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NATURAL CONVECTION IN A SHALLOW CAVITY WITH DIFFERENTIALLY HEATED END WALLS

PART ~~II~~ 2. NUMERICAL SOLUTIONS

by

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Supplementary Notes on Numerical Scheme
for Editor's File

$$F_j = 2\left(\frac{Y'_j}{\Delta Y}\right)^2$$

$$G_j = \left(\frac{Y'_j}{\Delta Y}\right)^2 - \frac{Y''_j}{\Delta Y}$$

The corresponding expression for the Jacobian is

$$J_{i,j}(\omega,\psi) = -\frac{Y'_j X'_i}{12\Delta X \Delta Y} H_{i,j}$$

where

$$\begin{aligned} H_{i,j} = \Big[& \omega_{i+1,j}(\psi_{i,j-1} + \psi_{i+1,j-1} - \psi_{i,j+1} - \psi_{i+1,j+1}) \\ & + \omega_{i,j+1}(\psi_{i+1,j} + \psi_{i+1,j+1} - \psi_{i-1,j} - \psi_{i-1,j+1}) \\ & + \omega_{i,j-1}(\psi_{i-1,j-1} + \psi_{i-1,j} - \psi_{i+1,j-1} - \psi_{i+1,j}) \\ & + \omega_{i-1,j}(\psi_{i-1,j+1} + \psi_{i,j+1} - \psi_{i-1,j-1} - \psi_{i,j-1}) \\ & + \omega_{i+1,j+1}(\psi_{i+1,j} - \psi_{i,j+1}) + \omega_{i-1,j-1}(\psi_{i-1,j} - \psi_{i,j-1}) \\ & + \omega_{i-1,j+1}(\psi_{i,j+1} - \psi_{i-1,j}) + \omega_{i+1,j-1}(\psi_{i,j-1} - \psi_{i+1,j}) \Big] \end{aligned}$$

Alternatively, when the arbitrarily discretized mesh is used in the horizontal direction ($A \leq 0.2$), the coefficients B_i, C_i and D_i become

$$B_i = \frac{2}{\Delta x_i(\Delta x_i + \Delta x_{i-1})}$$

$$C_i = 2\left(\frac{1}{\Delta x_i} + \frac{1}{\Delta x_{i-1}}\right) / (\Delta x_i + \Delta x_{i-1})$$

$$D_i = \frac{2}{x_{i-1}(x_i + x_{i-1})}$$

and the Jacobian is

$$J_{i,j}(\omega,\psi) = - H_{i,j} \frac{Y'_j}{6(\Delta Y\ (\Delta x_i + \Delta x_{i-1}))}$$

The appropriate boundary condition to be imposed on ω at the lower boundary of the cavity is

$$\omega_{i,1} = \frac{-F_1}{4} (8\psi_{i,2} - \psi_{i,3}) + O(\Delta Y)^2 \qquad \text{(S-4)}$$

Similar expressions may be derived for the vorticity on the other solid boundaries and for the temperature on the insulated surfaces by the method outlined in the paper.

II. NUMERICAL ALGORITHM

A typical iteration (n+1) was carried out as follows:

1. With known values of ψ^n, ω^n and θ^n at time step n, equation (S-1) was integrated ahead in time by one complete time step (two half time steps) to give θ^{n+1}.
2. θ^{n+1} was calculated on the upper and lower boundaries from the finite difference approximation of the insulating boundary condition.
3. Equation (S-2) was integrated for 1/2 time step using θ^{n+1}, ψ^n and ω^n to give $\omega^{n+1/2}$.
4. $\psi^{n+1/2}$ was determined by means of an A.D.I. iteration using an "over relaxation" factor of 1.5. This iteration was continued until equation (S-3) was satisfied at all grid points to within a prespecified error.
5. $\psi^{n+1/2}$ was used to evaluate $\omega^{n+1/2}$ on the boundaries using equation (S-4) and its counterpart on the remaining boundaries.
6. Equation (S-2) was integrated for another 1/2 time step using θ^{n+1}, $\psi^{n+1/2}$ and $\omega^{n+1/2}$ to give ω^{n+1}.

7. ψ^{n+1} was determined as in 4.

8. ω^{n+1} was evaluated on the boundaries from ψ^{n+1}.

9. Steps 1) to 8) were repeated until further iteration no longer changed the solution.