2. *Zhang (2017, 6 profiles)*



3.3. *Kramer and Chanson (2018, 34 profiles)*




(21) θ = 45.0°
q = 0.067 m²/s
x = SE5
(22) θ = 45.0°
q = 0.067 m²/s
x = SE6
(23) θ = 45.0°
q = 0.067 m²/s
x = SE7
(24) θ = 45.0°
q = 0.067 m²/s
x = SE8
(25) θ = 45.0°
q = 0.067 m²/s
x = SE9
(26) θ = 45.0°
q = 0.067 m²/s
x = SE10
(27) θ = 45.0°
q = 0.067 m²/s
x = SE11
(28) θ = 45.0°
q = 0.11 m²/s
x = SE5
(29) θ = 45.0°
q = 0.11 m²/s
x = SE6
(30) θ = 45.0°
q = 0.11 m²/s
x = SE7
(31) θ = 45.0°
q = 0.11 m²/s
x = SE8
(32) θ = 45.0°
q = 0.11 m²/s
x = SE9
(33) θ = 45.0°
q = 0.11 m²/s
x = SE10
(34) θ = 45.0°
q = 0.11 m²/s
x = SE11
y/y90
c̄

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