

Appendix A The operators in equations (3.12), (3.13)

$$L_1(q_0) = -\rho i \omega_0 \tau_0 + U i \alpha_0 \tau_0 + v_0 \frac{\partial \rho}{\partial Y} + \rho i \alpha_0 u_0 + \rho \frac{\partial v_0}{\partial y} + \rho i k_z w_0, \quad (A.1)$$

$$L_2(q_0) = -\rho i \omega_0 u_0 + \rho U i \alpha_0 u_0 + \rho v_0 \frac{\partial U}{\partial y} + i \alpha_0 p_0 + \frac{1}{\text{Re}} \frac{4}{3} \mu \alpha_0^2 u_0 + \frac{1}{\text{Re}} \frac{2}{3} \mu i \alpha_0 \frac{\partial v_0}{\partial y} \\ - \frac{1}{\text{Re}} \frac{2}{3} \mu \alpha_0 k_z w_0 - \frac{1}{\text{Re}} \mu \frac{\partial^2 u_0}{\partial y^2} - \frac{1}{\text{Re}} \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} \frac{\partial u_0}{\partial y} - \frac{1}{\text{Re}} \frac{\partial \mu}{\partial T} \frac{\partial^2 U}{\partial y^2} \theta_0 - \frac{1}{\text{Re}} \frac{\partial^2 \mu}{\partial T^2} \frac{\partial T}{\partial y} \frac{\partial U}{\partial y} \theta_0 \\ - \frac{1}{\text{Re}} \frac{\partial U}{\partial y} \frac{\partial \mu}{\partial T} \frac{\partial \theta_0}{\partial y} - \frac{1}{\text{Re}} \mu \frac{\partial v_0}{\partial y} i \alpha_0 - \frac{1}{\text{Re}} \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} i \alpha_0 v_0 + \frac{1}{\text{Re}} \mu k_z^2 u_0 + \frac{1}{\text{Re}} \mu \alpha_0 k_z w_0, \quad (A.2)$$

$$L_3(q_0) = -\rho i \omega_0 v_0 + \rho U i \alpha_0 v_0 + \frac{\partial p_0}{\partial y} - \frac{1}{\text{Re}} \mu i \alpha_0 \frac{\partial u_0}{\partial y} - \frac{1}{\text{Re}} \frac{\partial U}{\partial y} \frac{\partial \mu}{\partial T} i \alpha_0 \theta_0 + \frac{1}{\text{Re}} \mu \alpha_0^2 v_0 - \frac{1}{\text{Re}} \frac{4}{3} \mu \frac{\partial^2 v_0}{\partial y^2} \\ - \frac{1}{\text{Re}} \frac{4}{3} \frac{\partial v_0}{\partial y} \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} + \frac{1}{\text{Re}} \frac{2}{3} \mu i \alpha_0 \frac{\partial u_0}{\partial y} + \frac{1}{\text{Re}} \frac{2}{3} i \alpha_0 u_0 \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} + \frac{1}{\text{Re}} \frac{2}{3} \mu i k_z \frac{\partial w_0}{\partial y} \\ + \frac{1}{\text{Re}} \frac{2}{3} \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} i k_z w_0 + \frac{1}{\text{Re}} \mu k_z^2 v_0 - \frac{1}{\text{Re}} \mu i k_z \frac{\partial w_0}{\partial y}, \quad (A.3)$$

$$L_4(q_0) = -\rho i \omega_0 w_0 + \rho U i \alpha_0 w_0 + i k_z p_0 + \frac{1}{\text{Re}} \mu \alpha_0 k_z u_0 + \frac{1}{\text{Re}} \mu \alpha_0^2 w_0 - \frac{1}{\text{Re}} \mu i k_z \frac{\partial v_0}{\partial y} - \frac{1}{\text{Re}} \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} i k_z v_0 \\ - \frac{1}{\text{Re}} \mu \frac{\partial^2 w_0}{\partial y^2} - \frac{1}{\text{Re}} \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} \frac{\partial w_0}{\partial y} + \frac{1}{\text{Re}} \frac{4}{3} \mu k_z^2 w_0 - \frac{2}{3} \frac{\mu}{\text{Re}} \alpha_0 k_z u_0 + \frac{2}{3} \frac{\mu}{\text{Re}} i k_z \frac{\partial v_0}{\partial y}, \quad (A.4)$$

$$L_5(q_0) = -\rho i \omega_0 \theta_0 + \rho U i \alpha_0 \theta_0 + v_0 \rho \frac{\partial T}{\partial y} + (\gamma - 1) \left( i \alpha_0 u_0 + \frac{\partial v_0}{\partial y} + i k_z w_0 \right) - \frac{\gamma}{\sigma \text{Re}} \left\{ -\mu \alpha_0^2 \theta_0 + \mu \frac{\partial^2 \theta_0}{\partial y^2} \right. \\ \left. + \frac{\partial \theta_0}{\partial y} \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} + \frac{\partial^2 T}{\partial y^2} \frac{\partial \mu}{\partial T} \theta_0 + \left( \frac{\partial T}{\partial y} \right)^2 \frac{\partial^2 \mu}{\partial T^2} \theta_0 + \frac{\partial T}{\partial y} \frac{\partial \mu}{\partial T} \frac{\partial \theta_0}{\partial y} - \mu k_z^2 \theta_0 \right\} \\ - \frac{\gamma(\gamma - 1) M^2}{\text{Re}} \left\{ \frac{\partial \mu}{\partial T} \theta_0 \left( \frac{\partial U}{\partial y} \right)^2 + 2 \mu \frac{\partial U}{\partial y} \left( \frac{\partial u_0}{\partial y} + i \alpha_0 v_0 \right) \right\}, \quad (A.5)$$

$$L_6(q_0) = \gamma M^2 / p_0 - \rho \theta_0 - T \tau_0 = 0, \quad (A.6)$$

$$R_{11} = -U \tau_0 - \rho u_0, \quad (A.7)$$

$$R_{21} = -U \frac{\partial \tau_0}{\partial X} - u_0 \frac{\partial \rho}{\partial X} - V \frac{\partial \tau_0}{\partial Y} - \tau_0 \frac{\partial V}{\partial Y} - \tau_0 \frac{\partial U}{\partial X} - \rho \frac{\partial u_0}{\partial X}, \quad (A.8)$$

$$R_{12} = -\rho U u_0 - p_0 + \frac{\mu}{\text{Re}} \left\{ \frac{8}{3} i \alpha_0 u_0 + \frac{1}{3} \frac{\partial v_0}{\partial y} + \frac{1}{3} (i k_z w_0 + \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial T}{\partial Y}) \right\}, \quad (\text{A}_9)$$

$$R_{22} = -\rho U \frac{\partial u_0}{\partial X} - \frac{\partial p_0}{\partial X} - \rho \frac{dU}{dX} u_0 - U \frac{dU}{dX} \tau_0 - \rho V \frac{\partial u_0}{\partial Y} - V \tau_0 \frac{\partial U}{\partial Y} + \frac{\mu}{\text{Re}} \left\{ \frac{8}{3} i \alpha_0 \frac{\partial u_0}{\partial X} + \frac{4}{3} i u_0 \frac{d\alpha_0}{dX} \right. \\ \left. + \frac{1}{3} \frac{\partial^2 v_0}{\partial X \partial Y} + \frac{1}{3} \frac{\partial w_0}{\partial X} i k_z + \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial T}{\partial Y} \frac{\partial v_0}{\partial X} - \frac{2}{3} \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial T}{\partial X} \frac{\partial v_0}{\partial Y} - \frac{2}{3} \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial T}{\partial X} i k_z w_0 \right. \\ \left. + \frac{4}{3} \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial U}{\partial X} i \alpha_0 \theta_0 + \frac{4}{3} \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial T}{\partial X} i \alpha_0 u_0 - \frac{2}{3} \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial V}{\partial y} i \alpha_0 \theta_0 \right\}, \quad (\text{A}_{10})$$

$$R_{13} = -\rho U v_0 + \frac{\mu}{\text{Re}} \left\{ \frac{1}{3} \frac{\partial u_0}{\partial Y} + \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial U}{\partial y} \theta_0 + 2 i \alpha_0 v_0 - \frac{2}{3} \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial T}{\partial y} u_0 \right\}, \quad (\text{A}_{11})$$

$$R_{23} = -\rho U \frac{\partial v_0}{\partial x} - \rho V \frac{\partial v_0}{\partial Y} - \rho v_0 \frac{\partial V}{\partial y} + \frac{\mu}{\text{Re}} \left\{ \frac{1}{3} i \alpha_0 \frac{\partial u_0}{\partial y} + \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial U}{\partial Y} i \alpha_0 \theta_0 + 2 i \alpha_0 \frac{\partial v_0}{\partial x} + i \frac{d\alpha_0}{dX} v_0 \right. \\ \left. - \frac{2}{3} \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial T}{\partial y} \frac{\partial u_0}{\partial X} + \frac{1}{\mu} \frac{d\mu}{dT} \frac{dT}{dX} \frac{\partial u_0}{\partial Y} + \frac{1}{3} \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial^2 U}{\partial X \partial Y} \theta_0 + \frac{1}{\mu} \frac{d^2 \mu}{dT^2} \frac{\partial T}{\partial X} \frac{\partial U}{\partial y} \theta_0 \right. \\ \left. + \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial T}{\partial X} i \alpha_0 v_0 + \frac{1}{\mu} \frac{4}{3} \frac{d\mu}{dT} \frac{\partial^2 V}{\partial y^2} \theta_0 + \frac{1}{\mu} \frac{4}{3} \frac{d^2 \mu}{dT^2} \frac{\partial T}{\partial Y} \frac{\partial V}{\partial y} \theta_0 + \frac{1}{\mu} \frac{4}{3} \frac{d\mu}{dT} \frac{\partial V}{\partial Y} \frac{\partial \theta_0}{\partial Y} \right. \\ \left. - \frac{1}{\mu} \frac{2}{3} \frac{\partial U}{\partial X} \frac{d^2 \mu}{dT^2} \frac{\partial T}{\partial y} \theta_0 - \frac{1}{\mu} \frac{2}{3} \frac{\partial U}{\partial X} \frac{d\mu}{dT} \frac{\partial \theta_0}{\partial y} \right\}, \quad (\text{A}_{12})$$

Small frac

$$R_{14} = -\rho U w_0 + \frac{\mu}{\text{Re}} \left\{ \frac{1}{3} u_0 i k_z + 2 i \alpha_0 w_0 \right\}, \quad (\text{A}_{13})$$

$$R_{24} = -\rho U \frac{\partial w_0}{\partial x} - \rho V \frac{\partial w_0}{\partial y} + \frac{\mu}{\text{Re}} \left\{ \frac{1}{3} \frac{\partial u_0}{\partial X} i k_z + 2 i \alpha_0 \frac{\partial w_0}{\partial X} + i \frac{d\alpha_0}{dX} w_0 + \frac{1}{\mu} i k_z u_0 \frac{d\mu}{dT} \frac{\partial T}{\partial X} \right. \\ \left. - \frac{2}{3} \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial U}{\partial X} i k_z \theta_0 - \frac{2}{3} \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial V}{\partial y} i k_z \alpha_0 + \frac{1}{\mu} \frac{d\mu}{dT} \frac{\partial T}{\partial X} i \alpha_0 w_0 \right\}, \quad (\text{A}_{14})$$

$$R_{15} = -\rho U \theta_0 - (\gamma - 1) u_0 + \frac{\gamma}{\sigma \text{Re}} \mu 2 i \alpha_0 \theta_0 + \frac{[\gamma(\gamma - 1) M^2 / 2 \mu w_0]}{\text{Re}}, \quad (\text{A}_{15})$$

$$R_{25} = -\rho U \frac{\partial \theta_0}{\partial X} - (\gamma - 1) \frac{\partial u_0}{\partial x} + \frac{\gamma}{\sigma \text{Re}} \mu \left\{ 2 i \alpha_0 \frac{\partial \theta_0}{\partial X} + i \frac{d\alpha_0}{dX} \theta_0 + \frac{2}{\mu} \frac{d\mu}{dT} \frac{\partial T}{\partial X} i \alpha_0 \theta_0 \right\} \\ + \frac{\gamma(\gamma - 1) M^2}{\text{Re}} \mu \left\{ 2 \frac{\partial v_0}{\partial X} \frac{dU}{dY} + \frac{8}{3} \frac{\partial U}{\partial X} i \alpha_0 u_0 + \frac{8}{3} \frac{\partial V}{\partial y} \frac{\partial v_0}{\partial y} - \frac{4}{3} \frac{\partial V}{\partial y} i \alpha_0 u_0 - \frac{4}{3} \frac{\partial v_0}{\partial y} \frac{\partial U}{\partial X} \right\}$$

$$-\frac{4}{3} \frac{\partial U}{\partial X} (ik_z w_0 - \frac{4}{3} ik_z w_0 \frac{\partial V}{\partial Y}) - \tau_0 U \frac{\partial T}{\partial X} - u_0 \rho \frac{\partial T}{\partial X} - \rho V \frac{\partial \theta_0}{\partial y} - \tau_0 V \frac{\partial T}{\partial y} .$$

(A<sub>4</sub>16)