FOR EDITORIAL OFFICE FILES

reference p. 17

TABLE 3

C_{11}	$- 18F + 30L$
C_{12}	$(- 6E + 30K)x_2$
C_{21}	$- 6F + 30x_2^2J$
C_{22}	$(- 6E + 30K)x_2$
C_{31}	$(- 6E + 30K)x_3$
C_{32}	$30x_2x_3J$

reference p. 17

TABLE 4

D_{11}	$-\ 6F + 30x_2^2J$
D_{12}	$(-\ 6E + 30K)x_2$
D_{21}	$-\ 6F + 30x_2^2J$
D_{22}	$(-\ 18E + 30x_2^2I)x_2$
D_{31}	$30x_2x_3J$
D_{32}	$(-\ 6E + 30x_2^2I)x_3$

reference p. 17

TABLE 5

E_{111}	$[8BC - 24BH - 84GC + 180GH - 36r^2 GF + 60MC + 60\ ^2 KC - 180HM - 360r^2 GL - 180r^4 KF - 360x_2^2 x_3^2 KF]$
$(E_{112} + E_{121})$	$x_2 \ [8B^2 - 132BG - 84CF + 144HF + 216G^2 + 216r^2 F^2 + 60LC + 60BM + 60r^2 CJ + 60r^2 BK - 180GM - 180HL - 360r^2 FL - 360r^2 GK - 180r^4 KE - 180r^4 JF - 360x_2^2 x_3^2 KE - 360x_2^2 x_3^2 JF]$
E_{122}	$x_2^2 \ [- 108BF + 180GF + 180r^2 EF + 60BL + 60r^2 BJ - 180GL - 360r^2 FK - 180r^4 JE - 360x_2^2 x_3^2 JE]$
E_{211}	$x_2 \ [- 24BG + 108G^2 - 84FC + 144r^2 F^2 + 72FH + 36r^2 EG + 60LC - 60LH - 360r^2 KG + 60r^2 JC - 180r^4 JF]$
$(E_{212} + E_{221})$	$[324x_2^2 GF - 132x_2^2 BF + 8BC + 360x_2^2 r \ EF - 84x_2^2 EC + 60x_2^2 BL - 180x_2^2 LG + 60x_2^2 KC - 60x_2^2 KH - 360x_2^2 r \ KF - 360x_2^2 r^2 JG + 60x_2^2 r^2 BJ + 60x_2^2 r^2 CI - 180x_2^2 r^4 JE - 180x_2^2 r^4 FI]$
E_{222}	$x_2 \ [144x_2^2 F^2 + 8B^2 - 108x_2^2 BE + 180r^2 x_2^2 E^2 + 60x_2^2 BK - 180x_2^2 KG - 360x_2^2 r^2 FJ + 60x_2^2 r^2 BI - 180x_2^2 r^4 EI]$
E_{311}	$x_3 \ [- 24BG + 108G^2 + 144r^2 F^2 - 84FC + 72FH + 36r^2 EG + 60LC - 60LH - 360r^2 KG + 60r^2 JC - 180r^4 JF]$
$(E_{312} + E_{321})$	$x_2 x_3 \ [- 84EC - 132BF + 324GF + 360r^2 EF + 60BL - 180LG + 60KC - 60KH - 360r^2 KF - 360r^2 JG + 60r^2 BJ + 60r^2 CI - 180r^4 JE - 180r^4 FI]$
E_{322}	$x_3 \ [- 108x_2^2 BE + 144x_2^2 F^2 + 180x_2^2 r^2 E^2 + 60x_2^2 BK - 180x_2^2 KG - 360x_2^2 r^2 FJ + 60x_2^2 r^2 BI - 180x_2^2 r^4 EI]$

reference p. 17

TABLE 6

F_{111}	$[-14BC - 120CG + 90CM + 120r^2 CK - 30BH + 252GH - 270HM + 252r^2 GF - 540r^2 GL - 270r^4 FK - 18AF + 30AL - 18FD + 30DL - 12r^2 EC + 18r^4 EF]$
$F_{112} + F_{121}$	$x_2 \; [-144CF + 120CL + 120r^2 CJ + 14B^2 - 174BG + 90BM + 120r^2 BK + 198HF - 270HL + 288G^2 - 270GM + 180r^2 GE - 540r^2 GK + 288r^2 F^2 - 450r^2 FL - 270r^4 FJ - 270r^4 KE - 12AE + 60AK - 6ED - 12r^2 EB + 18r^4 E^2 + 30DK]$
F_{122}	$-6AF + x_2^2 \; [-144BF + 90BL + 120r^2 BJ + 234GF - 270GL + 216r^2 EF - 540r^2 FK - 270r^4 EF + 30AJ - 12EC + 30KC]$
F_{211}	$x_2 \; [-120CF + 90CL + 120r^2 CJ + 162HF - 270HL - 24GB + 135G^2 + 36r^2 GE - 540r^2 GK + 270r^2 F^2 - 270r^2 FJ - 6AE + 30AK - 6ED + 30DK]$
$(F_{212} + F_{221})$	$[14CB - 132x_2^2 CE - 168x_2^2 FB + 90x_2^2 BL + 120x_2^2 CK + 120x_2^2 r^2 BJ + 120x_2^2 r^2 CI + 496x_2^2 GF + 126x_2^2 EH - 270x_2^2 GL - 270x_2^2 HK + 612x_2^2 r^2 EF - 540x_2^2 r^2 FK - 540x_2^2 r^2 GJ - 270x_2^2 r^2 FI - 270x_2^2 r^2 EJ + 60x_2^2 AJ - 6DF + 30x_2^2 DJ - 6GC - 6r^2 FB - 12AF - 12r^2 CE - 6BH + 18GH + 36r^2 GF + 18r^4 EF]$
F_{222}	$x_2 \; [14B^2 - 150x_2^2 BE + 90x_2^2 BK + 120x_2^2 r^2 BI + 126x_2^2 GE - 270x_2^2 GK + 180x_2^2 F^2 - 450x_2^2 r^2 FJ + 342x_2^2 r^2 E^2 - 270x_2^2 r^2 EI - 18AE + 30x_2^2 AI - 6CF + 30x_2^2 CJ - 18r^2 EB - 6GB + 18G^2 + 36r^2 F^2]$
F_{311}	$x_3 \; [-120CF + 90CL + 120r^2 CJ + 162HF - 270HL - 24GB + 144G^2 - 540r^2 GK + 36r^2 GE + 270r^2 F^2 - 270r^2 FJ - 6AE + 30AK - 6ED - 30DK]$
$(F_{312} + F_{321}) x_3 x_2$	$[-132CE - 168FB + 90BL + 120r^2 BJ + 120r^2 CI + 450GF + 126EH - 270GL - 270HK + 612r^2 EF - 540r^2 FK - 450r^2 GJ - 270r^2 FI - 270r^2 EJ + 60AJ + 30DJ + 90KC + 36EG]$
F_{322}	$x_3 \; x_2^2 [-150BE + 90BK + 120r^2 BI + 126GE - 270GK + 144F^2 - 540r^2 FJ + 306r^2 E^2 - 270r^4 EI + 30AI + 30CJ + 36EF - 6AE]$

reference p. 19

TABLE 7

$64\pi^2\mu^2\ P_{111}$	$\left[4C^2 - 24CH + 36H^2 + 36r^2G^2\right]$
$64\pi^2\mu^2\ P_{211}$	$x_2\left[- 24GC + 36GH + 36r^2GF\right]$
$64\pi^2\mu^2\ P_{311}$	$x_3\left[- 24GC + 36GH + 36r^2GF\right]$
$64\pi^2\mu^2\ P_{112}$	$x_2\left[8BC - 24GC - 24BH + 72GH + 72r^2GF\right]$
$64\pi^2\mu^2\ P_{212}$	$x_2^2\left[- 24BG - 24FC + 36G^2 + 36FH + 36r^2GE + 36r^2F^2\right]$
$64\pi^2\mu^2\ P_{312}$	$x_2x_3\left[- 24BG - 24FC + 36G^2 + 36FH + 36r^2F^2 + 36r^2GE\right]$
$64\pi^2\mu^2\ P_{113}$	$x_2^2\left[4B^2 - 24GB + 36G^2 + 36r^2F^2\right]$
$64\pi^2\mu^2\ P_{213}$	$x_2^3\left[- 24BF + 36GF + 36r^2EF\right]$
$64\pi^2\mu^2\ P_{313}$	$x_2^2x_3\left[- 24BF + 36GF + 36r^2EF\right]$
$64\pi^2\mu^2\ P_{121}$	$x_2\left[- 24GC + 36GH + 36r^2GF\right]$
$64\pi^2\mu^2\ P_{221}$	$x_2^2\left[- 24GB - 24FC + 36G^2 + 36FH + 36r^2GE + 36r^2F^2\right]$
$64\pi^2\mu^2\ P_{321}$	$x_2^3\left[- 24BF + 36GF + 36r^2EF\right]$
$64\pi^2\mu^2\ P_{122}$	$\left[4C^2 - 24x_2^2CF + 36x_2^2r^2F^2 + 36x_2^2G^2\right]$
$64\pi^2\mu^2\ P_{222}$	$x_2\left[8BC - 24x_2^2CE - 24x_2^2BF + 72x_2^2r^2EF + 72x_2^2GF\right]$
$64\pi^2\mu^2\ P_{322}$	$x_2^2\left[4B^2 - 24x_2^2BE + 36x_2^2r^2E^2 + 36x_2^2F^2\right]$
$64\pi^2\mu^2\ P_{321}$	$x_2x_3\left[- 24FC + 36r^2F^2 + 36G^2\right]$
$64\pi^2\mu^2\ P_{322}$	$x_2^2x_3\left[- 24BF - 24CE + 72r^2EF + 72GF\right]$
$64\pi^2\mu^2\ P_{323}$	$x_2^3x_3\left[- 24BE + 36r^2E^2 + 36F^2\right]$

reference p. 19

$64\pi^2\mu^2 \; P_{131} \qquad x_3\left[- 24GC + 36GH + 36r^2GF\right]$

$64\pi^2\mu^2 \; P_{132} \qquad x_2x_3\left[- 24BG - 24FC + 36FH + 36G^2 + 36r^2F^2 + 36r^2GE\right]$

$64\pi^2\mu^2 \; P_{133} \qquad x_2^2x_3\left[- 24BF + 36GF + 36r^2EF\right]$

$64\pi^2\mu^2 \; P_{231} \qquad x_2x_3\left[- 24FC + 36r^2F^2 + 36G^2\right]$

$64\pi^2\mu^2 \; P_{232} \qquad x_2^2x_3\left[- 24CE - 24BF + 72r^2EF + 72GF\right]$

$64\pi^2\mu^2 \; P_{233} \qquad x_2^3x_3\left[- 24BE + 36r^2E^2 + 36F^2\right]$

$64\pi^2\mu^2 \; P_{331} \qquad \left[4C^2 - 24x_3^2CF + 36r^2x_3^2F^2 + 36x_3^2G^2\right]$

$64\pi^2\mu^2 \; P_{332} \qquad x_2\left[8BC - 24x_3^2BF - 24x_3^2CE + 72r^2x_3^2EF + 72x_3^2GF\right]$

$64\pi^2\mu^2 \; P_{333} \qquad x_2^2\left[4B^2 - 24x_3^2BE + 36r^2x_3^2E^2 + 36x_3^2F^2\right]$

reference p. 19

TABLE 8

$$64\pi^2\mu^2\, Q_{111} \quad \Big[2AB - 24AG + 30AM + 2BD - 24GD + 30DM - 6r^2FC + 30r^2CL + 4C^2 - 24CH + 36H^2 - 12r^2GB + 36r^2G^2\Big]$$

$$64\pi^2\mu^2\, Q_{211} \quad x_2\Big[- 24AF + 60AL - 18DF + 30DL + 12BC - 30GC + 30CM - 6r^2CE + 30r^2CK - 18r^2BF + 30r^2BL - 24BH + 72GH + 72r^2GF\Big]$$

$$64\pi^2\mu^2\, Q_{311} \quad \Big[2AB - 6AG + x_2^2\big(- 18CF + 30CL - 6AE + 30AK + 6B^2 - 6r^2BE + 30r^2BK - 30BG + 36G^2 + 36r^2F^2\big)\Big]$$

$$64\pi^2\mu^2\, Q_{112} \quad x_2\Big[- 12AF + 30AL - 12DF + 30DL - 30GC + 30r^2CK + 6BH + 36GH + 36r^2GF - 6r^2BF\Big]$$

$$64\pi^2\mu^2\, Q_{212} \quad \Big[- 6AG - 6r^2BG - 6HC + x_2^2\big(- 6AE - 6DE + 60AK + 30KD - 42CF + 30CL + 30r^2CJ - 18GB + 30r^2BK + 36HF + 36G^2 + 36r^2GE + 36r^2F^2 - 6r^2BE\big)\Big]$$

$$64\pi^2\mu^2\, Q_{312} \quad x_2\Big[- 12AF - 6r^2BF - 6GC + x_2^2\big(30CK + 30AJ - 30BF + 30r^2BJ + 36GF + 36EF\big)\Big]$$

$$64\pi^2\mu^2\, Q_{113} \quad x_3\Big[- 12AF - 12DF + 30AL + 30DL + 30r^2KC - 30GC + 6BH + + 36GH + 36r^2GF - 6r^2FB\Big]$$

$$64\pi^2\mu^2\, Q_{213} \quad x_2x_3\Big[- 6AE - 6DE + 30AK + 30DK - 42CF + 30CL + 30AK + 30r^2CJ + 30r^2BK - 18BG + 36HF + 36G^2 + 36r^2GE + 36r^2F^2 - 36r^2BE\Big]$$

$$64\pi^2\mu^2\, Q_{313} \quad x_3\Big[- 6AF + x_2^2\big(- 6EC + 30KC + 30AJ - 30BF + 30r^2BJ + 36GF + 36r^2EF\big)\Big]$$

reference p. 19

$64\pi^2\mu^2\, Q_{121}$ $\quad x_2\Big[-12AF + 30AL - 12DF + 30DL - 30GC + 30r^2CK + 6BH$
$+ 18GH + 36r^2GF + 18GH - 6r^2BF\Big]$

$64\pi^2\mu^2\, Q_{221}$ $\quad \Big[-6AG - 6r^2GB - 6HC + x_2^2\big(-6AE - 6ED + 60AK + 30KD$
$- 42CF + 30CL + 30r^2CJ - 18GB + 30r^2BK + 36HF + 36G^2$
$+ 36r^2GE + 36r^2F^2 - 6r^2BE\big)\Big]$

$64\pi^2\mu^2\, Q_{321}$ $\quad x_2\Big[-12AF - 6r^2BF - 6GC + x_2^2\big(30CK + 30AJ - 30BF + 30r^2BJ$
$+ 36GF + 36r^2EF\big)\Big]$

$64\pi^2\mu^2\, Q_{122}$ $\quad \Big[2AB - 6AG + 2BD - 6DG - 6r^2CF + 4C^2 + x_2^2\big(-6AE + 30AK$
$- 6DE + 30DK - 36CF + 30r^2CJ + 12GB + 36G^2 + 36r^2F^2\big)\Big]$

$64\pi^2\mu^2\, Q_{222}$ $\quad x_2\Big[-24AF - 6DF + 12BC - 6r^2CE - 18r^2BF - 18GC + x_2^2\big(60AJ$
$+ 30DJ - 48CE + 30r^2CI - 12BF + 30CK + 30r^2BJ + 72GF$
$+ 72r^2EF\big)\Big]$

$64\pi^2\mu^2\, Q_{322}$ $\quad \Big[2AB + x_2^2\big(-18CF - 24AE + 6B^2 - 18r^2BE\big) + x_2^4\big(30CJ + 30AI$
$- 30BE + 30r^2BI + 36F^2 + 36r^2E^2\big)\Big]$

$64\pi^2\mu^2\, Q_{321}$ $\quad x_2x_3\Big[-6AE + 30AK - 6ED + 30DK - 36CF + 30r^2CJ + 36G^2 + 36r^2F^2\Big]$

$64\pi^2\mu^2\, Q_{322}$ $\quad x_3\Big[-6AF - 6r^2BF - 6CG + x_2^2\big(60AJ + 30DJ - 48CE + 30CK +$
$+ 30r^2CI - 6BF + 30r^2BJ + 72GF + 72r^2EF\Big]$

$64\pi^2\mu^2\, Q_{323}$ $\quad x_2x_3\Big[-12EA - 6r^2BE - 6CF + x_2^2\big(30CJ + 30AI - 24BE + 30r^2BI$
$+ 36F^2 + 36r^2E^2\big)\Big]$

reference p. 19

$$64\pi^2\mu^2\, Q_{131} \qquad x_3\Big[-12AF + 30AL - 12ED + 30LD + 30r^2KC - 30GC + 6BH + 36GH + 36r^2GF - 6r^2BF\Big]$$

$$64\pi^2\mu^2\, Q_{132} \qquad x_3x_2\Big[-6E(A+D) + 30DK + 60AK - 42CF + 30CL + 30r^2BK + 30r^2JC - 18BG - 4B^2 + 36HF + 36G^2 + 36r^2GE + 36r^2F^2 - 6r^2BE\Big]$$

$$64\pi^2\mu^2\, Q_{133} \qquad x_3\Big[-6AF + x_2^2\big(30KC + 30AJ - 30BF + 30r^2BJ + 36GF + 36r^2EF\big)\Big]$$

$$64\pi^2\mu^2\, Q_{231} \qquad x_2x_3\Big[-6EA + 30AK - 6ED + 30DK - 36CF + 30r^2CJ + 36G^2 + 36r^2F^2\Big]$$

$$64\pi^2\mu^2\, Q_{232} \qquad x_3\Big[-6AF - 6CG - 6r^2BF + x_2^2\big(60AJ + 30DJ - 48CE + 30CK + 30r^2CI - 6BF + 30r^2BJ + 72GF + 72r^2EF\big)\Big]$$

$$64\pi^2\mu^2\, Q_{233} \qquad x_2x_3\Big[-12AE - 6r^2BE - 6CF + x_2^2\big(30CJ + 30AI - 24BE + 30r^2BI + 36F^2 + 36r^2E^2\big)\Big]$$

$$64\pi^2\mu^2\, Q_{331} \qquad \Big[(2B - 6G)(A+D) - 6r^2CF + 4C^2 + x_3^2\big(-6AE - 6DE + 30AK + 30DK - 36CF + 30r^2CJ + 12GB + 36G^2 + 36r^2F^2\big)\Big]$$

$$64\pi^2\mu^2\, Q_{332} \qquad x_2\Big[-12AF - 6DF + 12BC - 6GC - 6r^2CE - 6r^2BF + x_3^2\big(60AJ + 30DJ - 48CE + 30CK + 30r^2CI - 12BF + 30r^2BJ + 36GF + 72r^2EF\big)\Big]$$

$$64\pi^2\mu^2\, Q_{333} \qquad \Big[2AB - 6r^2AE + x_2^2\big(-6CF + 30x_3^2CJ + 30x_3^2AI + 6B^2 - 6r^2BE - 30x_3^2BE + 30x_3^2r^2BI - 36x_3^2F^2 + 36x_3^2r^2E^2\big)\Big]$$

reference p. 29

We begin by considering the evaluation of the first term of the integral (42) over ϕ, for i = 2 and i = 3, starting with $\bar{B}x_2x_3H_2$ and $\bar{B}x_2x_3H_3$. Since the coefficient $\bar{B}$ is independent of ϕ, we simply multiply H_2 and H_3 by x_2x_3, transform from (x_1,x_2,x_3) to the corresponding cylindrical coordinate system (x_1,r,ϕ)* and integrate over ϕ from 0 to 2 to obtain

$$\bar{B}\int_0^{2\pi} x_2x_3H_2 d\phi = \bar{B}\,\frac{\pi}{4}\Bigg[-32f_3^{(0)}x_1A_{11} + 8f_1^{(0)}x_1(A_{13}+A_{31}) + \varepsilon_1 \Big\{8x_1f_3^{(0)}(A_{11} - 5A_{22} + 2A_{33}) + 16f_1^{(0)}x_1(A_{13}+A_{31}) - 8f_2^{(0)}x_1A_{23} + 8f_1^{(0)}x_1A_{31} - 8x_1\frac{df_3^{(0)}}{dt}\Big\}\Bigg]\frac{1}{8\pi} + \bar{B}\,\frac{f_1^{(0)}f_3^{(0)}}{64\pi}\left\{12x_1\log\left(\frac{r^2}{4(1-x_1^2)}\right) + \frac{68}{3}x_1 + 8x_1\left(\frac{x_1^2}{1-x_1^2}\right)\right\} \quad (51)$$

and

$$\bar{B}\int_0^{2\pi} x_2x_3H_3 d\phi = \bar{B}\,\frac{\pi}{4}\Bigg[-32f_2^{(0)}x_1A_{33} + 8f_1^{(0)}x_1(A_{12}+A_{21}) + \varepsilon_1 \Big\{8f_2^{(0)}x_1(A_{11} + 2A_{22} - 5A_{33}) + 16f_1^{(0)}x_1(A_{12}+A_{21}) + 8f_1^{(0)}x_1A_{21} - 8f_3^{(0)}x_1A_{32} - 8x_1\frac{df_2^{(0)}}{dt}\Big\}\Bigg]\frac{1}{8\pi} + \bar{B}\,\frac{f_1^{(0)}f_2^{(0)}}{64\pi}\left\{12x_1\log\left(\frac{r^2}{4(1-x_1^2)}\right) + \frac{68}{3}x_1 + 8x_1\frac{x_1^2}{1-x_1^2}\right\} \quad (52)$$

in which the asymptotic forms of the coefficients $\bar{B}$, $\bar{C}$, . . . for small $r(r = 0(R_o) \ll 1)$ have been used. The latter terms of each expression arise from those contributions to H_i which are products of derivatives of the

---

*As before, $x_2 = r\cos\phi$ and $x_3 = r\sin\phi$.

disturbance velocity field with itself. On the other hand, the first terms result from contributions to H_i which are products of derivatives of the undisturbed velocity field with derivatives of the disturbance field.

The similar terms of (55) and (60) involving $(\bar{A} + x_2^2\bar{B})H_3$ and $(\bar{A} + x_3^2\bar{B})H_2$ are evaluated next for integration with respect to ϕ. Here, we simply integrate H_2 and H_3 over ϕ, since $\bar{A}$ is independent of ϕ and $x_2^2\bar{B}$ is asymptotically smaller than $\bar{A}$ and so neglected. The result is

$$\int_0^{2\pi} (\bar{A} + x_2^2\bar{B})H_3 d\phi \sim \bar{A}\, \frac{f_1^{(0)} f_3^{(0)}}{64\pi} \left[\frac{12x_1}{r^2} \log \left(\frac{r^2}{4(1 - x_1^2)}\right) - \frac{16x_1}{r^2} + \frac{32x_1}{r^2} \left(\frac{x_1^2}{1 - x_1^2}\right)\right] \quad (53)$$

and

$$\int_0^{2\pi} (\bar{A} + x_3^2\bar{B})H_2 d\phi \sim \bar{A}\, \frac{f_1^{(0)} f_2^{(0)}}{64\pi} \left[\frac{12x_1}{r^2} \log \left(\frac{r^2}{4(1 - x_1^2)}\right) - \frac{16x_1}{r^2} + \frac{32x_1}{r^2} \left(\frac{x_1^2}{1 - x_1^2}\right)\right] \quad (54)$$

again valid for small $r = 0(R_o) << 1$.

We now turn to evaluation of the surface integrals of equation (42) for $i = 2$ and $i = 3$. We proceed by evaluating the integrands at $r = R(x_1) << 1$, converting to the more convenient cylindrical coordinate system, and integrating with respect to ϕ. After a considerable algebraic effort, it may finally be shown

$$\int_0^{2\pi} R\sin\phi\ t_1 d\phi = \int_0^{2\pi} R\cos\phi\ t_1 d\phi \sim -\frac{2(\hat{n}_2 + \hat{n}_3)}{64\pi} \frac{x_1^2}{R(x_1)} \log\left(\frac{R^2(x_1)}{4(1 - x_1^2)}\right) \quad (55)$$

$$\int_0^{2\pi} x_1 t_3 d\phi = \int_0^{2\pi} x_1 t_2 d\phi \sim \frac{16(\hat{n}_2 + \hat{n}_3)}{64\pi} \frac{x_1^2}{R(x_1)} \log \left(\frac{R^2(x_1)}{4(1 - x_1^2)}\right) \quad (56)$$

The terms involving the undisturbed component of the 0(1) velocity field produce contributions which are 0(1) or smaller, and have hence been neglected in the equations (55) and (56).

In order to evaluate the torque from equation (42) for i = 2 and i = 3, we use the asymptotic evaluations of the various integral terms (51-56), the definitions of $\tilde{F}_i^j$ from equation (27), and the asymptotic (small r) forms for the coefficients $\bar{B}$ and $\bar{A}$ from table 2

$$\bar{B} \sim \frac{2x_1}{r^2} + 0(1)$$

$$\bar{A} \sim - x_1 \log \left(\frac{r^2}{4(1 - x_1^2)} \right)$$

Hence, combining all of these expressions, we obtain $G_2^{(1)}$ and $G_3^{(1)}$, equations (52 and (53).

reference p. 31

The functions $h_i(\theta_1, \phi_1, \omega_2, \omega_3)$

h_1	$8(\sin\theta_1 \cos^2\phi_1 - \omega_2)(\cos^2\theta_1 \sin\phi_1 \cos\phi_1) + (\sin^2\theta_1 \sin\phi_1 \cos\phi_1)$ $(\sin\theta_1 \sin^2\phi_1 - \sin\theta_1 \cos^2\phi_1) + \varepsilon_1\Big[2(\sin\theta_1 \cos^2\phi_1 - \omega_2)$ $(-\sin^2\theta_1 \sin\phi_1 \cos\phi_1 + 5\cos^2\theta_1 \sin\phi_1 \cos\phi_1 - 2\sin\phi_1 \cos\phi_1)$ $+ 2\sin^2\theta_1 \sin\phi_1 \cos\phi_1(\sin\theta_1 \sin^2\phi_1 - \sin\theta_1 \cos^2\phi_1)$ $- 2(\sin\theta_1 \cos\theta_1 \sin\phi_1 \cos\phi_1 - \omega_3)\cos\theta \sin^2\phi_1 - \sin^2\theta_1$ $\cos\phi_1 \sin\phi(\sin\theta_1 \cos^2\phi_1 - \omega_2) - 2 \frac{d}{dt} (\sin\theta_1 \cos^2\phi_1 - \omega_2)\Big]$
h_2	$-\frac{3}{32\pi} \sin^2\theta_1 \sin\phi_1 \cos\phi_1(\sin\theta_1 \cos\theta_1 \sin\phi_1 \cos\phi_1 - \omega_3)$
h_3	$8(-\sin\theta_1 \cos\theta_1 \sin\phi_1 \cos\phi_1 + \omega_3)\sin\phi_1 \cos\phi_1 + 2\sin^2\theta_1 \sin\phi_1$ $\cos\phi_1(\sin\theta_1 \cos\theta_1 \sin\phi_1 \cos\phi_1) + \varepsilon_1\Big[2(\sin\theta_1 \cos\theta_1 \sin\phi_1 \cos\phi_1$ $- \omega_3)(\sin^2\theta_1 \sin\phi_1 \cos\phi_1 + 2\cos^2\theta_1 \sin\phi_1 \cos\phi_1 - 5\sin\phi_1 \cos\phi_1)$ $+ 2\sin^2\theta_1 \sin\phi_1 \cos\phi_1(2\sin\theta_1 \cos\theta_1 \sin\phi_1 \cos\phi_1) + \sin^2\theta_1 \sin\phi_1$ $\cos\phi_1(\sin\theta_1 \cos\theta_1 \sin\phi_1 \cos\phi_1 - \omega_3) + 2(\sin\theta_1 \cos^2\phi_1 - \omega_2)$ $\cos\theta_1 \cos^2\phi_1 + 2 \frac{d}{dt} (\sin\theta_1 \cos\theta_1 \sin\phi_1 \cos\phi_1 - \omega_3)\Big]$
h_4	$\frac{3}{32\pi} \sin^2\theta_1 \sin\phi_1 \cos\phi_1(\sin\theta_1 \cos^2\phi_1 - \omega_2)$