Supplementary material of "Reanalysing the 2007-19 glaciological mass balance series of Mera Glacier, Nepal, Central Himalaya, using geodetic mass balance."

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S1: Distribution of the GNSS points surveyed in the field in November 2018 and used to co-register the October 2018 DEM

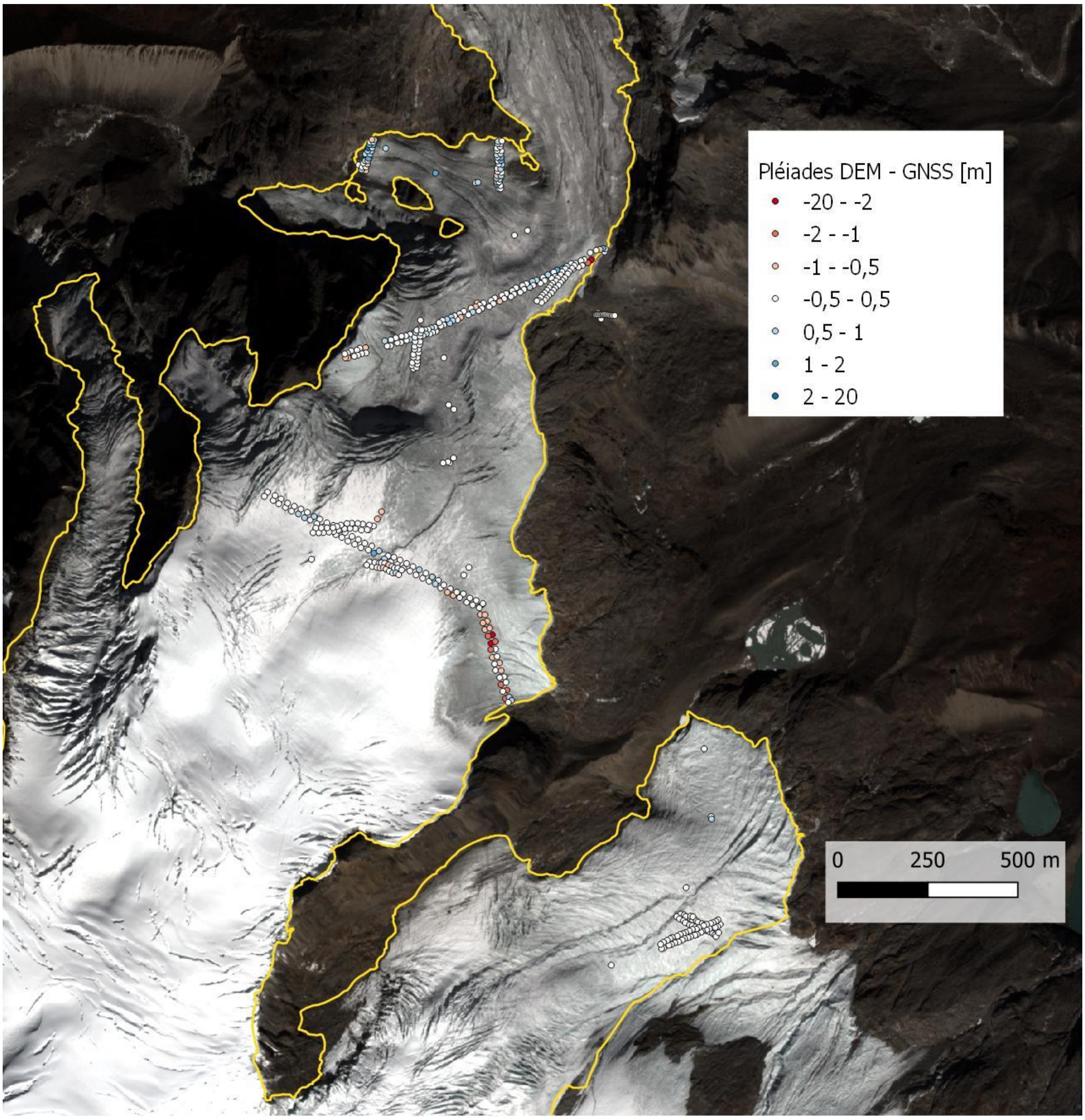

Fig. S1: Map showing the location of the 478 GNSS points surveyed in the field between 20 and 25 November 2018 and color-coded by difference with the pixel of the 4 m Pléiades 2018 DEM at the same location.

S2. The patch method

This method aims at empirically assessing the relationship between the uncertainty on the mean elevation change and the area used for averaging. This relationship is evaluated on stable terrain and assumed to hold on glacier terrain. We split the stable terrain (off-glacier terrain) into squared patches using a convolution algorithm with kernels of various sizes, corresponding to target area from 0.01 to 5 km^2. We sample the stable terrain so that the distribution of slope matches the slope distribution on Mera Glacier. We exclude patches with less than 30 % of valid data. We then draw the 95th percentile of the absolute value of the mean elevation change for each target area. We then fit a power law, to obtain a relationship between the uncertainty of the elevation change and the area (Figure S1). Note that the method is not well constrained for area larger than 1 km², as it is difficult to find continuous stable area of this size in our map of elevation difference around Mera Glacier.

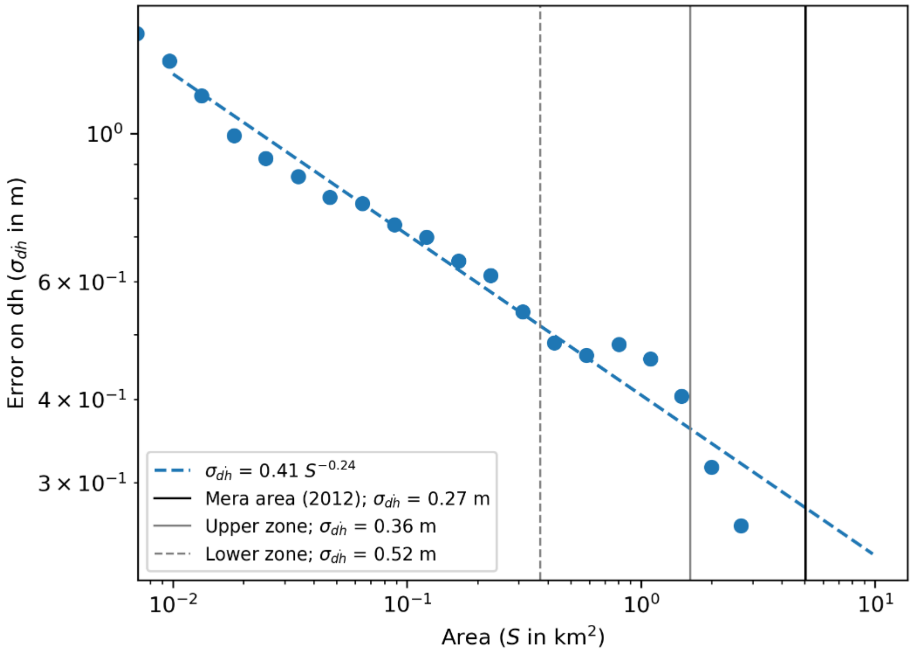

Fig. S2: 95th percentile of the mean absolute value of the mean elevation change for patches of stable terrain of a given area (dots). The dashed blue line is the power law fit and the vertical lines represent the area of Mera Glacier, of the upper zone and of the lower zone.

S3. Application of the nonlinear model (Vincent and others, 2018)

To compare with the results obtained with the glaciological profile method, the nonlinear model developed by Vincent and others (2018) and built from Lliboutry's (1974) linear model, has been applied to calculate the glacier-wide glaciological mass balance of Mera Glacier, and the associated random error.

S3.1. Application of the nonlinear model

The model assumes that the point mass balance can be decomposed into two independent variation terms, one spatial (α_i) and one temporal (β_t), which can be written as:

$$b_{i,t} = \alpha_i + \gamma_i \beta_t + \varepsilon_{i,t} \qquad \text{(S1)}$$

where $b_{i,t}$ is the mass balance recorded at site *i* for year *t*, α_i is the spatial effects at location *i* (i.e., the average balance at the site *i* over the whole study period), β_t is the annual deviation from this average balance (therefore $\sum \beta_t = 0$), and $\gamma_i = \frac{\sigma_i}{\sigma_{max}}$ is a scaling factor defined as the ratio of the standard deviation of mass balance at site *i* (σ_i) by the maximum standard deviation (σ_{max}) found from the point mass balance measurements (stakes and pits) on the glacier. The $\varepsilon_{i,t}$ term represents residuals corresponding to both measurement errors and discrepancies between the model and data. The surface of the glacier is divided into 100-m altitudinal bins. We assume that γ_i is only a function of altitude (z) and $\frac{\sigma_i}{\sigma_{max}}$ is found to vary linearly with z: $\gamma_i = \frac{\sigma_i}{\sigma_{max}} = (2.1 - 2.9 \times 10^{-4} z)/0.9$. See Vincent and others (2018) for details. Table S1 and Figure S2 give the results of the annual glacier-wide mass balances obtained from the nonlinear model, and compared with those of the profile method.

Table S1. Calibrated B_a for Mera Glacier, calculated from the profile method described in this present study and calculated from the nonlinear statistical model described in Vincent and others (2018). Also shown is the annual random error of this calibrated mass balance.

Years	07/08	08/09	09/10	10/11	11/12	12/13	13/14	14/15	15/16	16/17	17/18	18/19	Mean	SD
$B_{a,cal}$ (m w.e. a^{-1}) calibrated to match the 2012-18 geodetic mass balance														
with the profile method	0.15	-0.35	-0.75	0.26	-0.90	0.19	-0.41	-0.22	-0.42	-0.76	-0.92	-0.80	**-0.41**	**0.44**
with the nonlinear model	0.15	-0.21	-0.75	0.37	-0.85	0.23	-0.43	-0.12	-0.47	-0.79	-0.96	-0.88	**-0.39**	**0.47**
Annual random error*	±0.28	±0.26	±0.24	±0.22	±0.19	±0.16	±0.16	±0.16	±0.16	±0.16	±0.16	±0.19	**±0.20**	

*cf section S2.2 for the calculation of the annual random error

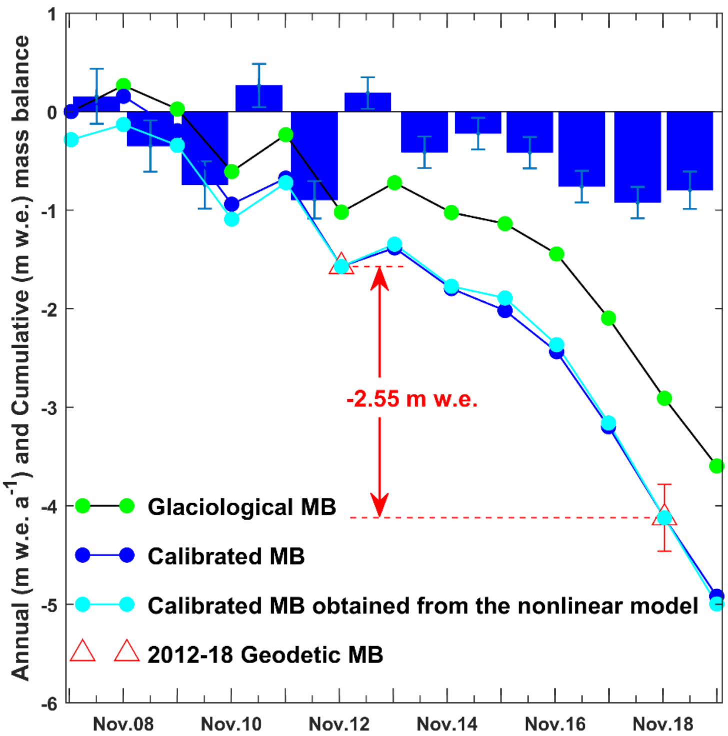

Fig. S3: Cumulative mass balance (MB) of Mera Glacier obtained with the profile method (black line with green dots) between November 2007 and November 2019, and calibrated (blue line with blue dots) to match the 2012-18 geodetic mass balance (red triangles). Also shown is the cumulative calibrated MB obtained from the nonlinear model of Vincent and others (2018) (cyan line with cyan dots). The blue histograms are the annual calibrated mass balances obtained with the profile method.

S3.2. Random error calculation of the calibrated annual glacier-wide mass balance

Combined with the geodetic mass balance B_g obtained by DEM differencing over N = 6 years (2012-18), the calibrated glacier-wide mass balance, $B_{a,cal}$, can be written as follow (Thibert and Vincent, 2009; Vincent and others, 2018):

$$B_{a,cal} = \frac{B_g}{N} + \beta_{mt} \pm \sqrt{\frac{\sigma_{B_g}^2}{N} + \sum_i s_i^2 \sigma_\varepsilon^2} \qquad \text{(S2)}$$

Where β_{mt} is the mean annual mass balance deviation for each year t of the series over the whole glacier ($\beta_{mt} = \beta_t \times \sum_i \gamma_i s_i$, therefore $\sum \beta_{mt}$ = 0), σ_{B_g} = 0.34 m w.e. is the error of the geodetic method over the period 2012-18 (see section 4.b of the main text), s_i terms are the relative areas of each 100-m altitudinal bin compared to the total glacier area (therefore $\sum s_i$ = 1), and σ_ε= 0.25 m w.e. a^{-1} is the standard deviation of the residual term of equation (S1) obtained with the nonlinear model. The random error term on the right-hand side of equation (S2) is the random error, $\sigma_{B_{a,cal}}$, of the calibrated glacier-wide annual mass balance over the calibration period 2012-18 and is equal to ±0.16 m w.e. a^{-1}.

Outside the calibration period 2012-18, considering that the bias is potentially not constant over time, the error $\sigma_{B_{a,cal}}$ is likely to be larger than ±0.16 m w.e. a^{-1}. To assess this additional uncertainty, let's first consider one more year in the series i.e. 2011-12, and let's apply the nonlinear model successively over the period 2012-18 (6 years) and then over the period 2011-18 (7 years). While applying the nonlinear model over the 2012-18 period, the annual and cumulative glacier-wide mass balances can be written as equations (S3) and (S4), respectively (Vincent and others, 2018):

$$\text{for any single year of 2012-18 period} \qquad B_{a,cal} = \alpha_{2012-18} + \beta_{mt} \qquad \text{(S3)}$$

$$\sum_{2012}^{2018} B_{a,cal} = 6 \times \alpha_{2012-18} + \sum_{2012}^{2018} \beta_{mt} = 6 \times \alpha_{2012-18} \qquad \text{(S4)}$$

Where $\alpha_{2012-18}$ is the annual area-weighted mean over the total glacier area of α_i spatial terms of equation (S1), for the 2012-18 period. By definition of the nonlinear model (equation S1), $\alpha_{2012-18}$ is the same for each year of the 2012-18 period, only the temporal term β_{mt} differs from year to year, with $\sum_{2012}^{2018} \beta_{mt} = 0$. Similarly, while applying the nonlinear model over the period 2011-18, the 2011-18 cumulative mass balance is:

$$\sum_{2011}^{2018} B_{a,cal} = 7 \times \alpha_{2011-18} \qquad \text{(S5)}$$

Given that the annual random error of the glacier-wide mass balance is known over the calibration period, we split the 2011-18 period into 2 sub-periods 2011-12 and 2012-18 and we combine equations (S3), (S4) and (S5):

$$\sum_{2011}^{2018} B_{a,cal} = B_{a,cal_{2011-12}} + \sum_{2012}^{2018} B_{a,cal} = \alpha_{2011-18} + \beta_{mt_{2011-2012}} + 6 \times \alpha_{2012-18} \qquad \text{(S6)}$$

Rewritten as:

$$7 \times \alpha_{2011-18} = \alpha_{2011-18} + \beta_{mt_{2011-12}} + 6 \times \alpha_{2012-18} \qquad \text{(S7)}$$

Which gives:

$$\alpha_{2011-18} = \alpha_{2012-18} + \frac{\beta_{mt_{2011-12}}}{6} \quad \text{(S8)}$$

By definition of the nonlinear model, the random error of the annual glacier-wide mass balance for the year 2011-12 is equal to that of $\alpha_{2011-18}$ in equation (S8) and is therefore obtained from this equation (S8):

$$\sigma_{B_{a,cal_{2011-12}}} = \sigma_{\alpha_{2011-18}} = \sqrt{\sigma^2_{\alpha_{2012-18}} + \frac{\sigma^2_{\beta_{mt}}}{6}} \quad \text{(S9)}$$

In our present case, $\sigma_{\alpha_{2012-18}}$ is the random error of the annual glacier-wide mass balance calibrated over the period 2012-18 i.e. 0.16 m w.e. a^{-1} and $\sigma_{\beta_{mt}}$ is the same whichever year is considered and is given by the standard deviation of the residual term of equation (S1) i.e. 0.25 m w.e. a^{-1}. The random error of the annual glacier-wide mass balance for the year 2011-12 is equal to 0.19 m w.e. a^{-1}, slightly larger than that for the years inside the calibration period. And the further away in time from the calibration period, the greater this random error. For instance, for the year 2010-11, two years away from the calibration period, equation (S8) becomes:

$$\alpha_{2010-18} = \alpha_{2012-18} + \frac{\beta_{mt_{2010-11}}}{6} + \frac{\beta_{mt_{2011-12}}}{6} \quad \text{(S10)}$$

Which gives:

$$\sigma_{B_{a,cal_{2010-11}}} = \sigma_{\alpha_{2010-18}} = \sqrt{\sigma^2_{\alpha_{2012-18}} + \frac{2 \times \sigma^2_{\beta_{mt}}}{6}} \quad \text{(S11)}$$

And more generally, when k years away in time from the calibration period, the random error of the corresponding annual glacier-wide mass balance is $\sqrt{\sigma^2_{\alpha_{2012-18}} + \frac{k \times \sigma^2_{\beta_{mt}}}{6}}$. Table S1 provides the annual random error of the calibrated annual glacier-wide mass balance for each year of the studied period.

S4. Comparison with Pokalde and West Changri Nup glaciers

Table S2. B_a, ELA, AAR and mass balance gradients *db/dz* for Mera, Pokalde and West Changri Nup glaciers. Table updated from Table 3 of Sherpa and others (2017).

Years	**07/08**	**08/09**	**09/10**	**10/11**	**11/12**	**12/13**	**13/14**	**14/15**	**15/16**	**16/17**	**17/18**	**18/19**	**Mean**	**SD**
							Mera Glacier (Elevation range: 4910-6390 m a.s.l.)							
$B_{a,cal}$ (m w.e. a^{-1})	0.15	-0.35	-0.75	0.26	-0.90	0.19	-0.41	-0.22	-0.42	-0.76	-0.92	-0.80	**-0.41**	**0.44**
ELA_{cal} (m)	5450	5611	5704	5356	5836	5493	5582	5453	5607	5748	5796	5782	**5618**	**157**
AAR_{cal}	0.67	0.44	0.35	0.81	0.23	0.62	0.50	0.69	0.48	0.33	0.28	0.29	**0.47**	**0.19**
db/dz_{Mera} (m w.e.(100 m)$^{-1}$a^{-1})	0.48	0.41	0.46	0.58	0.32	0.45[a]	0.45[a]	0.53	0.51	0.40	0.40	0.40	**0.45**	**0.08**
db/dz_{Naulek}(m w.e.(100 m)$^{-1}$a^{-1})	0.97	0.74	0.97	0.87[a]	0.72	0.87[a]	0.87[a]	0.95	0.87[a]	0.87[a]	0.87[a]	0.87[a]	**0.87**	**0.13**
							Pokalde Glacier (Elevation range: 5430-5690 m a.s.l.)							
B_a (m w.e. a^{-1})			-0.98	-0.02[b]	-1.12	-0.07	-1.23	-0.70	-0.46	-0.89	-1.29	-1.12	**-0.79**	**0.46**
ELA (m)			5634	-	5652	5578	5654	5616	5617	5622	5655	5718	**5638**	**39**
AAR			0.13	-	0.02	0.49	0.02	0.20	0.20	0.20	0.00	0.00	**0.16**	**0.22**
db/dz (m w.e. (100 m)$^{-1}$ a^{-1})			1.54		1.37	0.94	1.46	1.53	0.99	1.69	1.51	0.76	**1.31**	**0.33**
							West Changri Nup Glacier (Elevation range: 5330-5690 m a.s.l.)							
B_a (m w.e. a^{-1})				*-0.95*[c]	*-1.73*[c]	-0.92	-1.33	-1.28	-0.75	-2.56	-2.10	-1.69	**-1.48**	**0.60**
ELA (m)						5594	5619	5569	5554	5676	5616	5585	**5596**	**41**
AAR						0.13	0.03	0.17	0.22	0.00	0.03	0.15	**0.12**	**0.09**
db/dz (m w.e. (100 m)$^{-1}$ a^{-1})				1.59[d]	1.59[d]	1.03	1.16	1.98	1.60[a]	1.49	1.89	2.11	**1.60**	**0.38**

On Mera Glacier, mass balance gradients are distinguished between Mera and Naulek branches (referred as Mera and Naulek subscripts) (Wagnon and others, 2013). The mean and standard deviation (SD) for every variable are also shown.

[a] Applying a mean gradient because not enough stakes were visible.

[b] Calculated by the difference between 2010–2012 and 2011–2012 glacier-wide mass balances [*Ba* (2010–2011)=*Ba* (2010–2012) – *Ba*(2011–2012)], due to a lack of measurements in October 2011 where heavy snow falls had covered the stakes.

[c] Due to the lack of measurements in October 2011 where heavy snow falls had covered the stakes, 2010-11 and 2011-12 *Ba* was obtained from the 2010-12 mass balance (*Ba*(2010-12) = *Ba*(2010-11) + *Ba*(2011-12) = -2.68 m w.e.

[d] Mean value over the two-year 2010-2012 period.

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