To be held in editorial office

ADDENDUM TO PAPER BY R.C.T.RAINEY, "A NEW EQUATION FOR CALCULATING WAVE LOADS ON OFFSHORE STRUCTURES", JOURNAL OF FLUID MECHANICS, 1989

EFFECT OF POSITION INCREMENT ON LAST TERM IN (A3.1)

The incremental change in the last term in (A3.1) is the result of S' moving across the water surface; arguing as above, we see that this motion is given by cylinder motion times the linear mapping $\underline{\underline{S}}$, where $\underline{\underline{S}}$ leaves motion components parallel to the water surface unaltered, and maps unit motion components normal to the water surface to $\underline{n}' \tan\alpha$, where $\underline{n}'$ is the unit vector in the water surface and the plane of $\underline{n}$ and $\underline{\ell}$, see Figure 1. Thus the increment in ϕ_I can be written:

$$\delta\phi_I = \nabla\phi_I \cdot \underline{\underline{S}}\left(\delta\underline{x}_1 + \delta\underline{x}_2 \wedge \underline{r}\right) = \underline{v}\cdot\underline{\underline{S}}\left(\delta\underline{x}_1 + \delta\underline{x}_2 \wedge \underline{r}\right) \qquad \text{(A3.7)}$$

The increment in $\underline{v}$ is, from (A3.2):

$$\delta\underline{v} = \underline{\underline{v}}\,\underline{\underline{S}}\left(\delta\underline{x}_1 + \delta\underline{x}_2 \wedge \underline{r}\right) + \underline{v} \wedge \delta\underline{x}_2 \qquad \text{(A3.8)}$$

and exactly analogously the increment in $\underline{n}$ is:

$$\delta\underline{n} = \underline{\underline{n}}\,\underline{\underline{S}}\left(\delta\underline{x}_1 + \delta\underline{x}_2 \wedge \underline{r}\right) + \underline{n} \wedge \delta\underline{x}_2 \qquad \text{(A3.9)}$$

where we are writing $\underline{\underline{n}}$ for the gradient mapping of $\underline{n}$ [i.e. $\underline{\underline{n}}$ gives the change in $\underline{n}$ over a small displacement $\underline{\delta}$ as $\underline{\underline{n}}\,\underline{\delta}$]. Actually $\underline{\underline{n}}$ is only defined for small displacements within the water surface, i.e. normal to $\underline{n}$, such displacements being guaranteed by the definition of $\underline{\underline{S}}$. We can however define it as a 3-D mapping by setting $\underline{\underline{n}}\,\underline{n} = 0$, and also see that that it will be symmetric, by considering the surface as a function in Cartesian axes aligned with $\underline{n}$. The increment in $c/\cos\alpha$ can be written:

$$\delta(c/\cos\alpha) = (-c/\cos^2\alpha)\,\delta(\underline{l}.\underline{n}) = -c\,\underline{l}.\delta\underline{n}/\cos^2\alpha \tag{A3.10}$$

where we are writing $\underline{l}$ for the unit vector pointing out of the water along the cylinder axis, see Figure 1, and noting that from our structure-fixed viewpoint $\underline{l}$ is a constant. Finally, the increment in $\underline{u}$ can be evaluated by noting that the motion of S' from our structure-fixed viewpoint is $\underline{\underline{S}}'(\delta\underline{x}_1 + \delta\underline{x}_{2\wedge}\underline{r})$ where $\underline{\underline{S}}' = \underline{\underline{S}} - I$, so that:

$$\delta\underline{u} = \underline{w}_\wedge \underline{\underline{S}}'(\delta\underline{x}_1 + \delta\underline{x}_{2\wedge}\underline{r}) \tag{A3.11}$$

To assemble (A3.7) - (A3.11) we can write the increment in the last term in (A3.1) as:

$$-\rho\sum_p\left[\phi_I(\underline{v}-\underline{u}).\delta\underline{n}\,c/\cos\alpha + \phi_I(\delta\underline{v}-\delta\underline{u}).\underline{n}\,c/\cos\alpha + \left\{\delta\phi_I - \underline{l}.\delta\underline{n}\,\phi_I/\cos\alpha\right\}(\underline{v}-\underline{u}).\underline{n}\,c/\cos\alpha\right] \tag{A3.12}$$

The first term in (A3.12) is analogous to (A3.3) and so can be rearranged in the style of (A3.4):

$$-\rho\sum_p \phi_I\,c/\cos\alpha\left[\underline{\underline{S}}^T\underline{n}(\underline{v}-\underline{u}),\left\{\underline{\underline{r}}\,\underline{\underline{S}}^T\underline{n}(\underline{v}-\underline{u}) - \underline{n}_\wedge(\underline{v}-\underline{u})\right\}\right].\delta\underline{X} \tag{A3.13}$$

in which we note that $\underline{\underline{S}}$ is not symmetric. The second term in (A3.12) can similarly be written, noting that $\underline{\underline{V}}$ is symmetric but $\underline{\underline{W}} = \underline{w}_\wedge$ is skew-symmetric:

$$-\rho\sum_p \phi_I\,c/\cos\alpha\left[\left(\underline{\underline{S}}^T\underline{\underline{V}} + \underline{\underline{S}}'^T\underline{\underline{W}}\right)\underline{n},\left\{\underline{\underline{r}}\left(\underline{\underline{S}}^T\underline{\underline{V}} + \underline{\underline{S}}'^T\underline{\underline{W}}\right)\underline{n} - \underline{v}_\wedge\underline{n}\right\}\right].\delta\underline{X} \tag{A3.14}$$

The last term in (A3.12) becomes in a similar style:

$$-e\sum_{p}(\underline{v}-\underline{u}).\underline{n}\,c/\cos\alpha\left[\left\{\underline{\underline{S}}^{T}\underline{v}-\underline{\underline{S}}^{T}\underline{\underline{n}}\,\underline{\ell}\,\phi_{I}/\cos\alpha\right\},\left\{\underline{\underline{r}}\,\underline{\underline{S}}^{T}\underline{v}-\left(\underline{\underline{r}}\,\underline{\underline{S}}^{T}\underline{\underline{n}}\,\underline{\ell}-\underline{n}\wedge\underline{\ell}\right)\phi_{I}/\cos\alpha\right\}\right].\delta\underline{X} \qquad \text{(A3.15)}$$

This completes the expression of the constituent increments in δe as scalar products with $\delta\underline{X}$.