SUPPLEMENTARY MATERIAL: Can unconfined ice shelves provide buttressing via hoop stresses?

Martin G. WEARING,[1] Jonathan KINGSLAKE,[1] M. Grae WORSTER,[2]

[1] *Lamont-Doherty Earth Observatory, Columbia University, New York, USA*

[2] *Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, UK*

Correspondence: Martin Wearing <wearing@ldeo.columbia.edu>

FIGURES FOR ANTARCTIC ICE SHELVES

Figures S1 to S6, show a set of plots for each ice shelf considered in the main text, in which speed and thickness along flowlines are compared to model results along with the corresponding strain rates and buttressing number. In each figure: (a) The ice surface speed in the downstream region of the six ice shelves considered here, along with the flowlines and straight lines that approximate the unconfined and laterally spreading area of ice shelf. (b) Ice flow speed along flowlines (dashed) with radial flow from idealized models for range of rate factors (solid curves). The mean RMSE for each model fit is given in brackets in the legend. (c) Same as (b) but for ice thickness. (d) Radial and azimuthal strain rates along with their difference. Where the difference is negative, azimuthal strain is greater than radial strain and there is a positive contribution to hoop-stress buttressing. Different rate factors are represented by solid (A_{-10}), dashed (A_{-5}) and dotted (A_{-2}) curves. (e) The buttressing number along the length of the ice shelf, for a range of rate factors.

Figure S7 shows the sea-ice extent around the unconfined sections of ice shelves in September 2009 and is the equivalent to Figure 2 in the main text.

Figure S8a shows the ice temperature 10 m below the ice surface, from observations across the Antarctic Ice Sheet (Bohlander and Scambos, 2001). Across the ice shelves the temperature is expected to increase with depth approaching the temperature of the underlying ocean at the ice-shelf base (-2°C) (Cuffey and Paterson, 2010)(p.427). We estimate a depth-averaged temperature for the ice shelves, assuming a linear temperature profile, by taking the mean of the 10-m and basal temperature. This is plotted in Figure S8b. This is an upper estimate because in most places surface accumulation and basal melting will produce downward ice flow within the ice shelves, depressing the average temperature. Most ice shelf temperatures are below -5°C.

ALTERNATIVE METHODS FOR DETERMINING HOOP-STRESS CONTRIBUTIONS

Calculating contributions to hoop-stress buttressing

In Wearing (2016), regions of ice shelves are classified as contributing positively or negatively to hoop-stress buttressing, by determining the difference between along-flow and transverse-to-flow strain rates. In those areas where transverse-to-flow strain is greater than along-flow strain a positive contribution is made.

Approximating flowlines as model radial flowlines

An alternative approach to that presented in the main text, is to consider each flowline as a radial flowline. Examples of the results of this are shown in Figures S9 - S12, for flowlines 2 - 5 on Amery Ice Shelf in Figure S1; flowlines numbered from top to bottom.

In a similar manner to the idealized modelling with an annulus, here the thickness along a flowline is used as the thickness profile in the radial force-balance equation (equation (12) in main text). This gives the radial flow speed, and in turn the strain rates and buttressing. The radius of curvature at the upstream boundary is determined by the difference in flux between the beginning and end of the flowline, assume that the difference is only due to diverging flow (no net specific mass balance). (Alternatively the local specific mass balance could be calculated in an Eulerian frame using the flow divergence and thickness change between each location along flowlines.)

The results of this model are then compared to data collected along the flowlines. As in the annulus model, a range of rate factors can be used, here we show results for rate factors appropriate for ice at -5° C and -10° C. We compare the speed, strain rates, inferred stresses and buttressing along the observed and modelled flowline.

In all cases it is difficult to make clear comparisons between the data and model. This is exemplified in the plots of buttressing along flowlines derived from data using the ratio of extensional to driving stress. Along each flowline there are large and oscillating variations in the buttressing regardless of the choice of rate factor. The difficulty in comparing model and data may be partly because shear stresses are not accounted for in the model, but are present in the data. These shear stresses most likely result from lateral variations in ice thickness, which are not considered in the idealized model. In comparison, when using the annulus method a characteristic radial flowline is created, which effectively acts to smooth out the effects of lateral variations in thickness. In addition, errors resulting from the compilation of multiple sets of surface velocity measurements become prominent when gradients are taken. The best matches are for flowlines 2 and 5 when rate factor A_{-10} is used Figure S9b and S12b. In both these cases buttressing is close to zero (< 0.1), which agrees with the annulus model results.

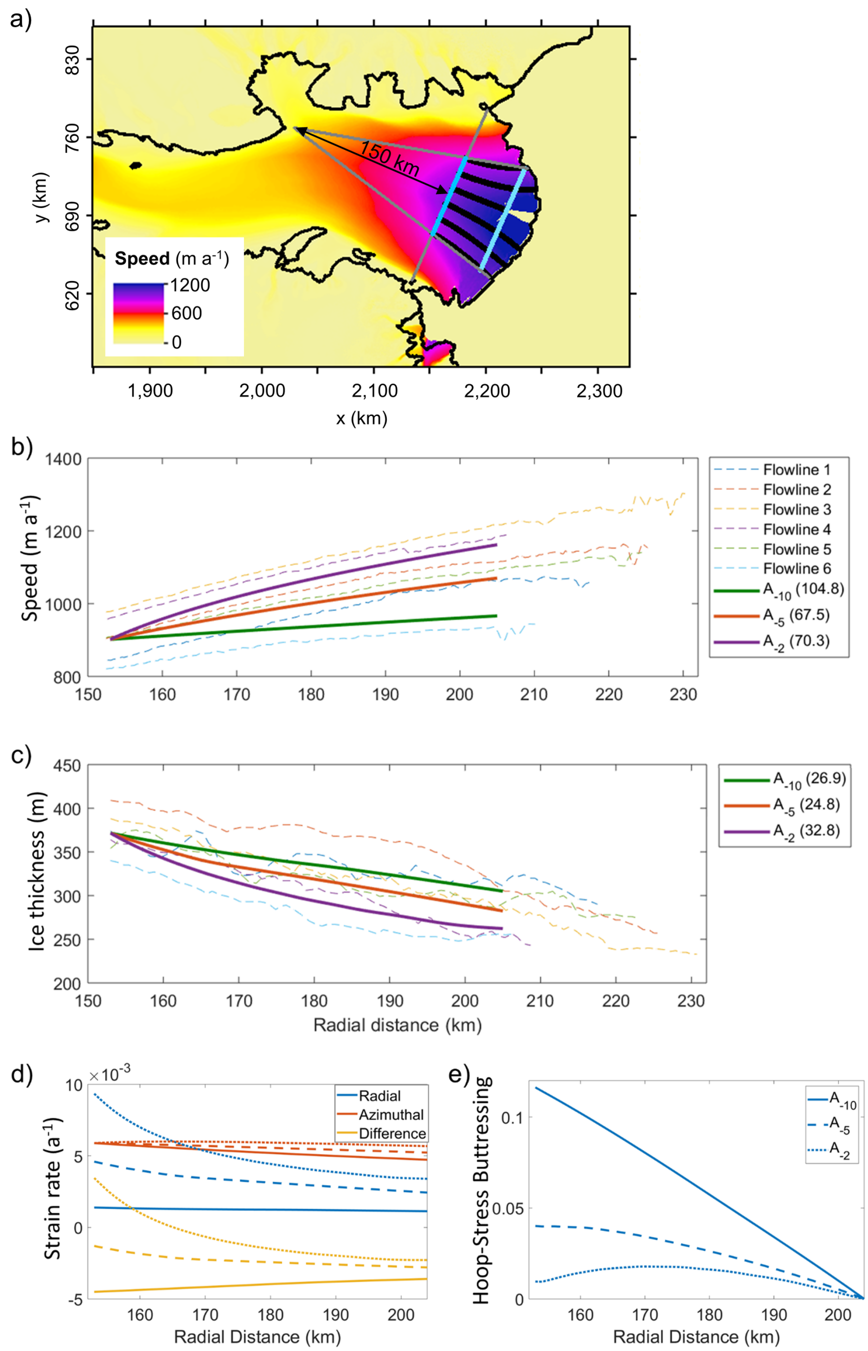

Fig. S1. Applying idealized model annulus to Amery Ice Shelf: (a) ice speed map with geometry of flowlines and upstream and downstream boundaries. (b) Flow speed along flowlines 1 - 6 (bottom to top in (a)) (dashed curves) with simulated speed (solid curves) at three rate factors (A_{-2}, A_{-5} and A_{-10}). (c) Same as (b) but for ice thickness. In both (b) and (c) the mean RMSE for each rate factor is given in the brackets in the legend. (d) Strain rates from model and (e) buttressing number along length of shelf. In (d) and (e); A_{-2} (dotted), A_{-5} (dashed) and A_{-10} (solid).

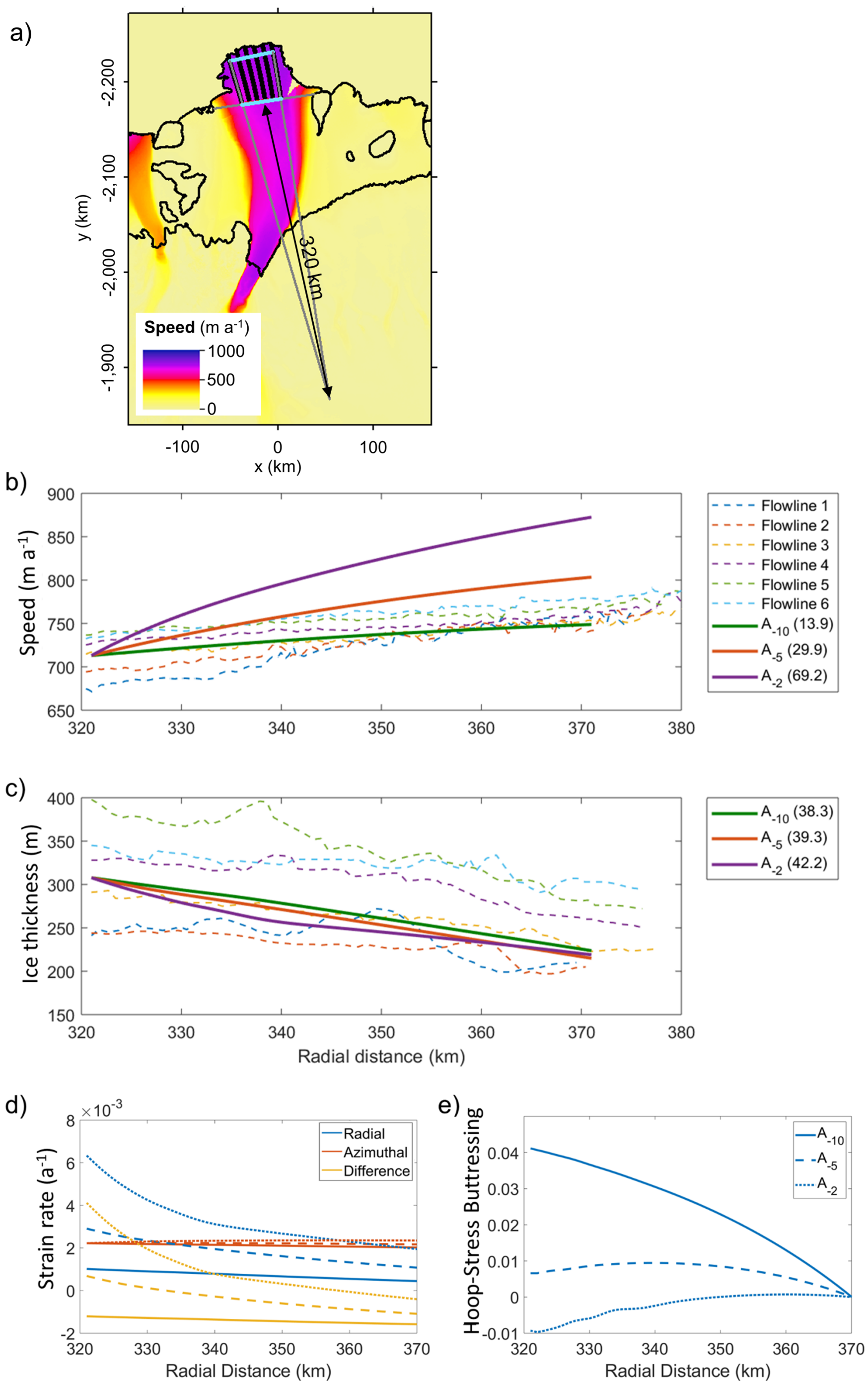

Fig. S2. Fimbul Ice Shelf: same as Figure S1

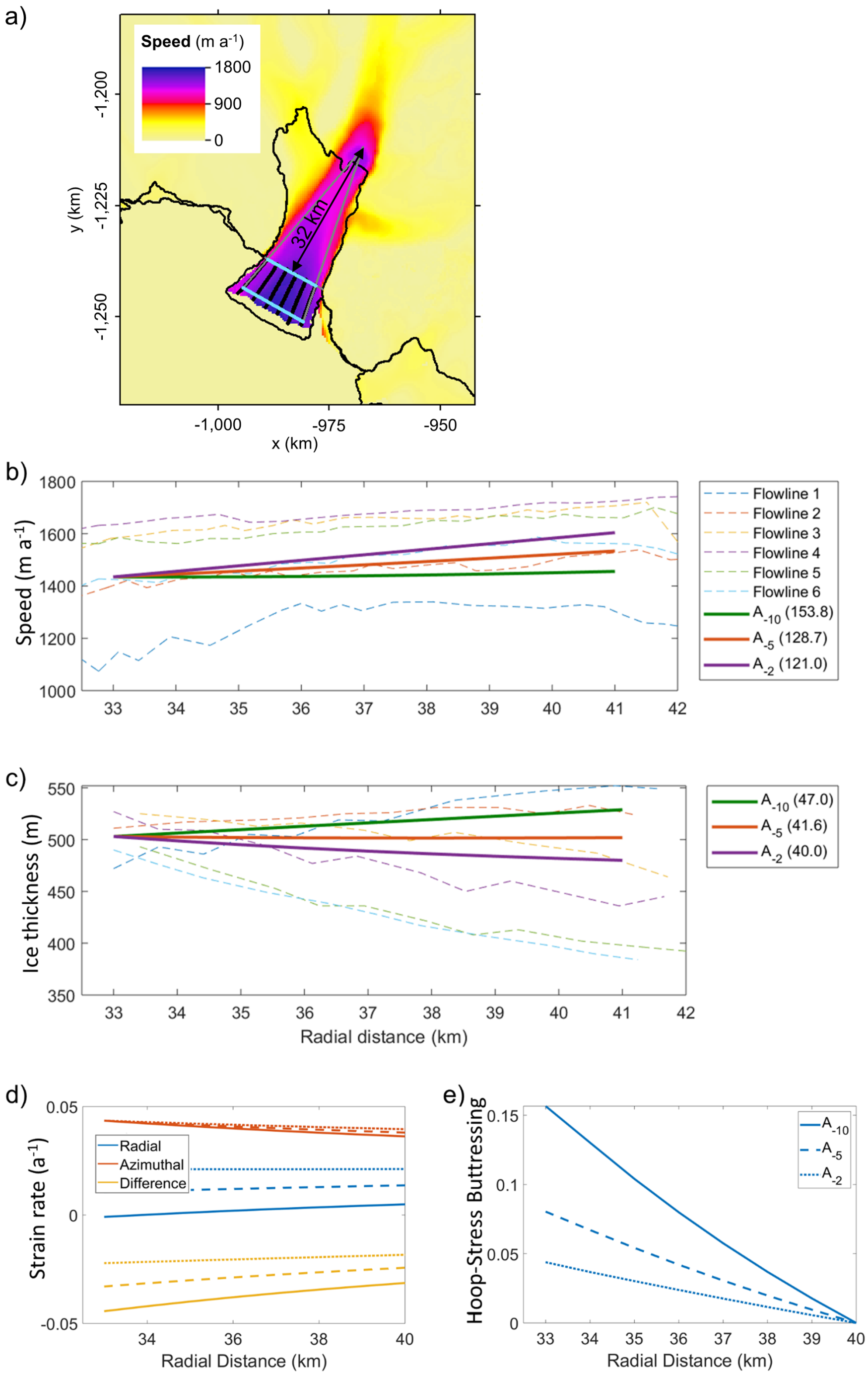

Fig. S3. Land Ice Shelf: same as Figure S1

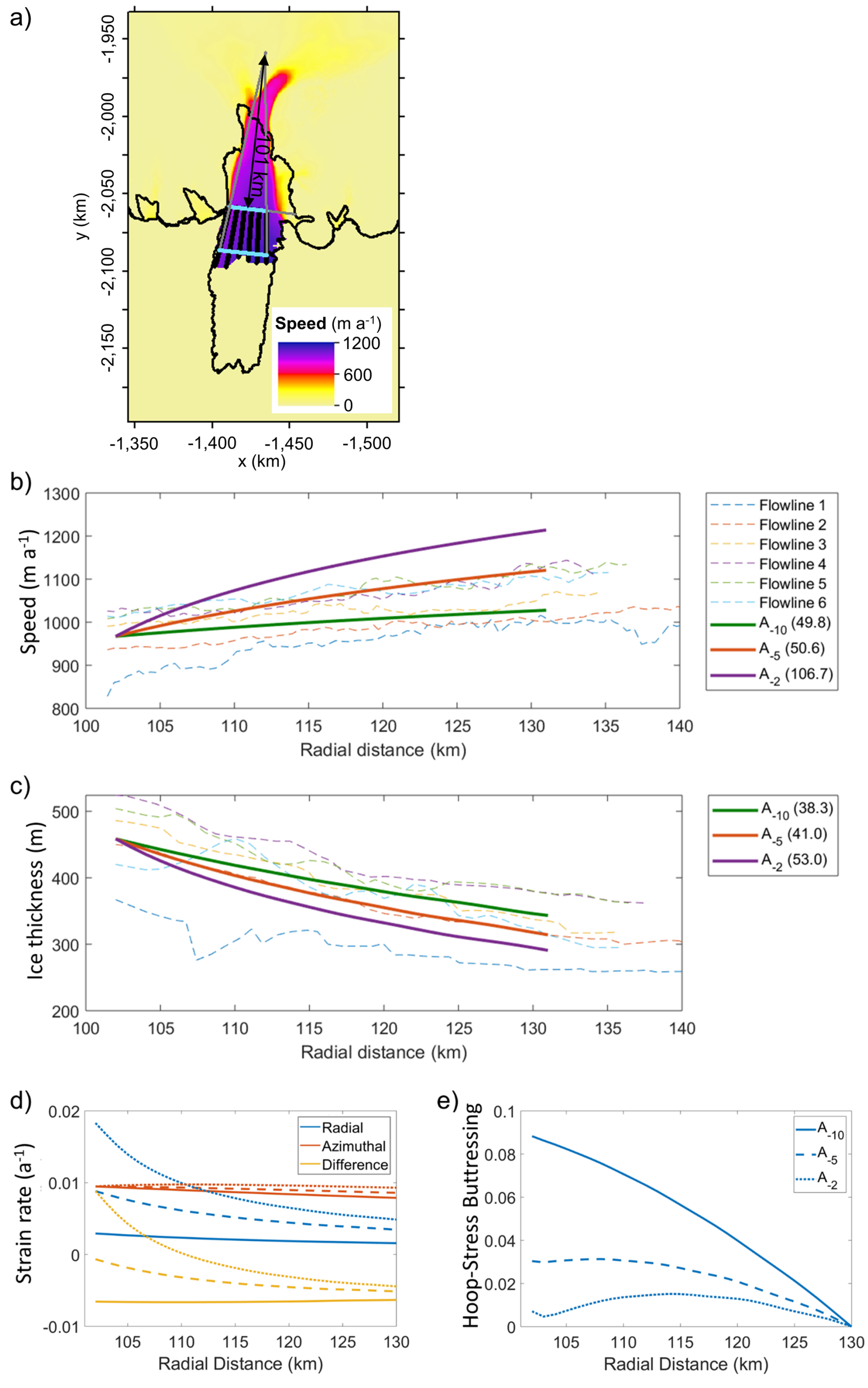

Fig. S4. Mertz Ice Tongue: same as Figure S1

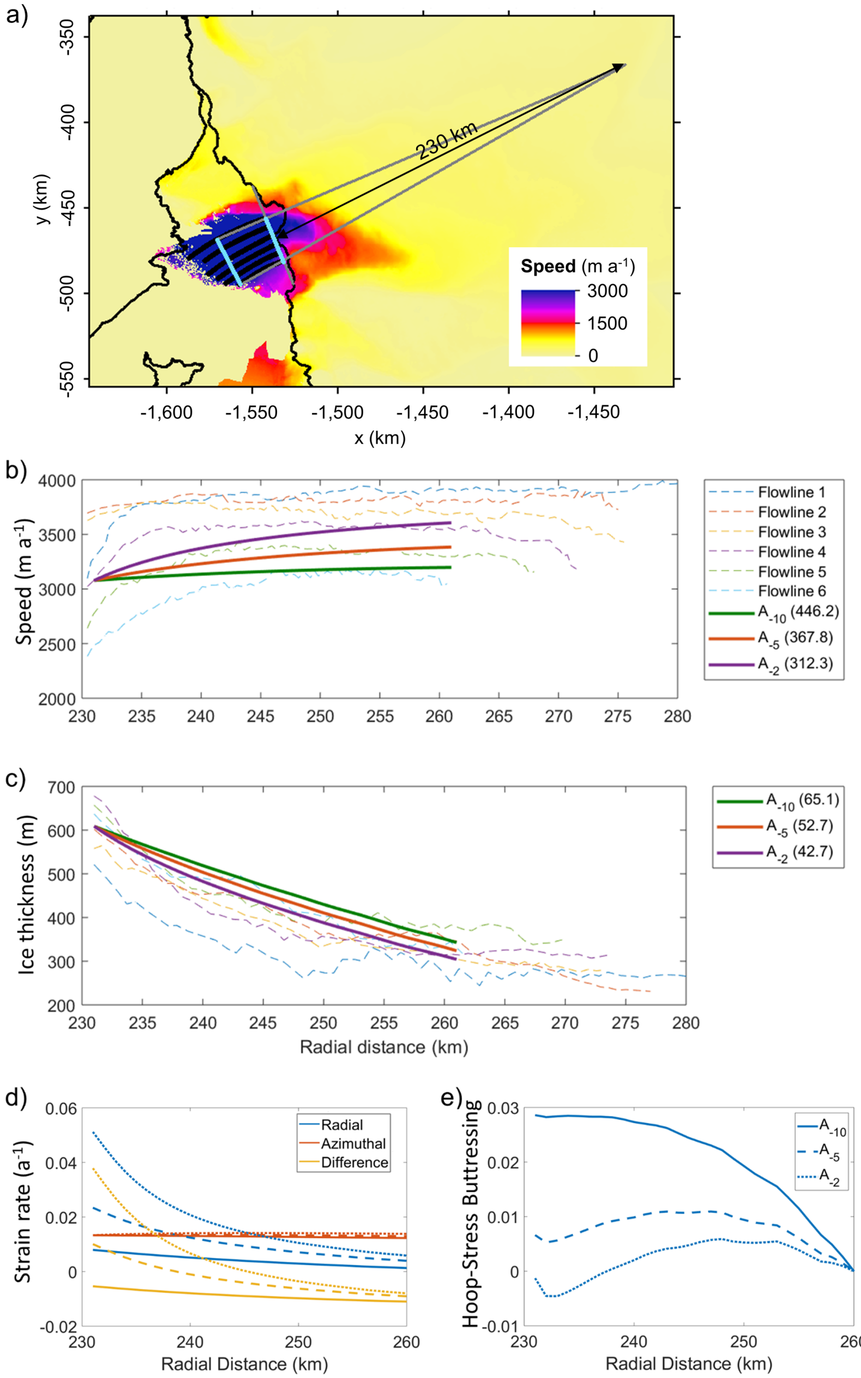

Fig. S5. Thwaites Ice Shelf: same as Figure S1

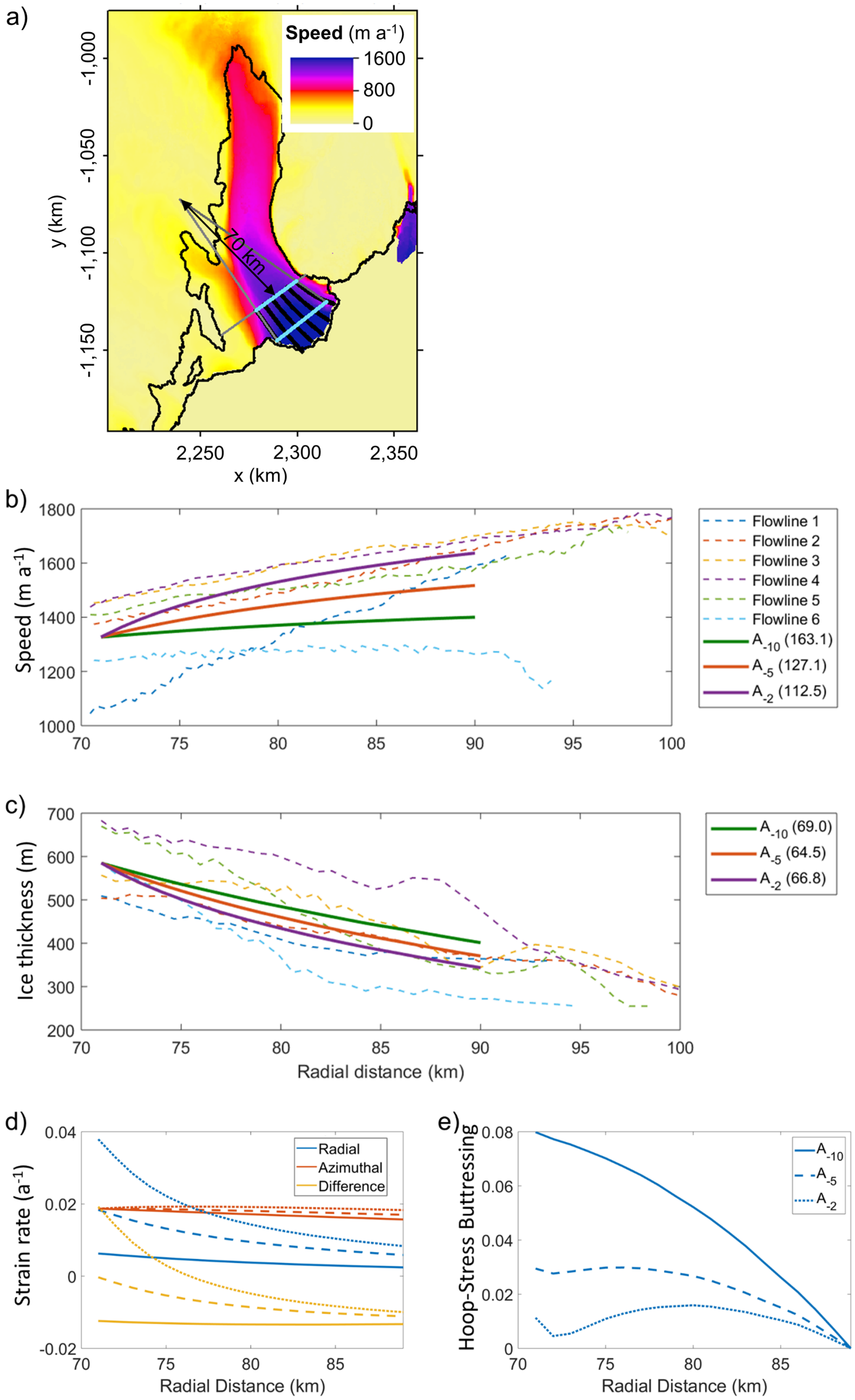

Fig. S6. Totten Ice Shelf: same as Figure S1

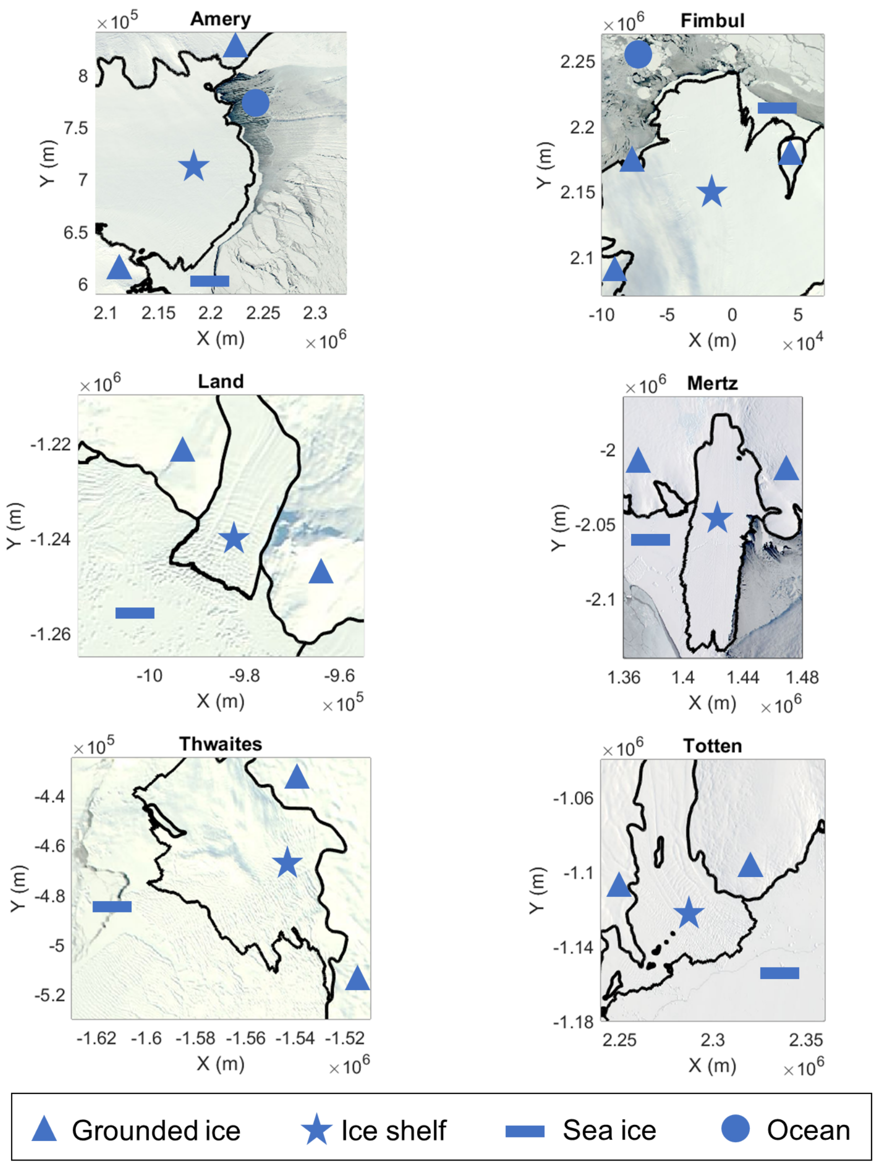

Fig. S7. Same as Figure 2 in main text but for September 2009: Worldview imagery of Amery, Fimbul, Land, Mertz, Thwaites and Totten ice shelves showing the extent of the laterally confined and unconfined regions of each ice shelf along with sea ice cover. The grounding line and ice-ocean boundary are denoted by a black line, with regions of grounded ice, ice shelf, open ocean and sea ice denoted with symbols shown in key.

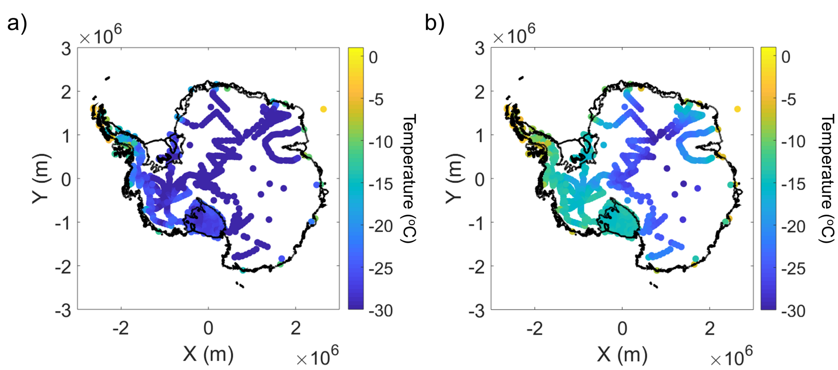

Fig. S8. (a): ice temperature 10 m below the ice surface, from observations across the Antarctic Ice Sheet (Bohlander and Scambos, 2001). (b) We estimate a depth-averaged temperature for the ice shelves, assuming a linear temperature profile, by taking the mean of the 10-m and basal temperature.

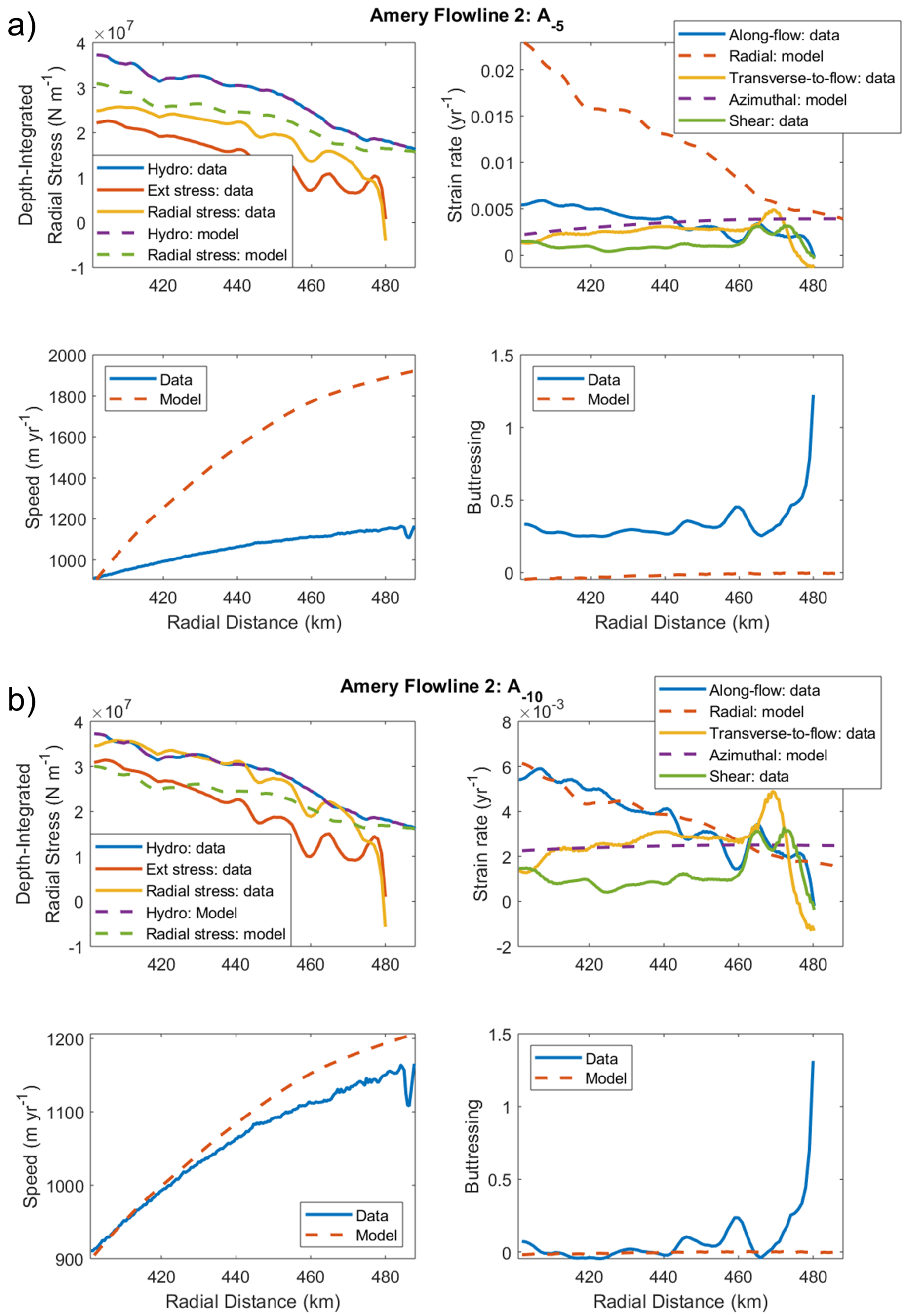

Fig. S9. Amery flowline 2: model output - dashed, flowline data - solid. (a) and (b) for rate factors A_{-5} and A_{-10} respectively. In each subfigure: (i) Depth-integrated stresses along the flowline, (ii) strain rates, (iii) ice speed and (iv) Buttressing number. Flowline data does not vary smoothly making it difficult to make direct comparisons with the model approximation.

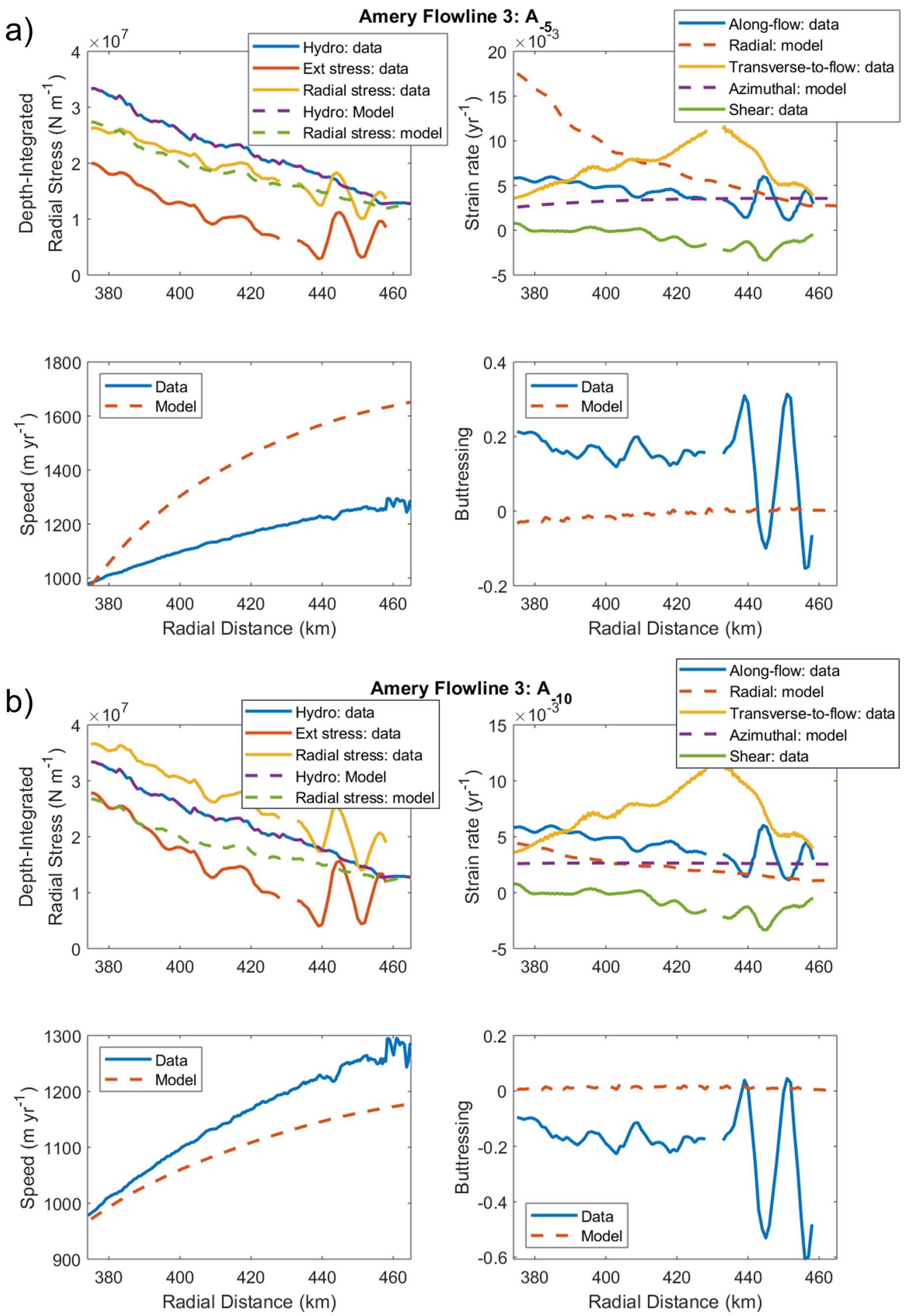

Fig. S10. Amery flowline 3: same as Figure S9

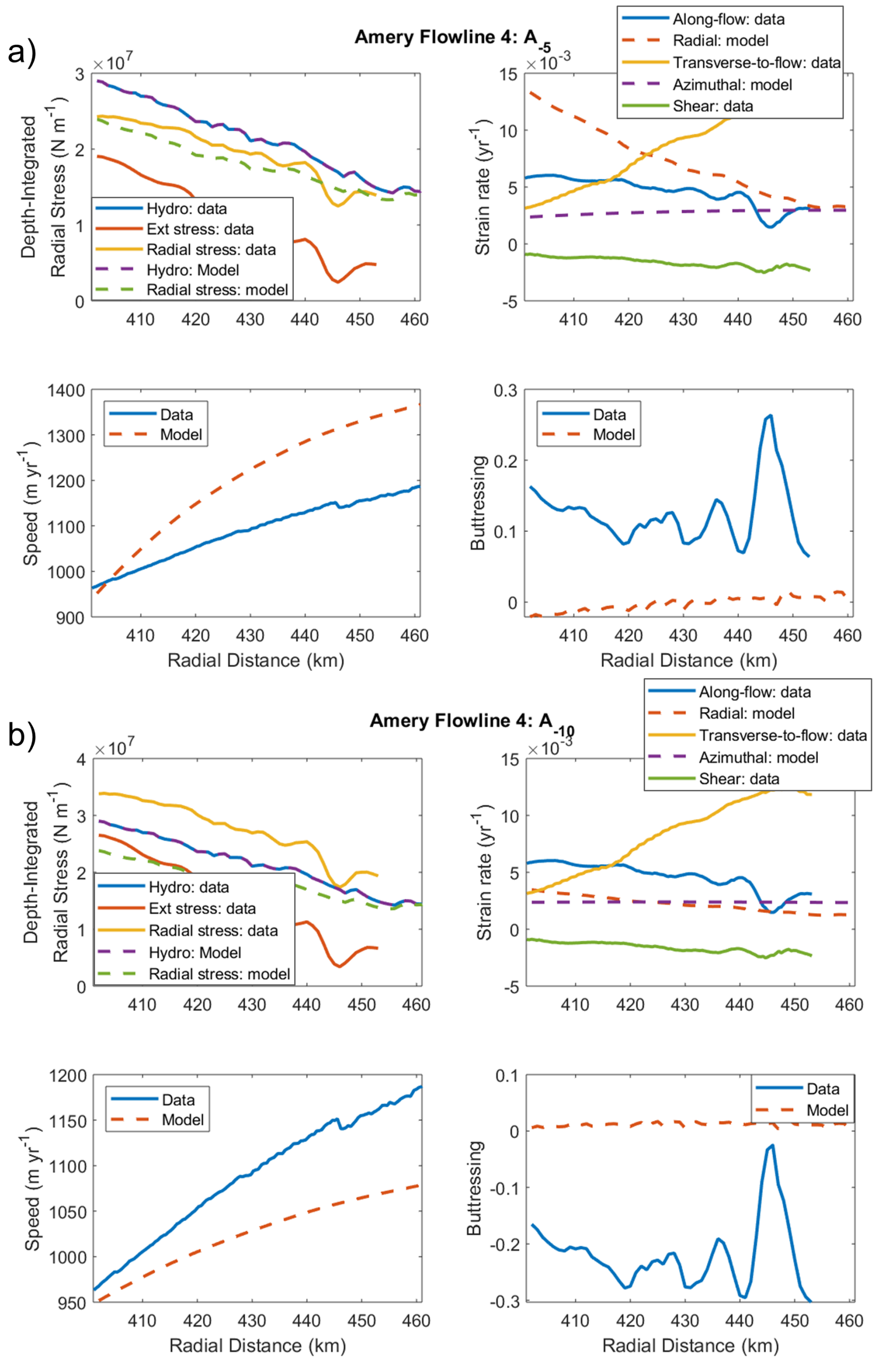

Fig. S11. Amery flowline 4: same as Figure S9

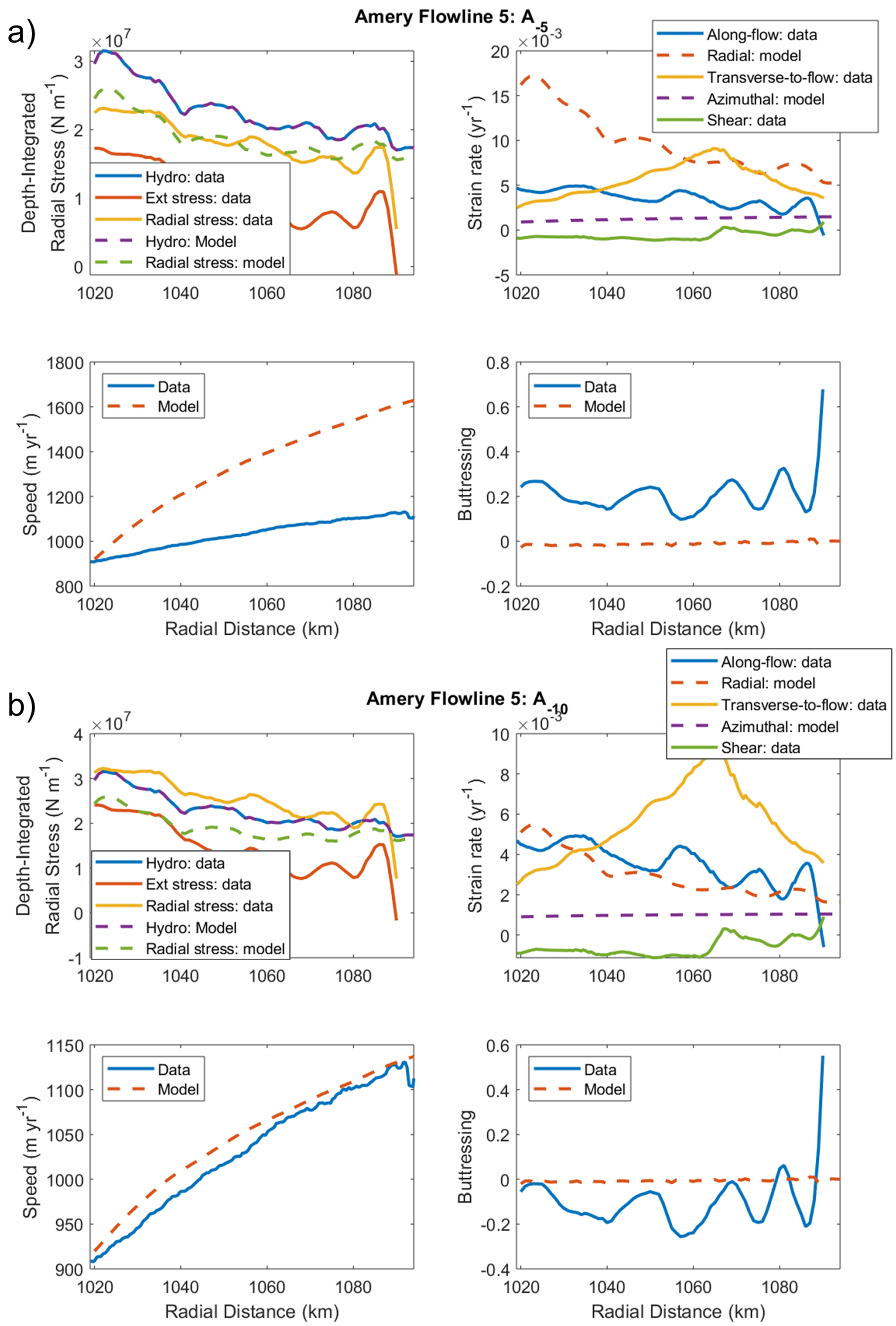

Fig. S12. Amery flowline 5: same as Figure S9

REFERENCES

Bohlander J and Scambos T (2001) THERMAP Antarctic Ice Sheet Temperature Data

Cuffey KM and Paterson WSB (2010) *The Physcis of Glaciers.* Elsevier, 4th edition, ISBN 978-0-12-369461-4

Wearing MG (2016) The Flow Dynamics and Buttressing of Ice Shelves. *PhD thesis, University of Cambridge.*