

#### Equations of model 4: Model for tick-borne infection with deer enclosure

$$\begin{aligned}
\dot{L}_Q^{IN} &= \frac{r^T}{1 + s^T T_{F_1}^{IN}} \sigma^A A_{F_1}^{IN} - \beta_1^L H_1^{IN} L_Q^{IN} - d^L L_Q^{IN} \\
\dot{L}_Q^{OUT} &= \frac{r^T}{1 + s^T T_{F_1}^{OUT}} \sigma^A A_{F_1}^{OUT} + \frac{r^T}{1 + s^T T_{F_2}^{OUT}} \sigma^A A_{F_2}^{OUT} - (\beta_1^L H_1^{OUT} + \beta_2^L H_2) L_Q^{OUT} - d^L L_Q^{OUT} \\
\dot{L}_{F_1}^{IN,e} &= \beta_1^L L_Q^{IN} H_1^{IN,i} + \beta_1^L L_Q^{IN} (H_1^{IN,s} + H_1^{IN,r}) \left( 1 - \exp \left\{ -\frac{\lambda N_{F_1}^{IN,i}}{H_1^{IN}} \right\} \right) + \\
&\quad - \sigma^L L_{F_1}^{IN,e} - \rho L_{F_1}^{IN,e} + \rho L_{F_1}^{OUT,e} \\
\dot{L}_{F_1}^{OUT,e} &= \beta_1^L L_Q^{OUT} H_1^{OUT,i} + \beta_1^L L_Q^{OUT} (H_1^{OUT,s} + H_1^{OUT,r}) \left( 1 - \exp \left\{ -\frac{\lambda N_{F_1}^{OUT,i}}{H_1^{OUT}} \right\} \right) + \\
&\quad - \sigma^L L_{F_1}^{OUT,e} + scale \rho L_{F_1}^{IN,e} - scale \rho L_{F_1}^{OUT,e} \\
\dot{L}_{F_1}^{IN,s} &= \left[ \beta_1^L (H_1^{IN,s} + H_1^{IN,r}) \exp \left\{ -\frac{\lambda N_{F_1}^{IN,i}}{H_1^{IN}} \right\} \right] L_Q^{IN} - \sigma^L L_{F_1}^{IN,s} - \rho L_{F_1}^{IN,s} + \rho L_{F_1}^{OUT,s} \\
\dot{L}_{F_1}^{OUT,s} &= \left[ \beta_1^L (H_1^{OUT,s} + H_1^{OUT,r}) \exp \left\{ -\frac{\lambda N_{F_1}^{OUT,i}}{H_1^{OUT}} \right\} \right] L_Q^{OUT} + \\
&\quad - \sigma^L L_{F_1}^{OUT,s} + scale \rho L_{F_1}^{IN,s} - scale \rho L_{F_1}^{OUT,s} \\
\dot{L}_{F_2}^{OUT,s} &= \beta_2^L H_2 L_Q^{OUT} - \sigma^L L_{F_2}^{OUT,s} \\
\dot{N}_Q^{IN,i} &= m^L \sigma^L L_{F_1}^{IN,e} - d^N N_Q^{IN,i} - \beta_1^N H_1^{IN} N_Q^{IN,i} \\
\dot{N}_Q^{OUT,i} &= m^L \sigma^L L_{F_1}^{OUT,e} - d^N N_Q^{OUT,i} - (\beta_1^N H_1^{OUT} + \beta_2^N H_2) N_Q^{OUT,i} \\
\dot{N}_Q^{IN,s} &= m^L \sigma^L L_{F_1}^{IN,s} - d^N N_Q^{IN,s} - \beta_1^N H_1^{IN} N_Q^{IN,s} \\
\dot{N}_Q^{OUT,s} &= m^L \sigma^L (L_{F_1}^{OUT,s} + L_{F_2}^{OUT,s}) - d^N N_Q^{OUT,s} - (\beta_1^N H_1^{OUT} + \beta_2^N H_2) N_Q^{OUT,s} \\
\dot{N}_{F_1}^{IN,i} &= \beta_1^N H_1^{IN} N_Q^{IN,i} - \sigma^N N_{F_1}^{IN,i} - \rho N_{F_1}^{IN,i} + \rho N_{F_1}^{OUT,i} \\
\dot{N}_{F_1}^{OUT,i} &= \beta_1^N H_1^{OUT} N_Q^{OUT,i} - \sigma^N N_{F_1}^{OUT,i} + scale \rho N_{F_1}^{IN,e} - scale \rho L_{F_1}^{OUT,e} \\
\dot{N}_{F_1}^{IN,e} &= \beta_1^N N_Q^{IN,s} H_1^{IN,i} + \beta_1^N N_Q^{IN,s} (H_1^{IN,s} + H_1^{IN,r}) \left( 1 - \exp \left\{ -\frac{\lambda N_{F_1}^{IN,i}}{H_1^{IN}} \right\} \right) + \\
&\quad - \sigma^N N_{F_1}^{IN,e} - \rho N_{F_1}^{IN,e} + \rho N_{F_1}^{OUT,e}
\end{aligned}$$

$$\begin{aligned}
\dot{N}_{F_1}^{OUT,e} &= \beta_1^N N_Q^{OUT,s} H_1^{OUT,i} + \beta_1^N N_Q^{OUT,s} (H_1^{OUT,s} + H_1^{OUT,r}) \left( 1 - \exp \left\{ -\frac{\lambda N_{F_1}^{OUT,i}}{H_1^{OUT}} \right\} \right) + \\
&\quad - \sigma^N N_{F_1}^{OUT,e} + scale \rho L_{N_1}^{IN,e} - scale \rho L_{N_1}^{OUT,e} \\
\dot{N}_{F_1}^{IN,s} &= \left[ \beta_1^N (H_1^{IN,s} + H_1^{IN,r}) \exp \left\{ -\frac{\lambda N_{F_1}^{IN,i}}{H_1^{IN}} \right\} \right] N_Q^{IN,s} - \sigma^N N_{F_1}^{IN,s} - \rho N_{F_1}^{IN,s} + \rho N_{F_1}^{OUT,s} \\
\dot{N}_{F_1}^{OUT,s} &= \left[ \beta_1^N (H_1^{OUT,s} + H_1^{OUT,r}) \exp \left\{ -\frac{\lambda N_{F_1}^{OUT,i}}{H_1^{OUT}} \right\} \right] N_Q^{OUT,s} + \\
&\quad - \sigma^N N_{F_1}^{OUT,s} + scale \rho N_{F_1}^{IN,s} - scale \rho N_{F_1}^{OUT,s} \\
\dot{N}_{F_2}^{OUT,i} &= \beta_2^N H_2 N_Q^{OUT,i} - \sigma^N N_{F_2}^{OUT,i} \\
\dot{N}_{F_2}^{OUT,s} &= \beta_2^N H_2 N_Q^{OUT,s} - \sigma^N N_{F_2}^{OUT,s} \\
\dot{A}_Q^{IN,i} &= m^N \sigma^N (N_{F_1}^{IN,i} + N_{F_1}^{IN,e}) - d^A A_Q^{IN,i} - \beta_1^A H_1^{IN} A_Q^{IN,i} \\
\dot{A}_Q^{OUT,i} &= m^N \sigma^N (N_{F_1}^{OUT,i} + N_{F_2}^{OUT,i} + N_{F_1}^{OUT,e}) - d^A A_Q^{OUT,i} - (\beta_1^A H_1^{OUT} + \beta_2^A H_2) A_Q^{OUT,i} \\
\dot{A}_Q^{IN,s} &= m^N \sigma^N N_F^{IN,s} - d^A A_Q^{IN,s} - \beta_1^A H_1^{IN} A_Q^{IN,s} \\
\dot{A}_Q^{OUT,s} &= m^N \sigma^N (N_{F_1}^{OUT,s} + N_{F_2}^{OUT,s}) - d^A A_Q^{OUT,s} - (\beta_1^A H_1^{OUT} + \beta_2^A H_2) A_Q^{OUT,s} \\
\dot{A}_{F_1}^{IN} &= \beta_1^A H_1^{IN} (A_Q^{IN,i} + A_Q^{IN,s}) - \sigma^A A_{F_1}^{IN} - \rho A_{F_1}^{IN} + \rho A_{F_1}^{OUT} \\
\dot{A}_{F_1}^{OUT} &= \beta_1^A H_1^{OUT} (A_Q^{IN,i} + A_Q^{IN,s}) - \sigma^A A_{F_1}^{OUT} + scale \rho A_{F_1}^{IN} - scale \rho A_{F_1}^{OUT} \\
\dot{A}_{F_2}^{OUT} &= \beta_2^A H_2 (A_Q^{OUT,i} + A_Q^{OUT,s}) - \sigma^A A_{F_2}^{OUT} \\
\dot{H}_1^{IN,s} &= r_1 H_1^{IN} - d_1(H_1) H_1^{IN,s} - (\beta_1^N N_Q^{IN,i} + \beta_1^A A_Q^{IN,i}) H_1^{IN,s} + \rho H_1^{OUT,s} - \rho H_1^{IN,s} \\
\dot{H}_1^{IN,i} &= (\beta_1^N N_Q^{IN,i} + \beta_1^A A_Q^{IN,i}) H_1^{IN,s} - (d_1(H_1) + \gamma_1 + \alpha_1) H_1^{IN,i} + \rho H_1^{OUT,i} - \rho H_1^{IN,i} \\
\dot{H}_1^{IN,r} &= \gamma_1 H_1^{IN,i} - d_1(H_1) H_1^{IN,r} + \rho H_1^{OUT,r} - \rho H_1^{IN,r} \\
\dot{H}_1^{OUT,s} &= r_1 H_1^{OUT} - d_1(H_1) H_1^{OUT,s} - (\beta_1^N N_Q^{OUT,i} + \beta_1^A A_Q^{OUT,i}) H_1^{OUT,s} + \\
&\quad - scale \rho H_1^{OUT,s} + scale \rho H_1^{IN,s} \\
\dot{H}_1^{OUT,i} &= (\beta_1^N N_Q^{OUT,i} + \beta_1^A A_Q^{OUT,i}) H_1^{OUT,s} - (d_1(H_1) + \gamma_1 + \alpha_1) H_1^{OUT,i} + \\
&\quad - scale \rho H_1^{OUT,i} + scale \rho H_1^{IN,i} \\
\dot{H}_1^{OUT,r} &= \gamma_1 H_1^{OUT,i} - d_1(H_1) H_1^{OUT,r} - scale \rho H_1^{OUT,r} + scale \rho H_1^{IN,r}
\end{aligned}$$