Hydrodynamics of quantum corrections to the Coulomb interaction via the third rank tensor evolution equation: Application to the Langmuir waves and the spin-electron-acoustic waves: SUPPLEMENTARY MATERIAL

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A. Basic definitions, microscopic Hamiltonian of the system, and the general structure of the continuity equation

Description of collective behavior of quantum plasmas can be started with the concentration or the number density of particles [1], [2], [3]

$$n=\int dR\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i)\Psi^{\dagger}(R,t)\Psi(R,t), \tag{1}$$

which is the first collective variable in our model. Other collective variables appear during the derivation. Equation (1) contains the following notations $dR=\prod_{i=1}^{N}d\mathbf{r}_i$ is the element of volume in $3N$ dimensional configurational space, with N is the number of electrons. Symbol † means the Hermitian conjugation.

If we consider the separate spin evolution hydrodynamics we need to split concentration of electrons on two parts: $n_e=n_{\uparrow}+n_{\downarrow}$. This separation is made in accordance with the structure of the wave function [2], [3], [4], [5]

$$n_s=\int dR\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i)\Psi^{*}_s(R,t)\Psi_s(R,t), \tag{2}$$

where

$$\Psi(R,t)=\left(\begin{array}{c}\Psi_{\uparrow}(R,t)\\ \Psi_{\downarrow}(R,t)\end{array}\right). \tag{3}$$

Symbol * means the complex conjugation. Sum in equation (2) is made for all electrons. The probability to have a specified spin projection is kept in the wave function. Spin polarization of each electron can be partial.

Evolution of functions (1) and (2) is calculated using the Schrodinger equation $\imath\hbar\partial_t\Psi=\hat{H}\Psi$ with Hamiltonian

$$\hat{H}=\sum_{i=1}^{N}\left(\frac{\hat{\mathbf{p}}_i^2}{2m_i}\right)+\frac{1}{2}\sum_{i,j\neq i}\frac{q_e^2}{\mid\mathbf{r}_i-\mathbf{r}_j\mid}, \tag{4}$$

where m_i is the mass of i-th particle, $\hat{\mathbf{p}}_i=-\imath\hbar\nabla_i$ is the momentum of i-th particle.

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Current of particles with the spin projection s appear in the continuity equation in the following form [2], [3], [4], [5]:

$$\mathbf{j}_s=\int dR\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i)\times$$

$$\times\frac{1}{2m_i}(\Psi^{*}_s(R,t)\hat{\mathbf{p}}_i\Psi_s(R,t)+c.c.), \tag{5}$$

with *c.c.* is the complex conjugation.

B. General structure of the momentum balance equation

Definition of current (5) allows to derive the Euler equation for the current (momentum density) evolution using the Schrodinger equation with Hamiltonian (4):

$$\partial_t j^{\alpha}_s+\partial_\beta\Pi^{\alpha\beta}_s=\frac{1}{m}F^{\alpha}_{int}, \tag{6}$$

where

$$\Pi^{\alpha\beta}_s=\int dR\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i)\frac{1}{4m^2}[\Psi^{*}_s(R,t)\hat{p}^{\alpha}_i\hat{p}^{\beta}_i\Psi_s(R,t)$$

$$+\hat{p}^{\alpha *}_i\Psi^{*}_s(R,t)\hat{p}^{\beta}_i\Psi_s(R,t)+c.c.] \tag{7}$$

is the momentum flux, and

$$F^{\alpha}_{int}=-\int(\partial^{\alpha}U(\mathbf{r}-\mathbf{r}'))n_{2,ss'}(\mathbf{r},\mathbf{r}',t)d\mathbf{r}' \tag{8}$$

is the force field for the Coulomb interaction, with the two-particle concentration

$$n_2(\mathbf{r},\mathbf{r}',t)$$

$$=\int dR\sum_{i,j=1,j\neq i}^{N}\delta(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{r}'-\mathbf{r}_j)\Psi^{*}_s(R,t)\Psi_s(R,t), \tag{9}$$

and the Coulomb interaction potential

$$U(\mathbf{r}-\mathbf{r}')=\frac{q_e^2}{\mid\mathbf{r}-\mathbf{r}'\mid}. \tag{10}$$

The Euler equation (6) is obtained in Refs. [2], [3], [4], [5].

The Euler equation has simple structure. It shows that the time evolution of the current or the momentum density $\mathbf{j}$ is caused by two mechanisms. One of them is the kinetic momentum flux presented on the left-hand side. It is related to the motion of particles being in the fixed states. The second mechanism is the interaction. Same structure repeats itself in other hydrodynamic equations for the physical quantities with the higher rank tensors. The evolution of the chosen quantities caused by its flux and due to the interaction.

C. General structure of equation for the second rank tensor

Extending the set of hydrodynamic equations we can derive the equation for the momentum flux evolution. Consider the time evolution of the momentum flux (7) using the Schrodinger equation with Hamiltonian (4) and derive the momentum flux evolution equation

$$\partial_t \Pi^{\alpha\beta}_{s} + \partial_\gamma M^{\alpha\beta\gamma}_{s} = \frac{1}{m}(F^{\alpha\beta} + F^{\beta\alpha}), \tag{11}$$

where the force field tensor

$$F^{\alpha\beta} = -\int [\partial^\alpha U(\mathbf{r}-\mathbf{r}')] j^{\beta}_{2,ss'}(\mathbf{r},\mathbf{r}',t) d\mathbf{r}' \tag{12}$$

represents the interaction,

$$M^{\alpha\beta\gamma}_{s} = \int dR \sum_{i=1}^{N} \delta(\mathbf{r}-\mathbf{r}_i) \frac{1}{8m_i^3} \Biggl[\Psi^*_s(R,t) \hat{p}^\alpha_i \hat{p}^\beta_i \hat{p}^\gamma_i \Psi_s(R,t)$$

$$+ \hat{p}^{\alpha *}_i \Psi^*_s(R,t) \hat{p}^\beta_i \hat{p}^\gamma_i \Psi_s(R,t) + \hat{p}^{\alpha *}_i \hat{p}^{\gamma *}_i \Psi^*_s(R,t) \hat{p}^\beta_i \Psi_s(R,t)$$

$$+ \hat{p}^{\gamma *}_i \Psi^*_s(R,t) \hat{p}^\alpha_i \hat{p}^\beta_i \Psi_s(R,t) + c.c. \Biggr] \tag{13}$$

is the flux of the momentum flux, and two-particle current-concentration function:

$$\mathbf{j}_{2,ss'}(\mathbf{r},\mathbf{r}',t) = \int dR \sum_{i,j\neq i} \delta(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{r}'-\mathbf{r}_j) \times$$

$$\times \frac{1}{2m_i} (\Psi^*_s(R,t) \hat{\mathbf{p}}_i \Psi_s(R,t) + c.c.). \tag{14}$$

If quantum correlations are dropped function $j^{\alpha}_{2}(\mathbf{r},\mathbf{r}',t)$ splits on product of the current $j^{\alpha}(\mathbf{r},t)$ and the concentration $n(\mathbf{r}',t)$.

D. General structure of equation for the third rank tensor

General structure of the evolution equation for the third rank tensor $M^{\alpha\beta\gamma}_{s}$ is:

$$\partial_t M^{\alpha\beta\gamma}_{s} + \partial_\delta R^{\alpha\beta\gamma\delta}_{s}$$

$$= \frac{1}{m}(F^{\alpha\beta\gamma}_{Q} + F^{\beta\alpha\gamma} + F^{\beta\alpha\gamma} + F^{\beta\alpha\gamma}), \tag{15}$$

where

$$F^{\alpha\beta\gamma}_{Q} = \frac{\hbar^2}{4m^3} \int [\partial^\alpha \partial^\beta \partial^\gamma U(\mathbf{r}-\mathbf{r}')] n_2(\mathbf{r},\mathbf{r}',t) d\mathbf{r}' \tag{16}$$

is the quantum part of interaction reported in this paper,

$$F^{\alpha\beta\gamma} = -\int [\partial^\alpha U(\mathbf{r}-\mathbf{r}')] \Pi^{\beta\gamma}_{2}(\mathbf{r},\mathbf{r}',t) d\mathbf{r}' \tag{17}$$

is the quasi-classic part of interaction,

$$R^{\alpha\beta\gamma\delta}_{s} = \int dR \sum_{i=1}^{N} \delta(\mathbf{r}-\mathbf{r}_i) \frac{1}{16m_i^4} \Biggl[\Psi^*_s(R,t) \hat{p}^\alpha_i \hat{p}^\beta_i \hat{p}^\gamma_i \hat{p}^\delta_i \Psi_s(R,t)$$

$$+ \hat{p}^{\alpha *}_i \Psi^*_s(R,t) \hat{p}^\beta_i \hat{p}^\gamma_i \hat{p}^\delta_i \Psi_s(R,t) + \hat{p}^{\alpha *}_i \hat{p}^{\gamma *}_i \hat{p}^{\delta *}_i \Psi^*_s(R,t) \hat{p}^\beta_i \Psi_s(R,t)$$

$$+ \hat{p}^{\gamma *}_i \Psi^*_s(R,t) \hat{p}^\alpha_i \hat{p}^\beta_i \hat{p}^\delta_i \Psi_s(R,t) + \hat{p}^{\alpha *}_i \hat{p}^{\beta *}_i \hat{p}^{\gamma *}_i \Psi^*_s(R,t) \hat{p}^\delta_i \Psi_s(R,t)$$

$$+ \hat{p}^{\alpha *}_i \hat{p}^{\delta *}_i \Psi^*_s(R,t) \hat{p}^\beta_i \hat{p}^\gamma_i \Psi_s(R,t) + \hat{p}^{\alpha *}_i \hat{p}^{\gamma *}_i \Psi^*_s(R,t) \hat{p}^\beta_i \hat{p}^\delta_i \Psi_s(R,t)$$

$$+ \hat{p}^{\gamma *}_i \hat{p}^{\delta *}_i \Psi^*_s(R,t) \hat{p}^\alpha_i \hat{p}^\beta_i \Psi_s(R,t) + c.c. \Biggr] \tag{18}$$

is the flux of $M^{\alpha\beta\gamma}_{s}$ and

$$\Pi^{\alpha\beta}_{2,ss'}(\mathbf{r},\mathbf{r}',t) = \int dR \sum_{i,j\neq i} \delta(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{r}'-\mathbf{r}_j) \frac{1}{4m_i^2} \times$$

$$\times (\Psi^*_s(R,t) \hat{p}^\alpha_i \hat{p}^\beta_i \Psi_s(R,t) + (\hat{p}^\alpha_i \Psi_s(R,t))^* \hat{p}^\beta_i \Psi_s(R,t) + c.c.). \tag{19}$$

If quantum correlations are dropped function $\Pi^{\alpha\beta}_{2}(\mathbf{r},\mathbf{r}',t)$ splits on product of the momentum flux $\Pi^{\alpha\beta}(\mathbf{r},t)$ and the concentration $n(\mathbf{r}',t)$.

General untruncated form of the equation for the "thermal" part of the third rank tensor (the part defined in the comoving frame) has the following form

$$\partial_t \tilde{Q}^{\alpha\beta\gamma}_{s} + \partial_\delta (v^\delta_s \tilde{Q}^{\alpha\beta\gamma}_{s}) + \tilde{Q}^{\alpha\gamma\delta}_{s} \partial_\delta v^\beta_s + \tilde{Q}^{\beta\gamma\delta}_{s} \partial_\delta v^\alpha_s$$

$$+ \tilde{Q}^{\alpha\beta\delta}_{s} \partial_\delta v^\gamma_s + \partial_\delta (P^{\alpha\beta\gamma\delta}_{s} + T^{\alpha\beta\gamma\delta}_{s}) = \frac{\hbar^2}{4m^3} q_e n_s \partial^\alpha \partial^\beta \partial^\gamma \Phi$$

$$+\frac{1}{mn}[(p_s^{\alpha\beta}+T_s^{\alpha\beta})\partial^\delta(p_s^{\gamma\delta}+T_s^{\gamma\delta})+(p_s^{\alpha\gamma}+T_s^{\alpha\gamma})\times$$

$$\times\partial^\delta(p_s^{\beta\delta}+T_s^{\beta\delta})+(p_s^{\beta\gamma}+T_s^{\beta\gamma})\partial^\delta(p_s^{\alpha\delta}+T_s^{\alpha\delta})], \quad (20)$$

where $\tilde{Q}_s^{\alpha\beta\gamma}=Q_s^{\alpha\beta\gamma}+T_s^{\alpha\beta\gamma}$,

$$T_s^{\alpha\beta\gamma}=-\frac{\hbar^2}{12m^2}n_s(\partial^\alpha\partial^\beta v_s^\gamma+\partial^\alpha\partial^\gamma v_s^\beta+\partial^\beta\partial^\gamma v_s^\alpha) \quad (21)$$

is the third rank tensor which is the analog of the quantum Bohm potential, the fourth rank tensor $P_s^{\alpha\beta\gamma\delta}$ is constructed on the thermal velocities or the velocities in the local frame, basically the fourth rank tensor $P_s^{\alpha\beta\gamma\delta}$ is the analog of the pressure tensor with higher tensor rank,

$$T_{s,lin}^{\alpha\beta\gamma\delta}=\frac{\hbar^4}{8m^4}\sqrt{n_s}\partial^\alpha\partial^\beta\partial^\gamma\partial^\delta\sqrt{n_s}$$

$$+2p_s^{\alpha\beta}T_s^{\gamma\delta}/n_s+2p_s^{\alpha\gamma}T_s^{\beta\delta}/n_s+2p_s^{\alpha\delta}T_s^{\beta\gamma}/n_s$$

$$+2p_s^{\beta\gamma}T_s^{\alpha\delta}/n_s+2p_s^{\beta\delta}T_s^{\alpha\gamma}/n_s+2p_s^{\gamma\delta}T_s^{\alpha\beta}/n_s. \quad (22)$$

is the main part of the fourth rank tensor which is the analog of the quantum Bohm potential.

It is necessary to omit the term proportional to the spatial derivatives of the fourth and second rank kinetic tensors in accordance with estimations presented in Ref. [6], but the term $\partial_\delta P_s^{\alpha\beta\gamma\delta}$ is kept to get some rough estimations of the fourth rank pressure-like tensor contribution.

E. Equilibrium expressions for the pressure and pressure-like third and fourth rank tensors

Perturbations of pressure tensor and the third rank tensor can be found from the corresponding equations of evolution. However, their equilibrium values are found via the equilibrium distribution function chosen in the form of the Fermi step function:

$$p_{0s}^{\alpha\beta}=m\int_0^{p_{Fs}}p^\alpha p^\beta\frac{d^3p}{(2\pi\hbar)^3}, \quad (23)$$

and

$$Q_{0s}^{\alpha\beta\gamma}=\int_0^{p_{Fs}}p^\alpha p^\beta p^\gamma\frac{d^3p}{(2\pi\hbar)^3}=0. \quad (24)$$

The equation of state for the thermal part (or the Pauli blocking part) of the fourth rank tensor is also found via the equilibrium distribution function chosen in the form of the Fermi step function:

$$P_{0s}^{\alpha\beta\gamma\delta}=\int_0^{p_{Fs}}p^\alpha p^\beta p^\gamma p^\delta\frac{d^3p}{(2\pi\hbar)^3}, \quad (25)$$

where symbol p with no indexes is the momentum, $p_{Fs}=(6\pi^2 n_{0s})^{1/3}\hbar$ is the partial Fermi momentum.

F. Fourth rank quantum Bohm potential

The fourth rank tensor, which is similar in nature with the quantum Bohm potential, appears in the equation for evolution of the third rank tensor equation (20). It has rather complex form. Therefore, it is not demonstrated in the main part of the paper.

Hence, this tensor is given as the superposition of three parts:

$$T_s^{\alpha\beta\gamma\delta}=\sum_{i=1}^{3}T_{si}^{\alpha\beta\gamma\delta}. \quad (26)$$

The first part can be approximately written via the concentration of fermions:

$$T_{s1}^{\alpha\beta\gamma\delta}=\frac{\hbar^4}{8m^4}\Biggl[\sqrt{n}\partial^\alpha\partial^\beta\partial^\gamma\partial^\delta\sqrt{n}$$

$$+\partial^\alpha\partial^\beta\sqrt{n}\cdot\partial^\gamma\partial^\delta\sqrt{n}+\partial^\alpha\partial^\gamma\sqrt{n}\cdot\partial^\beta\partial^\delta\sqrt{n}+\partial^\alpha\partial^\delta\sqrt{n}\cdot\partial^\beta\partial^\gamma\sqrt{n}$$

$$-\partial^\alpha\sqrt{n}\cdot\partial^\beta\partial^\gamma\partial^\delta\sqrt{n}-\partial^\beta\sqrt{n}\cdot\partial^\alpha\partial^\gamma\partial^\delta\sqrt{n}$$

$$-\partial^\gamma\sqrt{n}\cdot\partial^\alpha\partial^\beta\partial^\delta\sqrt{n}-\partial^\delta\sqrt{n}\cdot\partial^\alpha\partial^\beta\partial^\gamma\sqrt{n}\Biggr]. \quad (27)$$

Similar approximation is used for the quantum Bohm potential in equation (5) in the main text.

The second part of tensor $T_s^{\alpha\beta\gamma\delta}$ contains the traditional quantum Bohm potential:

$$T_{s2}^{\alpha\beta\gamma\delta}=2\Biggl[v^\gamma v^\delta T_{micro}^{\alpha\beta}+v^\gamma\langle u^\delta t^{\alpha\beta}\rangle+v^\delta\langle u^\gamma t^{\alpha\beta}\rangle+\langle u^\gamma u^\delta t^{\alpha\beta}\rangle\Biggr]$$

$$+2\Biggl[v^\beta v^\delta T_{micro}^{\alpha\gamma}+v^\beta\langle u^\delta t^{\alpha\gamma}\rangle+v^\delta\langle u^\beta t^{\alpha\gamma}\rangle+\langle u^\beta u^\delta t^{\alpha\gamma}\rangle\Biggr]$$

$$+2\Biggl[v^\gamma v^\beta T_{micro}^{\alpha\delta}+v^\gamma\langle u^\beta t^{\alpha\delta}\rangle+v^\beta\langle u^\gamma t^{\alpha\delta}\rangle+\langle u^\gamma u^\beta t^{\alpha\delta}\rangle\Biggr]$$

$$+2\Biggl[v^\alpha v^\delta T_{micro}^{\beta\gamma}+v^\alpha\langle u^\delta t^{\beta\gamma}\rangle+v^\delta\langle u^\alpha t^{\beta\gamma}\rangle+\langle u^\alpha u^\delta t^{\beta\gamma}\rangle\Biggr]$$

$$+2\Biggl[v^\alpha v^\gamma T_{micro}^{\beta\delta}+v^\gamma\langle u^\alpha t^{\beta\delta}\rangle+v^\alpha\langle u^\gamma t^{\beta\delta}\rangle+\langle u^\alpha u^\gamma t^{\beta\delta}\rangle\Biggr]$$

$$+2\Biggl[v^\alpha v^\beta T_{micro}^{\gamma\delta}+v^\alpha\langle u^\beta t^{\gamma\delta}\rangle+v^\beta\langle u^\alpha t^{\gamma\delta}\rangle+\langle u^\alpha u^\beta t^{\gamma\delta}\rangle\Biggr], \quad (28)$$

where

$$t^{\alpha\beta}=\frac{\hbar^2}{4m^2}(a\partial^\alpha\partial^\beta a-\partial^\alpha a\cdot\partial^\beta a), \quad (29)$$

$$T^{\alpha\beta}_{micro}=\frac{\hbar^2}{4m^2}\int dR\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i)t^{\alpha\beta}$$

$$=\int dR\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i)(a\partial^\alpha\partial^\beta a-\partial^\alpha a\cdot\partial^\beta a), \qquad (30)$$

where $T^{\alpha\beta}_{micro}=\langle t^{\alpha\beta}\rangle$. Equation (30) approximately gives the quantum Bohm potential $T^{\alpha\beta}_{micro}\approx T^{\alpha\beta}$. However, other expressions like $\langle u^\alpha t^{\beta\delta}\rangle$ has no simple expression. Using the theorem on average we can make the following approximation $\langle u^\alpha t^{\beta\delta}\rangle=(T^{\alpha\beta}/n)\langle a^2u^\alpha\rangle=0$, since $\langle a^2u^\alpha\rangle=0$ by definition of the thermal velocity. Moreover, we find $\langle u^\alpha u^\beta t^{\gamma\delta}\rangle\approx(T^{\gamma\delta}/n)p^{\alpha\beta}$. These expressions are used as the equation of state for the described functions.

The third part of tensor $T^{\alpha\beta\gamma\delta}_s$ is constructed of the velocities

$$T^{\alpha\beta\gamma\delta}_{s3}=\frac{\hbar^2}{24m^2}\Biggl[nv^\alpha[\partial^\beta\partial^\gamma v^\delta+\partial^\gamma\partial^\delta v^\beta+\partial^\beta\partial^\delta v^\gamma]$$

$$+nv^\beta[\partial^\alpha\partial^\gamma v^\delta+\partial^\alpha\partial^\delta v^\gamma+\partial^\gamma\partial^\delta v^\alpha]$$

$$+nv^\gamma[\partial^\alpha\partial^\beta v^\delta+\partial^\alpha\partial^\delta v^\beta+\partial^\beta\partial^\delta v^\alpha]$$

$$+nv^\delta[\partial^\alpha\partial^\beta v^\gamma+\partial^\alpha\partial^\gamma v^\beta+\partial^\beta\partial^\gamma v^\alpha]$$

$$+v^\alpha[\langle a^2\partial^\beta\partial^\gamma u^\delta\rangle+\langle a^2\partial^\beta\partial^\delta u^\gamma\rangle+\langle a^2\partial^\gamma\partial^\delta u^\beta\rangle]$$

$$+v^\beta[\langle a^2\partial^\alpha\partial^\gamma u^\delta\rangle+\langle a^2\partial^\alpha\partial^\delta u^\gamma\rangle+\langle a^2\partial^\gamma\partial^\delta u^\alpha\rangle]$$

$$+v^\gamma[\langle a^2\partial^\alpha\partial^\beta u^\delta\rangle+\langle a^2\partial^\alpha\partial^\delta u^\beta\rangle+\langle a^2\partial^\beta\partial^\delta u^\alpha\rangle]$$

$$+v^\delta[\langle a^2\partial^\alpha\partial^\beta u^\gamma\rangle+\langle a^2\partial^\alpha\partial^\gamma u^\beta\rangle+\langle a^2\partial^\beta\partial^\gamma u^\alpha\rangle]$$

$$+[\langle a^2u^\alpha\partial^\beta\partial^\gamma u^\delta\rangle+\langle a^2u^\alpha\partial^\beta\partial^\delta u^\gamma\rangle+\langle a^2u^\alpha\partial^\gamma\partial^\delta u^\beta\rangle]$$

$$+[\langle a^2u^\beta\partial^\alpha\partial^\gamma u^\delta\rangle+\langle a^2u^\beta\partial^\alpha\partial^\delta u^\gamma\rangle+\langle a^2u^\beta\partial^\gamma\partial^\delta u^\alpha\rangle]$$

$$+[\langle a^2u^\gamma\partial^\alpha\partial^\beta u^\delta\rangle+\langle a^2u^\gamma\partial^\alpha\partial^\delta u^\beta\rangle+\langle a^2u^\gamma\partial^\beta\partial^\delta u^\alpha\rangle]$$

$$+[\langle a^2u^\delta\partial^\alpha\partial^\beta u^\gamma\rangle+\langle a^2u^\delta\partial^\alpha\partial^\gamma u^\beta\rangle+\langle a^2u^\delta\partial^\beta\partial^\gamma u^\alpha\rangle]\Biggr]. \qquad (31)$$

Approximate equation of state for $T^{\alpha\beta\gamma\delta}_{s3}$ has the following form

$$T^{\alpha\beta\gamma\delta}_{s3,appr}=\frac{\hbar^2}{24m^2}\Biggl[nv^\alpha[\partial^\beta\partial^\gamma v^\delta+\partial^\gamma\partial^\delta v^\beta+\partial^\beta\partial^\delta v^\gamma]$$

$$+nv^\beta[\partial^\alpha\partial^\gamma v^\delta+\partial^\alpha\partial^\delta v^\gamma+\partial^\gamma\partial^\delta v^\alpha]$$

$$+nv^\gamma[\partial^\alpha\partial^\beta v^\delta+\partial^\alpha\partial^\delta v^\beta+\partial^\beta\partial^\delta v^\alpha]$$

$$+nv^\delta[\partial^\alpha\partial^\beta v^\gamma+\partial^\alpha\partial^\gamma v^\beta+\partial^\beta\partial^\gamma v^\alpha]\Biggr]. \qquad (32)$$

It is equal to zero in the linear approximation for the macroscopically motionless plasmas since it is nonlinear on the velocity field.

G. Title of the developed approximation

Various extended hydrodynamics can be developed. Suggested model is called 20-moment hydrodynamics. We have five traditional moments: concentration n, projections of momentum nv_x, nv_y, nv_z, and energy density (or the temperature) $\varepsilon=p^{\beta\beta}/3$. Six functions are in the pressure tensor $p^{\alpha\beta}$, but one of them is taken for the energy density. Three functions are in the energy current. Their account leads to the traditional 13-moments approximation. Furthermore, the symmetric third rank tensor $Q^{\alpha\beta\gamma}$ has 10 independent elements, but three of them give the energy current. Therefore, the account of the third rank tensor $Q^{\alpha\beta\gamma}$ makes the model 20-moment hydrodynamics, where twenty independent functions are used to describe each species.

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