Footnote-referenced material of

On the Resonant Triad Interaction in Flows Over Rigid and Flexible Boundaries

by M. D. Thomas

$$
\begin{aligned}
F_1 = {} & \frac{1}{4}\mathrm{i}\alpha A_3 A_2^* \exp(\alpha c_\mathrm{i} t)(\frac{\alpha^2}{\gamma^2}-2)\phi_3(\phi_2^{*\prime\prime\prime}-\gamma^2\phi_2^{*\prime}) \\
& +(\frac{\alpha^2}{\gamma^2}-3)\phi_3'(\phi_2^{*\prime\prime}-\gamma^2\phi_2^*)-2\phi_2^{*\prime}(\phi_3''-\alpha^2\phi_3) \\
& -\phi_2^*(\phi_3'''-\alpha^2\phi_3')-\frac{2\alpha\beta}{\gamma^2}(\phi_3\hat{v}_2^{*\prime\prime}+\phi_3'\hat{v}_2^{*\prime}+\gamma^2\phi_3\hat{v}_2^*)
\end{aligned} \tag{A1}
$$

$$
\begin{aligned}
F_2 = {} & \frac{1}{4}\mathrm{i}\alpha A_3 A_1^* \exp(\alpha c_\mathrm{i} t)(\frac{\alpha^2}{\gamma^2}-2)\phi_3(\phi_1^{*\prime\prime\prime}-\gamma^2\phi_1^{*\prime}) \\
& +(\frac{\alpha^2}{\gamma^2}-3)\phi_3'(\phi_1^{*\prime\prime}-\gamma^2\phi_1^*)-2^{*\prime}\phi_1(\phi_3''-\alpha^2\phi_3) \\
& -\phi_1^*(\phi_3'''-\alpha^2\phi_3')-\frac{2\alpha\beta}{\gamma^2}(\phi_3\hat{v}_1^{*\prime\prime}+\phi_3'\hat{v}_1^{*\prime}+\gamma^2\phi_3\hat{v}_1)
\end{aligned} \tag{A2}
$$

$$
\begin{aligned}
F_3 = {} & \frac{1}{2}\mathrm{i}\alpha A_2 A_1 \exp\{\alpha(\tilde{c}_\mathrm{i}-c_\mathrm{i})t\}(3-\frac{\alpha^2}{\gamma^2})\phi_1'(\phi_1''-\gamma^2\phi_1) \\
& +\phi_1(\phi_1'''-\gamma^2\phi_1')+\frac{2\alpha\beta}{\gamma^2}\left(\hat{v}_1(\phi_1''-\gamma^2\phi_1)+\phi_1'\hat{v}_1'\right) \\
& -\frac{2\beta}{\alpha}(\phi_1''\hat{v}_1+2\phi_1'\hat{v}_1'+\phi_1\hat{v}_1'')-4\frac{\beta^2}{\gamma^2}\hat{v}_1\hat{v}_1'.
\end{aligned} \tag{A3}
$$

$$
\mu_1^{(3)} = f_1 + \frac{\bar{u}'(0)}{\mathrm{i}\alpha c} f_2 \tag{A4}
$$

$$
\mu_2^{(3)} = -R\{\mathrm{i}\alpha c\mu_1^{(3)} + f_3 - \mathrm{i}\alpha f_4 + c^{-1}[m\alpha^2(c^2-c_0^2)+\mathrm{i}\alpha c d - S]f_2\} \tag{A5}
$$

$$
\mu_1^{(1)} = \frac{\alpha}{2\gamma}\lambda_1 + \frac{\beta}{\gamma}\lambda_2 + \frac{\bar{u}'(0)}{\mathrm{i}\gamma\tilde{c}}\lambda_3 \tag{A6}
$$

$$
\begin{aligned}
\mu_2^{(1)} = {} & R\{\mathrm{i}\gamma\lambda_4 - \frac{\alpha}{2\gamma}\lambda_5 - \frac{\beta}{\gamma}\lambda_6 - \frac{1}{2}\mathrm{i}\alpha\tilde{c}\mu_1^{(1)} \\
& -2\frac{\gamma}{\alpha\tilde{c}}[\frac{1}{4}m\alpha^2(\tilde{c}^2-\frac{\alpha}{2\gamma}c_0^2)+\mathrm{i}\frac{\alpha}{2}\tilde{c}d - S]\lambda_3\}
\end{aligned} \tag{A7}
$$

$$
\begin{aligned}
\sigma_3 = {} & \frac{1}{\mathrm{i}\alpha c R}\phi_3'(0)[\psi_3''(0)+(\alpha^2+\mathrm{i}\alpha R c)\psi_3(0)] \\
& -\frac{1}{\bar{u}'(0)R}\phi_3'(0)\psi_3(0)[\mathrm{i}\alpha R(2\mathrm{i}\alpha c m - d)-\frac{\mathrm{i}}{\alpha}B_3] \\
& +\int_0^\infty(\phi_3''-\alpha^2\phi_3)\psi_3\,dz
\end{aligned} \tag{A8}
$$

$$
\begin{aligned}
\sigma_1 = {} & \frac{2}{\mathrm{i}\alpha\tilde{c}\tilde{R}}\phi_1'(0)[\psi_1''(0)+(\gamma^2+\mathrm{i}\gamma\tilde{R}\tilde{c})\psi_1(0)] \\
& -\frac{2\gamma}{\alpha\bar{u}'(0)\tilde{R}}\phi_1'(0)\psi_1(0)[\mathrm{i}\gamma R(\mathrm{i}\alpha\tilde{c}m - d)-\frac{\mathrm{i}}{\gamma}B_1] \\
& +\int_0^\infty(\phi_1''-\gamma^2\phi_1)\psi_1\,dz
\end{aligned} \tag{A9}
$$

$$
\begin{aligned}
\zeta_3 = & \int_0^\infty F_3\psi_3\,dz + \psi_3(0)[\mathrm{i}\alpha c f_1 + \bar{u}'(0) f_2 + f_3 - \mathrm{i}\alpha f_4 + \frac{1}{\mathrm{i}\alpha R} B_3 f_2] \\
& - \frac{1}{R}[f_1 + \frac{\bar{u}'(0)}{\mathrm{i}\alpha c} f_2][\psi_3''(0) + (\alpha^2 + \mathrm{i}\alpha R c)\psi_3(0)]
\end{aligned} \tag{A10}
$$

$$
\begin{aligned}
\zeta_1 = & -\frac{1}{\tilde{R}}[\frac{\alpha}{2\gamma}\lambda_1 + \frac{\beta}{\gamma}\lambda_2 + \frac{\bar{u}'(0)}{\mathrm{i}\gamma\tilde{c}}\lambda_3][\psi_1''(0) + (\gamma^2 + \mathrm{i}\gamma\tilde{R}\tilde{c})\psi_1(0)] \\
& + \int_0^\infty F_1\psi_1\,dz - \frac{R}{\tilde{R}}\psi_1(0)\{\mathrm{i}\gamma\lambda_4 - \frac{\alpha}{2\gamma}\lambda_5 - \frac{\beta}{\gamma}\lambda_6 \\
& - \mathrm{i}\frac{\alpha}{2\gamma}[\frac{\alpha}{2}\tilde{c}\lambda_1 + \beta\tilde{c}\lambda_2 - \mathrm{i}\bar{u}'(0)\lambda_3] - \frac{1}{\mathrm{i}\gamma R} B_1\lambda_3\}
\end{aligned} \tag{A11}
$$

where

$$
f_1 = \eta_1\left(\frac{\beta}{\gamma}\hat{v}_1'(0) - \frac{\alpha}{2\gamma}\phi_1''(0)\right) \tag{A12}
$$

$$
f_2 = \mathrm{i}\alpha\eta_1\left(\frac{\beta}{\gamma}\hat{v}_1(0) - \frac{\alpha}{2\gamma}\phi_1'(0) - \frac{1}{2}\bar{u}'(0)\eta_1\right) \tag{A13}
$$

$$
\begin{aligned}
f_3 = & \frac{\mathrm{i}\alpha}{2\gamma}\tilde{c}\eta_1(\frac{\alpha}{2}\phi_1''(0) - \beta\hat{v}_1'(0)) - \mathrm{i}\frac{\alpha}{2}\eta_1 p_1'(0) + \mathrm{i}\frac{\beta}{\gamma}\bar{u}'(0)\eta_1\left(\beta\phi_1'(0) + \frac{\alpha}{2}\hat{v}_1'(0)\right) \\
& - \frac{\mathrm{i}}{\gamma^2}\left((\frac{\alpha^3}{8} - \frac{\alpha\beta^2}{2}){\phi_1'}^2(0) + \alpha\beta^2{\hat{v}_1'}^2(0) + \left(\beta^3 - \frac{3\alpha^2\beta}{4}\right)\phi_1'(0)\hat{v}_1(0)\right. \\
& \left. - \gamma\phi_1(0)\left(\frac{\alpha}{2}\phi_1''(0) - \beta\hat{v}_1'(0)\right)\right) \\
& + \frac{1}{\gamma R}\eta_1\left(\frac{\alpha}{2}(\phi_1''''(0) - \gamma^2\phi_1''(0)) - \beta(\hat{v}_1'''(0) - \gamma^2\hat{v}_1'(0))\right)
\end{aligned} \tag{A14}
$$

$$
\begin{aligned}
f_4 = & -\frac{1}{R}\eta_1\left(Rp_1'(0) + \mathrm{i}(\frac{3\alpha^2 + 4\beta^2}{\gamma})\phi_1''(0)\right. \\
& \left. -\mathrm{i}\frac{\alpha\beta}{\gamma}\hat{v}_1'(0) + \mathrm{i}\gamma(\frac{\alpha^2}{4} - \beta^2)\phi_1(0)\right)
\end{aligned} \tag{A15}
$$

$$
\lambda_1 = -\frac{1}{2}\left(\frac{\alpha}{2\gamma}\eta_3\phi_1^{*\prime\prime}(0) - \frac{\beta}{\gamma}\eta_3\hat{v}_1^{*\prime}(0) + {\eta_1}^*\phi_3''(0)\right) \tag{A16}
$$

$$
\lambda_2 = \frac{1}{2}\eta_3\left(\frac{\beta}{\gamma}\phi_1^{*\prime\prime}(0) + \frac{\alpha}{2\gamma}\hat{v}_1^{*\prime}(0)\right) \tag{A17}
$$

$$
\lambda_3 = -\mathrm{i}\frac{\alpha}{4}\left(\bar{u}'(0)\eta_1^*\eta_3 - 2\frac{\beta}{\gamma}\hat{v}_1^*(0)\eta_3 + \phi_3'(0)\eta_1^* + (\frac{\alpha^2 - 2\gamma^2}{\gamma\alpha})\phi_1^{*\prime}(0)\eta_3\right) \tag{A18}
$$

$$
\begin{aligned}
\lambda_4 = & -\frac{1}{2}(\eta_1^* p_3'(0) + \eta_3 p_1^{*\prime}(0)) + \mathrm{i}\gamma\eta_3\phi_1^{*\prime\prime}(0) - \mathrm{i}\alpha\eta_1^*\phi_3''(0) \\
& - \frac{1}{2}\mathrm{i}\alpha\eta_3\left(\frac{\alpha}{2\gamma}(\phi_1^{*\prime\prime}(0) + \gamma^2\phi_1^*(0)) - \frac{\beta}{\gamma}\hat{v}_1^{*\prime}(0)\right) \\
& + \mathrm{i}\frac{\alpha}{4}\eta_1^*(\phi_3''(0) + \alpha^2\phi_3(0))
\end{aligned} \tag{A19}
$$

$$
\begin{aligned}
\lambda_5 =& -\frac{1}{2R}\Big(\eta_1^*(\phi_3''''(0)-\alpha^2\phi_3''(0)) \\
&+\frac{\alpha}{2\gamma}\eta_3(\phi_1^{*''''}(0)-\gamma^2\phi_1^{*''}(0))-\frac{\beta}{\gamma}\eta_3(\hat{v}_1^{*'''}(0)-\gamma^2\hat{v}_1^{*'}(0))\Big) \\
&+\mathrm{i}\frac{\alpha}{4}\left(\eta_3 p_1^{*'}(0)-2\eta_1^* p_3'(0)-\tilde{c}^*\eta_3(\frac{\alpha}{2\gamma}\phi_1^{*''}(0)-\beta\hat{v}_1^{*'}(0))+2c\eta_1^*\phi_3''(0)\right) \\
&-\frac{\mathrm{i}}{2}\Big(\frac{\alpha^2}{4\gamma}\phi_1^{*'}(0)\phi_3'(0)-\frac{\alpha\beta}{2\gamma}\hat{v}_1^*(0)\phi_3'(0)+\gamma\phi_1^*(0)\phi_3''(0)-\frac{\alpha^2}{2\gamma}\phi_1^{*''}(0)\phi_3(0) \\
&+\frac{\alpha\beta}{\gamma}\hat{v}_1^{*'}(0)\phi_3(0)+\frac{\beta^2}{\gamma}\bar{u}'(0)\eta_3\phi_1^{*'}(0)+\frac{\alpha\beta}{2\gamma}\bar{u}'(0)\eta_3\hat{v}_1^*(0)\Big)
\end{aligned}
\tag{A20}
$$

$$
\begin{aligned}
\lambda_6 =& \mathrm{i}\frac{\alpha}{4\gamma}\left(\tilde{c}^*\eta_3(\beta\phi_1^{*''}(0)+\frac{\alpha}{2}\hat{v}_1^{*'}(0))-2\phi_3(0)(\beta\phi_1^{*''}(0)+\frac{\alpha}{2}\hat{v}_1^{*'}(0))\right) \\
&-\frac{1}{2}\eta_3\Big(\mathrm{i}\beta p_1^{*'}(0)+\frac{\beta}{\gamma R}(\phi_1^{*''''}(0)-\gamma^2\phi_1^{*''}(0)) \\
&+\frac{\alpha}{2\gamma R}(\hat{v}_1^{*'''}(0)-\gamma^2\hat{v}_1^{*'}(0))\Big)
\end{aligned}
\tag{A21}
$$