8 Appendix: The MF-VAR when the Frequency Mis-Match Ends in 2012

The MF-VAR of Koop et al. (2020) can be written as:

$$\begin{bmatrix} y_t^{UK} \\ y_t^{Q} \end{bmatrix} = \begin{bmatrix} \Phi_{qc} \\ \Phi_{ac} \end{bmatrix} + \begin{bmatrix} \Phi_{qq,1} & \Phi_{qa,1} \\ \Phi_{aq,1} & \Phi_{aa,1} \end{bmatrix} \begin{bmatrix} y_{t-1}^{UK} \\ y_{t-1}^{Q} \end{bmatrix} + \ldots + \begin{bmatrix} \Phi_{qq,7} & \Phi_{qa,7} \\ \Phi_{aq,7} & \Phi_{aa,7} \end{bmatrix} \begin{bmatrix} y_{t-7}^{UK} \\ y_{t-7}^{Q} \end{bmatrix} + \epsilon_t, \tag{3}$$

where ϵ_t is i.i.d. $N(0, \Sigma_t)$. Σ_t follows the same multivariate stochastic volatility process as in Koop et al. (2020).

If we group the coefficients in the MF-VAR into blocks as:

$$\Phi_{qq} = \begin{bmatrix} \Phi_{qq,1}, & \Phi_{qq,2}, & \Phi_{qq,3} & ,\ldots, \Phi_{qq,7} \end{bmatrix}, \tag{4}$$

$$\Phi_{qa} = \begin{bmatrix} \Phi_{qa,1}, & \Phi_{qa,2}, & \Phi_{qa,3} & ,\ldots, \Phi_{qa,7} \end{bmatrix}, \tag{5}$$

$$\Phi_{aq} = \begin{bmatrix} \Phi_{aq,1} & ,\Phi_{aq,2} & ,\Phi_{aq,3} & ,\ldots, \Phi_{aq,7} \end{bmatrix}, \tag{6}$$

$$\Phi_{aa} = \begin{bmatrix} \Phi_{aa,1} & ,\Phi_{aa,2}, & \Phi_{aa,3} & ,\ldots, \Phi_{aa,7} \end{bmatrix}, \tag{7}$$

then we can define the part of the MF-VAR relating to the regional data as:

$$\mathbf{s}_t = \Gamma_s \mathbf{s}_{t-1} + \Gamma_z \mathbf{y}_{t-p:t-1}^{UK} + \Gamma_c + u_{a,t}, \tag{8}$$

where $\mathbf{s}_t = (y_t^{Q'}, y_{t-1}^{Q'}, y_{t-2}^{Q'}, \ldots, y_{t-7}^{Q'})'$ is a $z \times 1 = R \times 7$ vector containing the regional variables and their lags and $\mathbf{y}_{t-p:t-1}^{UK} = (y_{t-1}^{UK}, \ldots, y_{t-7}^{UK})'$ contains lags of the UK variables. The coefficient matrices in this equation have the form:

$$\Gamma_s = \begin{bmatrix} \Phi_{qq} & 0 \\ \mathbf{I} & 0 \end{bmatrix}_{z \times z}, \Gamma_z = \begin{bmatrix} \Phi_{aq} \\ 0 \end{bmatrix}_{z \times p}, \Gamma_c = \begin{bmatrix} \Phi_{ac} \\ 0 \end{bmatrix}_{z \times 1}. \tag{9}$$

For UK GVA growth, we have

$$y_t^{UK} = \Lambda_{qs} \mathbf{s}_t + \Phi_{qq} \mathbf{y}_{t-p:t-1}^{UK} + \Phi_{ac} + u_{q,t}, \tag{10}$$

where

$$\Lambda_{qs} = \begin{bmatrix} 0 & \Phi_{qa} \end{bmatrix}_{1 \times z}.$$

Combining these together into one MF-VAR we have

$$y_t = \Lambda_{as}\mathbf{s}_t + \Lambda_z \mathbf{y}^{UK}_{t-p:t-1} + \Phi_{qc}, \tag{11}$$

where

$$\Lambda_{as} = \begin{bmatrix} 0 & \Phi_{qa} \\ & M \end{bmatrix}, \Lambda_z = \begin{bmatrix} \Phi_{qq} \\ 0 \end{bmatrix}, \tag{12}$$

$$M = \begin{bmatrix} \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{3}{4} & 0 & 1 & 0 & \frac{3}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{3}{4} & 0 & 1 & 0 & \frac{3}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \end{bmatrix}. \tag{13}$$

Note that the matrix M imposes the inter-temporal restriction.

Finally, the cross-sectional restriction gives us an additional measurement equation for the MF-VAR. We have

$$y^{UK}_t = \mathbf{R}\mathbf{s}_t + \eta_t, \eta_t \sim N(0, \sigma^2_{cs}), \tag{14}$$

where

$$\mathbf{R} = \begin{bmatrix} \frac{1}{R} & \dots & \frac{1}{R} & 0 & \dots & 0 \end{bmatrix}_{1\times z}. \tag{15}$$

As discussed in Koop et al. (2020), the cross-sectional restriction can be expected to only hold approximately and this explains the presence of the error, η_t, added to this restriction (e.g. GVA from the UK's continental shelf, UKCS, is included in UK GVA but not in any of our regions). We use the same prior for σ^2_{cs} as in Koop et al. (2020) which reflects a view that the approximation error is small.

The model just described is that of Koop et al. (2020). It can be seen that y^Q_t, which is regional quarterly GVA growth, appears in the vector of variables y_t (and, thus, in $\mathbf{s_t}$). It is not observed and, thus, treated as states to be estimated by the MF-VAR. In the present paper, the same holds true, prior to 2012. However, from 2012 onwards the RSTI data exist and y^Q_t is observed. This can be accommodated by having the M matrix in (13), which imposes the inter-temporal restriction, change in 2012. To be specific, prior to 2012 the model is as above. From 2012 onwards, we use the same model but with:

$$M = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 & 0 & 0 & 0 \\ & & & & & & & \end{bmatrix}. \tag{16}$$

That is, the first R columns of M are now an identity matrix (and the remaining columns are zeros). The inter-temporal restriction, which specifies the relationship between quarterly and annual output figures, is no longer needed since the quarterly figures are directly observed. That is, from 2012 we have quarterly RSTI data and the quarterly output data from NISRA and the Scottish

Government; so if we were to want any annual regional data, the annual data they add up to would simply be their annual aggregation. For the English regions, these annual data will be the same as the annual regional data that the ONS report. For Northern Ireland and Scotland, as explained in section 3, the quarterly output data will aggregate to annual estimates from NISRA and the Scottish Government.

In summary, we use a model which is the same as Koop et al. (2020), and all specification, prior and computational details are as described there. But we then extend it, as described above, to allow for the change in 2012 when the quarterly RSTI data also become available.