Supplementary Material: Simulating ice shelf extent using damage mechanics

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1 VELOCITY BOUNDARY FOR NO-SLIP WALLS

With no-slip walls, a uniform downstream velocity creates a singularity at the influx corners of the domain. We remedy this by adopting a velocity profile that goes to zero at the walls (at vertical positions $y = \pm w/2$),

$$u(x = 0, y) \equiv u_0[y^m - (w/2)^m], \tag{S1}$$

with $m = 3$. This velocity has an across-channel average of $\bar{u_0} = u_0\,[1 - 1/(m + 1)]$, which we use to generate a 1D, analytic ice tongue with the same incoming flux to investigate how side-wall buttressing affects the locations and thicknesses of fully-damaged termini.

To determine the effect our choice of $m = 3$ (the same as the Glen Flow Law exponent) has on the simulation results, we ran one Amery-like simulation with $m = 10$. The increased influx results in thicker ice across the ice shelf and a slightly longer center line length to the fully-damaged terminus (316 km instead of 305 km), with a thickness difference of 3 m.

2 FITTING DETAILS

We determine the simulation parameters — grounding line thickness and velocity, uniform melt rate, and temperature — and generate uncertainties for the Erebus Glacier Tongue and Drygalski Ice Tongue simulations using a modified least squares algorithm against observed thicknesses and velocities. We consider residuals $r(\theta)$ representing the uncertainty-weighted L2-norm between observations y and model results $x(\theta)$, evaluated at the set of parameters θ. In our analysis, we found that the traditional Levenberg-Marquardt step tended to get stuck in a region of the parameter space where the ice was infinitely-cold and stiff, a phenomenon known in fitting sciences known as parameter evaporation. One possible solution involves accounting for the curvature of the model in the parameter space by using a first-order approximation to the second derivative of the model results x with respect to the parameters θ, called Geodesic-Accelerated Levenberg-Marquardt (Transtrum and others, 2011). This fitting yields the optimal parameter sets recorded in Table 1, and results in the profiles shown in Figs 1 and 2.

For Drygalski Ice Tongue, the uncertainties in both the thickness (Blankenship and others, 2012) and velocity (Wuite and others, 2009) measurements are small, but taken along different points of the ice shelf. We chose to estimate the error from this misalignment by assuming a 50 m a^{-1} error in the velocity measurements, though this choice mostly affects the propagated uncertainties in parameters and model results, as described below, because the fit is dominated by the data-dense thickness profile.

To estimate the variance in the best-fit model's predictions of mass-balance and fully-damaged terminus locations, L_{max} and L_r, respectively, we first estimate the error in the fitted parameters. Traditionally, the posterior covariance is approximated with the curvature of the cost fitting canyon at the best-fit parameters, approximated with $\frac{1}{2}(J^T J)^{-1}$, where J is the Jacobian of the residuals r with respect to the parameters θ, evaluated numerically. The parameter variances are the diagonals of this matrix. We follow this method and then use the full covariance matrix, along with the best-fit parameters, to form a multivariate normal distribution of parameters that fit the data. We sample 5000 parameter sets from this distribution and compute analytic values for L_{max} and L_r, to form a separate distribution for each parameter set. From these distributions, we may directly compute standard deviations, which are the errors in each prediction. We follow this procedure for the ice tongues but not for the ice shelves, as the close agreement between the simulated thickness, velocity, and damage profiles of the ice tongues and their analytic counterparts allows us to quickly evaluate these expressions without rerunning the simulation.

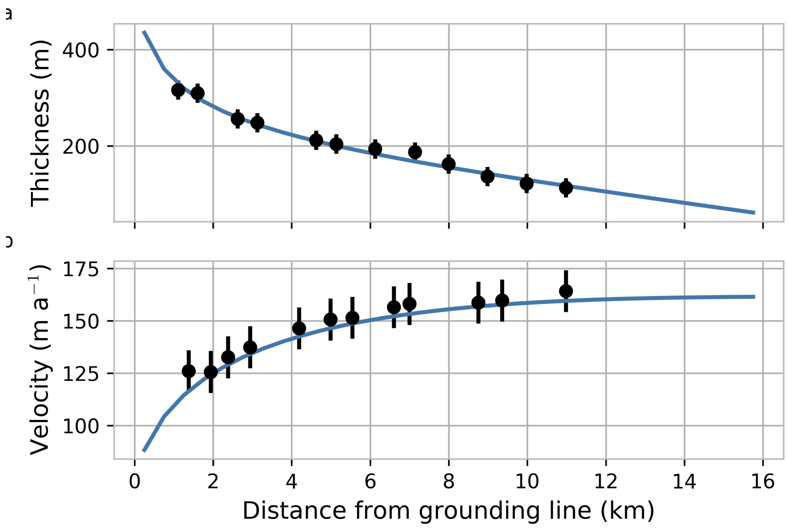

Fig. 1. a) Thickness and b) velocity profiles associated with the best fitting model parameters for Erebus Glacier Tongue, shown against the data from Holdsworth (1974).

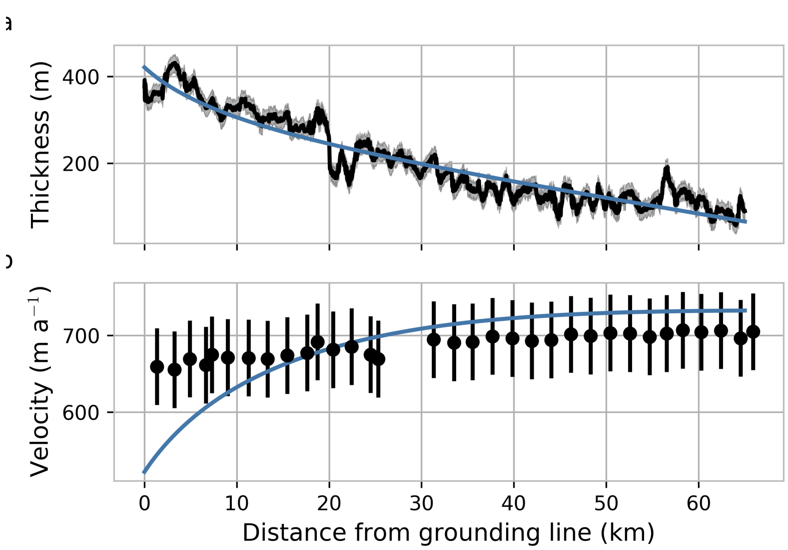

Fig. 2. Same as Figure 1 for the unembayed 67 km long Drygalski Ice Tongue, with data from (Blankenship and others, 2012) and (Wuite and others, 2009). As the velocity and thickness recordings were not taken along the same trackline, we have estimated the error in the velocity to be 50 m a^{-1}.

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