Supplementary information for

Controls on rapid supraglacial lake drainage in West Greenland: An Exploratory Data Analysis approach

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Calculating the ice-surface principal strain rate and the von Mises yield criterion

To calculate the ice-surface principal strain rate, $\dot{\mathcal{E}}$, we followed Poinar and others (2015), which involved differencing the 250 m resolution ice-velocity data to derive the strain rate tensor ($\dot{\mathcal{E}}_{ij}$), then calculating $\dot{\mathcal{E}}$ following Cuffey and Paterson (2010):

$$\dot{\mathcal{E}} = \frac{\partial u}{\partial x}\cos^2\theta + \frac{\partial v}{\partial y}\sin^2\theta, \tag{S1}$$

with θ derived by:

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}}{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}}\right). \tag{S2}$$

To calculate the von Mises yield criterion, σ_V, we used the methods of Clason and others (2015). We first derived the strain rate tensor as above, and from this calculated the second invariant or the effective strain ($\dot{\mathcal{E}}_E$), measuring the total magnitude of the tensor's diagonal components (Cuffey and Paterson, 2010). These values were then used to derive the stress tensors (σ_{ij}) using Nye's (1957) constitutive relation:

$$\sigma_{ij} = A^{[-1/n]}\dot{\mathcal{E}}_E^{\;(1-n)/n}\dot{\mathcal{E}}_{ij}, \tag{S3}$$

where A is the creep factor, which we assigned as 9.3×10^{-16} s^{-1} kPa^{-3}, consistent with ice at -5 ˚C for a creep exponent (n) that was set at 3 (Cuffey and Paterson, 2010). The stress tensors were then used to derive σ_V:

$$\sigma_V = (\sigma_1\sigma_1) + (\sigma_3\sigma_3) - (\sigma_1\sigma_3), \tag{S4}$$

where σ_1 and σ_3 represented the maximum and minimum principal stresses, respectively. These were calculated with:

$$\sigma_1 = \sigma_{max} = \frac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right) + \sqrt{\left[\frac{1}{2}\left(\sigma_{xx} - \sigma_{yy}\right)\right] + \tau_{xy}^2} \tag{S5}$$

and

$$\sigma_3 = \sigma_{min} = \frac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right) + \sqrt{\left[\frac{1}{2}\left(\sigma_{xx} - \sigma_{yy}\right)\right] + \tau_{xy}^2}, \tag{S6}$$

where σ_{xx}, σ_{yy} and τ_{xy} were the longitudinal, transverse and shear stresses, respectively.

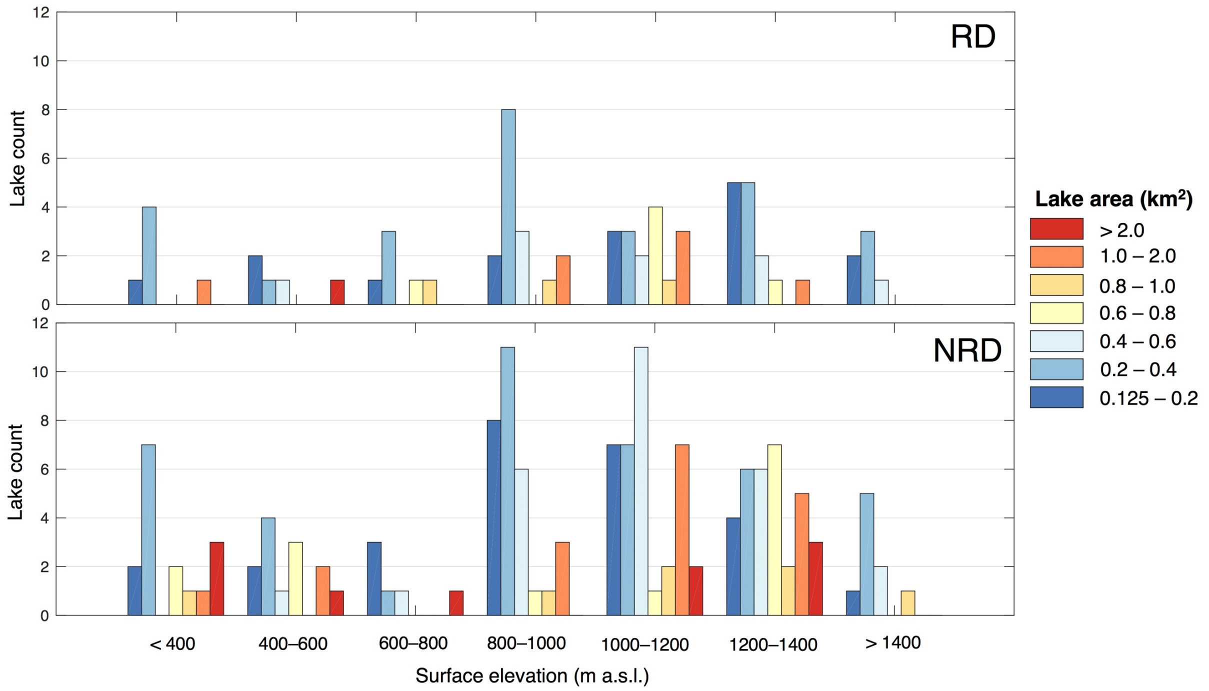

Fig. S1. The distribution of rapidly draining (RD) and non-rapidly draining (NRD) lakes by surface-elevation band (from Howat and others, 2014), colour-coded by lake area immediately prior to drainage for the RD lakes and by lake area when lakes reached their maximum extent for the NRD lakes.

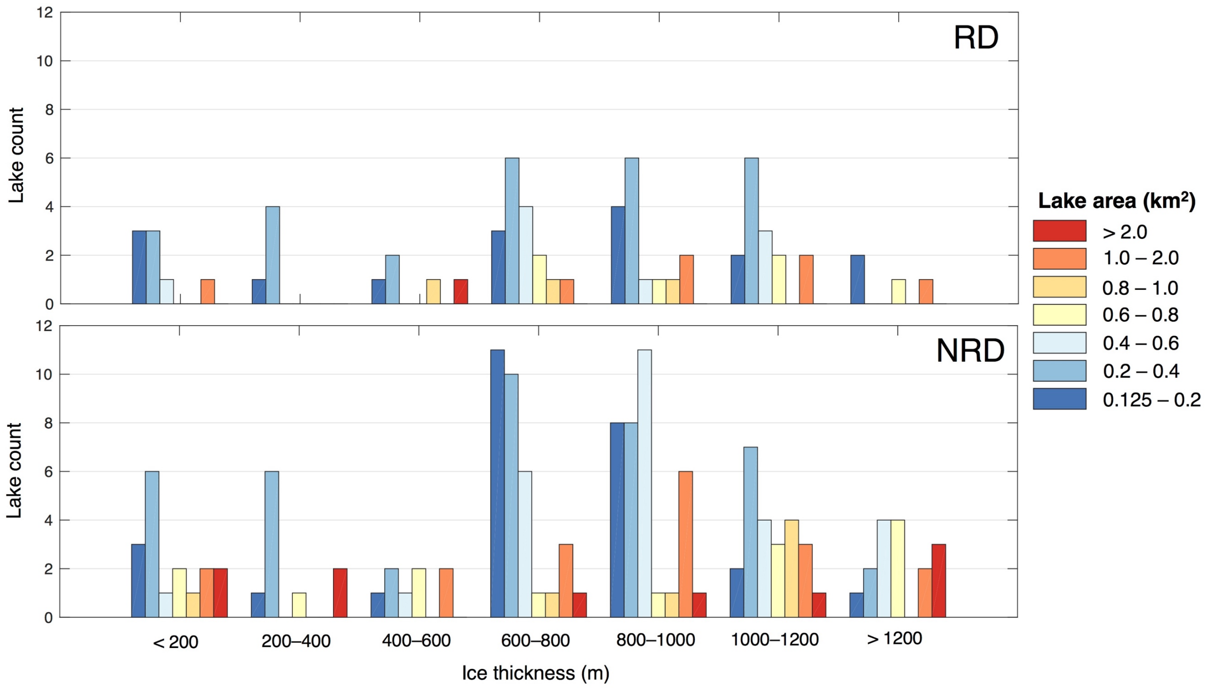

Fig. S2. The distribution of rapidly draining (RD) and non-rapidly draining (NRD) lakes by ice-thickness band (from Morlighem and others, 2014), colour-coded by lake area immediately prior to drainage for RD lakes and by lake area when lakes reached their maximum extent for NRD lakes.

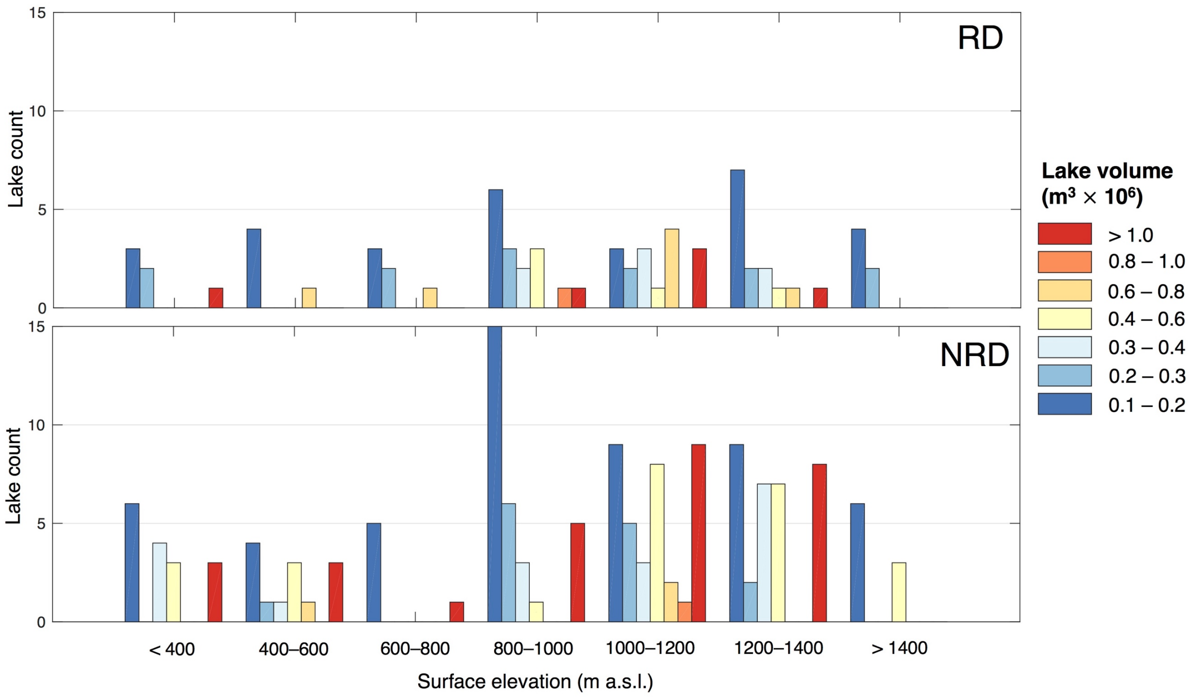

Fig. S3. The distribution of rapidly draining (RD) and non-rapidly draining (NRD) lakes by surface-elevation band (from Howat and others, 2014), colour-coded by lake volume prior to drainage for the RD lakes and by lake volume when lakes reached their maximum extent for the NRD lakes.

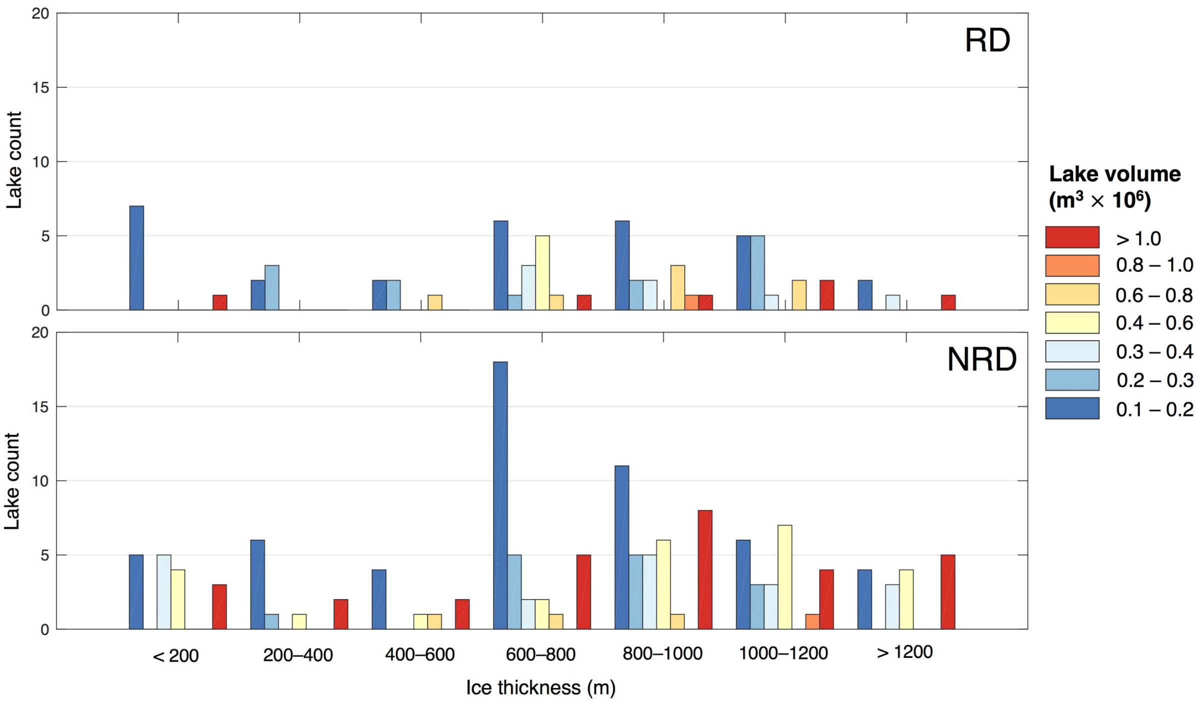

Fig. S4. The distribution of rapidly draining (RD) and non-rapidly draining (NRD) lakes by ice-thickness band (from Morlighem and others, 2014), colour-coded by the lake volume immediately prior to drainage for RD lakes and by lake volume when lakes reached their maximum extent for NRD lakes.

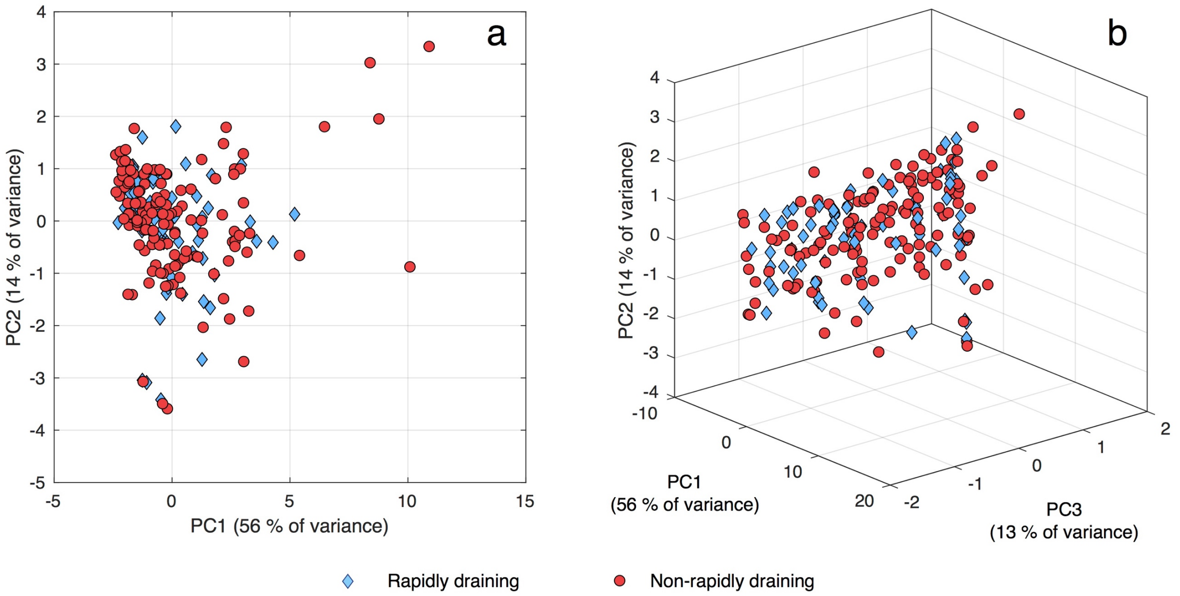

Fig. S5. Principal component (PC) scores for (a) PCs 1 and 2, and (b) PCs 1–3 for rapidly draining lakes (blue diamonds) and non-rapidly draining (red circles) lakes for variables within the hydro-morphological factor categories of analysis (see section 2.2 and Table 2). The percentage figures indicate the amount of variance in the data explained by the PC. The tight clustering of the rapidly draining and non-rapidly draining lake PC scores shows the statistical similarity of the potential controlling factors for the two lake types.

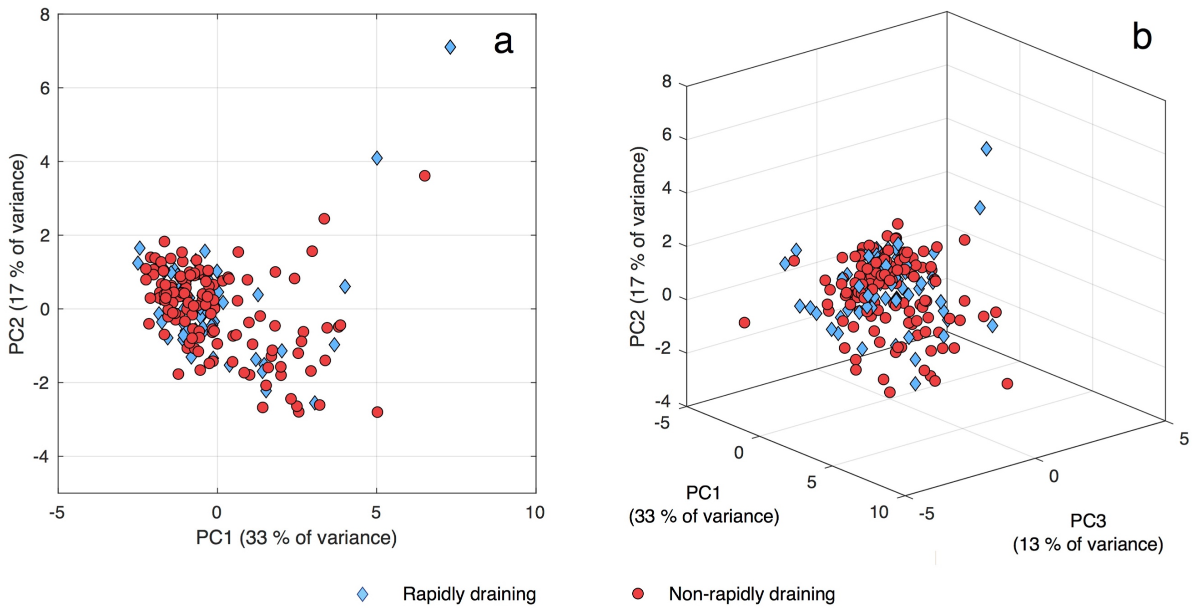

Fig. S6. Principal component (PC) scores for (a) PCs 1 and 2, and (b) PCs 1–3 for rapidly draining (blue diamonds) and non-rapidly draining (red circles) lakes for variables within the glaciological factor category of analysis (see section 2.2 and Table 2). The percentage figures indicate the amount of variance in the data explained by the PC. The tight clustering of the rapidly draining and non-rapidly draining lake PC scores shows the statistical similarity of the potential controlling factors for the two lake types.

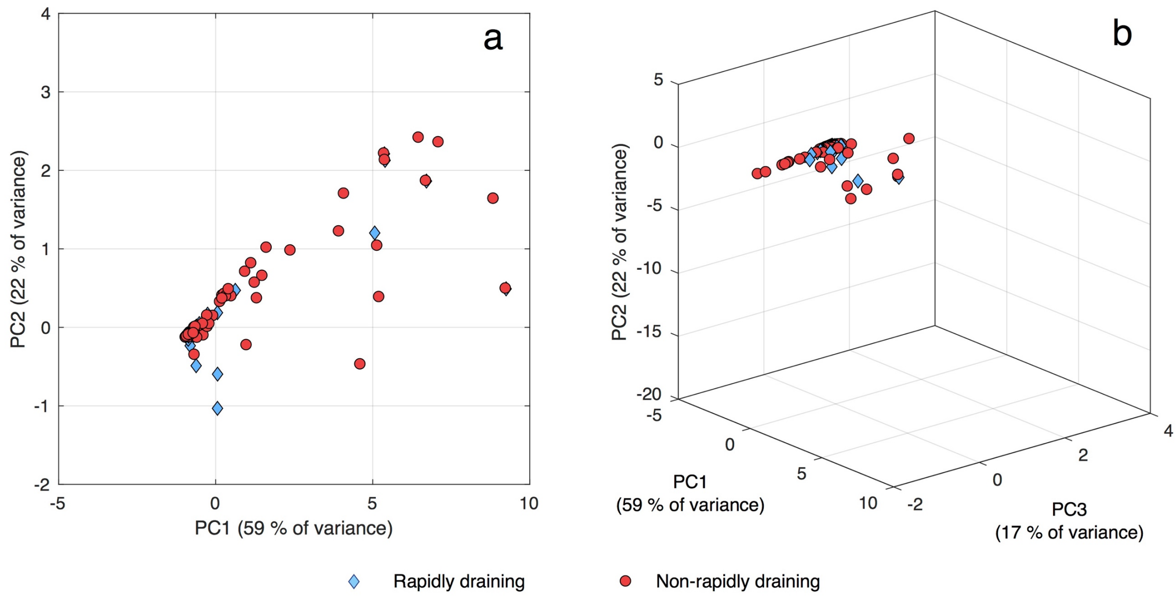

Fig. S7. Principal component (PC) scores for (a) PCs 1 and 2, and (b) PCs 1–3 for rapidly draining (blue diamonds) and non-rapidly draining (red circles) lakes for variables within the surface-mass-balance factor category of analysis (see section 2.2 and Table 2). The percentage figures indicate the amount of variance in the data explained by the PC. The tight clustering of the rapidly draining and non-rapidly draining lake PC scores shows the statistical similarity of the potential controlling factors for the two lake types.

Table S1. Results of unpaired two-sample Student's t-tests (degrees of freedom = 211) for the potential controlling factors for the rapidly draining (RD) and non-rapidly draining (NRD) lake types. The reader is referred to Table 2 for details of the potential controlling factors. t-test results that are statistically significant at above the 95 % confidence interval (i.e. $p < 0.05$) are highlighted in bold text; p value marked with a * narrowly misses the 95 % confidence interval, being significant at only the 94.6 % confidence interval.

Potential controlling factor	**RD lake mean**	**NRD lake mean**	t **value**	p **value**
Lake volume	4.1×10^5	7.4×10^5	-1.934	0.054*
Lake area	**0.47**	**0.69**	**-2.261**	**0.025**
Ice thickness	752	764	-0.234	0.815
Ice-surface elevation	978	964	0.271	0.786
Ice-bed elevation	226	200	1.104	0.271
Lake eccentricity	0.74	0.73	0.215	0.830
Lake orientation	41	38	0.595	0.553
Maximum lake water depth	1.29	1.35	-0.432	0.666
Mean lake water depth	0.77	0.80	-0.449	0.654
Standard deviation of lake water depth	0.31	0.32	-0.225	0.822
Lake mean filling rate	3.66	3.99	-0.271	0.787
Ice-surface slope	1.40	1.61	-1.001	0.318
Background winter ice velocity	346	240	1.300	0.195
Principal strain rate	0.04	0.00	1.510	0.132
von Mises yield criterion	4.3×10^4	4.4×10^4	-0.075	0.940
Melt within catchment on day	2.3×10^4	3.0×10^4	-0.792	0.429
Rain within catchment on day	3.0×10^3	4.6×10^2	1.440	0.151
Melt within catchment on previous day	2.3×10^4	2.9×10^4	-0.697	0.487
Rain within catchment on previous day	7.1×10^3	1.2×10^3	1.545	0.124
Cumulative melt within catchment	1.2×10^6	1.4×10^6	-0.472	0.637
Cumulative rain within catchment	1.0×10^5	1.2×10^5	-0.310	0.757
Cumulative runoff within catchment	6.6×10^7	7.5×10^7	-0.333	0.740
Difference between cumulative catchment runoff and lake volume	6.5×10^7	7.4×10^7	-0.321	0.749

Table S2. Eigenvectors (i.e. loadings) for the first three principal components (PCs) to show the contribution of the original 23 potential controlling factors to each PC. Data are presented after the potential controlling factor variables were normalised to correct for their different scales and units (see section 2.3.1 for details). Negative eigenvectors are shown in red text, indicating a negative relationship of the original variable to the PC. The reader is referred to Table 2 for details of the potential controlling factors.

Potential controlling factor	**PC1**	**PC2**	**PC3**
Ice thickness	-0.293	-0.120	0.189
Ice-surface elevation	-0.260	-0.176	0.312
Ice-bed elevation	0.080	-0.117	0.261
Lake eccentricity	0.044	-0.016	-0.109
Lake orientation	-0.022	-0.021	0.064
Lake volume	-0.136	0.405	-0.036
Lake area	-0.097	0.394	-0.031
Maximum lake water depth	-0.209	0.370	0.081
Mean lake water depth	-0.200	0.359	0.103
Standard deviation of lake water depth	-0.210	0.334	0.141
Lake mean filling rate	-0.159	0.312	0.074
Ice-surface slope	0.278	0.136	-0.194
Background winter ice velocity	0.034	0.030	-0.455
Principal strain rate	0.010	0.060	-0.336
von Mises yield criterion	0.105	0.144	-0.441
Melt within catchment on day	0.244	0.112	0.101
Rain within catchment on day	0.184	0.033	0.190
Melt within catchment on previous day	0.261	0.118	0.116
Rain within catchment on previous day	0.197	0.036	0.187
Cumulative melt within catchment	0.315	0.146	0.163
Cumulative rain within catchment	0.267	0.105	0.133
Cumulative runoff within catchment	0.313	0.146	0.155
Difference between catchment runoff and lake volume	0.314	0.144	0.156

Table S3. Results of bivariate correlation analysis of lake area against other potential controlling factors for rapidly draining (RD) and non-rapidly draining (NRD) lake types. ρ is the Spearman's rank correlation coefficient. Correlations significant at above the 95 % confidence interval (i.e. $p < 0.05$) are highlighted in bold text. Negative correlations are shown in red text. The reader is referred to Table 2 for details of the potential controlling factors.

Potential controlling factor	**RD ρ**	**RD p**	**NRD ρ**	**NRD p**
Ice thickness	0.132	0.279	0.185	**0.026**
Ice-surface elevation	-0.021	0.861	0.075	0.375
Ice-bed elevation	-0.249	**0.039**	-0.208	**0.012**
Lake eccentricity	0.079	0.517	-0.102	0.224
Lake orientation	0.121	0.321	0.106	0.205
Maximum lake water depth	0.588	**0.000**	0.673	**0.000**
Mean lake water depth	0.379	**0.001**	0.584	**0.000**
Standard deviation of lake water depth	0.570	**0.000**	0.646	**0.000**
Lake mean filling rate	0.617	**0.000**	0.687	**0.000**
Ice-surface slope	0.088	0.473	-0.117	0.162
Background winter ice velocity	0.064	0.599	0.119	0.154
Principal strain rate	-0.058	0.645	0.007	0.937
von Mises yield criterion	0.200	0.107	0.132	0.123
Melt within catchment on day	-0.025	0.841	-0.008	0.922
Rain within catchment on day	-0.156	0.200	0.067	0.427
Melt within catchment on previous day	0.094	0.444	0.015	0.857
Rain within catchment on previous day	-0.043	0.723	0.116	0.167
Cumulative melt within catchment	-0.024	0.846	0.216	**0.009**
Cumulative rain within catchment	-0.019	0.877	0.219	**0.008**
Cumulative runoff within catchment	-0.024	0.844	0.262	**0.002**
Difference between cumulative catchment runoff and lake volume	-0.060	0.627	0.167	**0.045**

Table S4. Results of bivariate correlation analysis of lake volume against other potential controlling factors for rapidly draining (RD) and non-rapidly draining (NRD) lake types. ρ is the Spearman's rank correlation coefficient. Correlations significant at above the 95 % confidence interval (i.e. $p < 0.05$) are highlighted in bold text. Negative correlations are shown in red text. The reader is referred to Table 2 for details of the potential controlling factors.

Potential controlling factor	RD ρ	RD p	NRD ρ	NRD p
Ice thickness	0.176	0.149	0.203	**0.015**
Ice-surface elevation	0.032	0.797	0.109	0.195
Ice-bed elevation	-0.231	0.056	-0.162	0.052
Lake eccentricity	-0.108	0.376	-0.176	**0.035**
Lake orientation	0.116	0.344	0.038	0.652
Maximum lake water depth	0.889	**0.000**	0.888	**0.000**
Mean lake water depth	0.773	**0.000**	0.844	**0.000**
Standard deviation of lake water depth	0.847	**0.000**	0.833	**0.000**
Lake mean filling rate	0.741	**0.000**	0.724	**0.000**
Ice-surface slope	-0.044	0.723	-0.184	**0.027**
Background winter ice velocity	-0.053	0.668	0.036	0.665
Principal strain rate	-0.135	0.279	0.031	0.719
von Mises yield criterion	0.078	0.536	0.065	0.445
Melt within catchment on day	-0.078	0.521	-0.024	0.773
Rain within catchment on day	-0.185	0.129	0.049	0.557
Melt within catchment on previous day	0.021	0.863	-0.005	0.950
Rain within catchment on previous day	-0.169	0.164	0.024	0.772
Cumulative melt within catchment	-0.084	0.490	0.176	**0.035**
Cumulative rain within catchment	-0.098	0.421	0.198	**0.017**
Cumulative runoff within catchment	-0.078	0.522	0.219	**0.008**
Difference between cumulative catchment runoff and lake volume	-0.113	0.353	0.126	0.131