When they don't bite, we smell money: understanding malaria bednet misuse

Supplementary Online Material

This supplementary document provides detailed theoretical analysis of a two-player ITN (insecticide-treated net) use game. In Sections 2 and 3, the Nash equilibrium and the Pareto equilibrium are defined, and the distributions of the two equilibria are shown. In Section 4, the distribution of Pareto efficient Nash equilibria is shown.

1 Distribution of Nash equilibria

In the ITN use game, the two players $i \in \{1, 2\}$ have the common set of pure strategies $\Sigma := \{\mathrm{T}, \mathrm{F}\}$, and each player chooses a strategy $\sigma_i \in \Sigma$. As shown in Figure 2, the profile $(\sigma_1, \sigma_2) \in \Sigma^2$ determines the players' infection probabilities, labor productivities, and expected payoffs.

The Nash equilibrium is the set of profiles from which any player has no incentive to deviate. At the Nash equilibrium, each player's strategy is the best response against the other player's strategy. Let $\mathcal{B}_i(\sigma_1, \sigma_2)$ be the proposition that σ_i is the best response against σ_j, $i \neq j$, $j \in \{1, 2\}$. The propositions $\mathcal{B}_1(\sigma_1, \sigma_2)$ and $\mathcal{B}_2(\sigma_1, \sigma_2)$ are defined as

$$\mathcal{B}_1(\sigma_1, \sigma_2) \leftrightarrow \forall \sigma_1' \in \Sigma,\ U_1(\sigma_1, \sigma_2) \geq U_1(\sigma_1', \sigma_2),$$
$$\mathcal{B}_2(\sigma_1, \sigma_2) \leftrightarrow \forall \sigma_2' \in \Sigma,\ U_2(\sigma_1, \sigma_2) \geq U_2(\sigma_1, \sigma_2'),$$

where $U_i(\sigma_1, \sigma_2) \in [0, \infty)$ denotes the expected payoff of the i-th player at the profile (σ_1, σ_2). Let $\mathcal{N}(\sigma_1, \sigma_2)$ be the proposition that the profile (σ_1, σ_2) is a Nash equilibrium. The Nash equilibrium is defined as

$$\mathcal{N}(\sigma_1, \sigma_2) \leftrightarrow \mathcal{B}_1(\sigma_1, \sigma_2) \wedge \mathcal{B}_2(\sigma_1, \sigma_2),$$

where the operator $\wedge$ denotes the logical conjunction of propositions.

The all-F profile (F, F) is a Nash equilibrium if

$$[U_1(\mathrm{F}, \mathrm{F}) \geq U_1(\mathrm{T}, \mathrm{F})] \wedge [U_2(\mathrm{F}, \mathrm{F}) \geq U_2(\mathrm{F}, \mathrm{T})].$$

The first and second inequalities correspond to the best responses $\mathcal{B}_1(\mathrm{F},\mathrm{F})$ and $\mathcal{B}_2(\mathrm{F},\mathrm{F})$, respectively. Since $U_1(\mathrm{F},\mathrm{F}) = U_2(\mathrm{F},\mathrm{F})$ and $U_1(\mathrm{T},\mathrm{F}) = U_2(\mathrm{F},\mathrm{T})$, the best responses of the two players are equivalent. Solving $\mathcal{B}_1(\mathrm{F},\mathrm{F})$ with respect to P gives the interval of the all-F Nash equilibrium, that is

$$\mathcal{N}(\mathrm{F},\mathrm{F}) \leftrightarrow P \in [0, P_L],\ P_L := \frac{\beta - 1}{\beta - \alpha_1\alpha_2} \in (0,1).$$

Note that $\alpha_1 \in (0,1)$, $\alpha_2 \in (0,1)$, and $\beta \in (1,\infty)$.

The all-T profile (T,T) is a Nash equilibrium if

$$[U_1(\mathrm{T},\mathrm{T}) \geq U_1(\mathrm{F},\mathrm{T})] \wedge [U_2(\mathrm{T},\mathrm{T}) \geq U_2(\mathrm{T},\mathrm{F})].$$

Similar to the all-F profile, the best responses of the two players are equivalent. Solving $\mathcal{B}_1(\mathrm{T},\mathrm{T})$ with respect to P gives the interval of the all-T Nash equilibrium, that is

$$\mathcal{N}(\mathrm{T},\mathrm{T}) \leftrightarrow P \in [P_R, 1],\ P_R := \frac{P_L}{\alpha_2} \in \left(0, \frac{1}{\alpha_2}\right).$$

Note that $P_L < P_R$. If $\alpha_2 < P_L$, P_R exceeds one, and the all-T Nash equilibrium is crowded out of the domain $[0,1]$.

At the profiles (T,F) and (F,T), the player with the strategy F free rides on the community effect provided by the player with the strategy T, without abandoning the benefit from the alternative use of ITNs. For the free-rider profiles,

$$\mathcal{N}(\mathrm{T},\mathrm{F}) \leftrightarrow [U_1(\mathrm{T},\mathrm{F}) \geq U_1(\mathrm{F},\mathrm{F})] \wedge [U_2(\mathrm{T},\mathrm{F}) \geq U_2(\mathrm{T},\mathrm{T})],$$
$$\mathcal{N}(\mathrm{F},\mathrm{T}) \leftrightarrow [U_1(\mathrm{F},\mathrm{T}) \geq U_1(\mathrm{T},\mathrm{T})] \wedge [U_2(\mathrm{F},\mathrm{T}) \geq U_2(\mathrm{F},\mathrm{F})].$$

$\mathcal{B}_1(\mathrm{T},\mathrm{F})$ and $\mathcal{B}_2(\mathrm{T},\mathrm{F})$ are equivalent to $\mathcal{B}_2(\mathrm{F},\mathrm{T})$ and $\mathcal{B}_1(\mathrm{F},\mathrm{T})$, respectively. Hence, the free-rider Nash equilibria $\mathcal{N}(\mathrm{T},\mathrm{F})$ and $\mathcal{N}(\mathrm{F},\mathrm{T})$ are equivalent. Solving $\mathcal{N}(\mathrm{T},\mathrm{F})$ with respect to P gives the interval of the free-rider Nash equilibria, that is

$$\mathcal{N}(\mathrm{T},\mathrm{F}) \leftrightarrow \mathcal{N}(\mathrm{F},\mathrm{T}) \leftrightarrow P \in [P_L, P_R] \cap [0,1],$$

where the operator $\cap$ produces the intersection of sets.

2 Distribution of Pareto equilibria

The Pareto equilibrium is the set of profiles at which any player cannot increase its payoff without decreasing the other player's payoff. At the Pareto equilibrium, the vector of the two players' expected payoffs is efficient from the viewpoint of public welfare, which means that any player's strategy is not harmful for the other player. The Pareto equilibrium is not always the Nash equilibrium, and the mismatch between the two equilibria is called a *social dilemma* (SD). In the social dilemma, each player's best response to the other player results in an inefficient expected payoff vector.

Let $\mathbf{u}(\sigma_1, \sigma_2) := (U_1(\sigma_1, \sigma_2), U_2(\sigma_1, \sigma_2))$ be the expected payoff vector at the profile (σ_1, σ_2). A vector $\mathbf{u}(\sigma_1', \sigma_2')$ is Pareto superior to the other vector $\mathbf{u}(\sigma_1, \sigma_2)$ if

$$\mathbf{u}(\sigma_1', \sigma_2') \in \Phi_{\mathrm{S}}(\sigma_1, \sigma_2) := \{[U_1(\sigma_1, \sigma_2), \infty) \times [U_2(\sigma_1, \sigma_2), \infty)\} \setminus \mathbf{u}(\sigma_1, \sigma_2).$$

$\Phi_{\mathrm{S}}(\sigma_1, \sigma_2)$ indicates the Pareto superior region to $\mathbf{u}(\sigma_1, \sigma_2)$. The operator $\times$ produces the Cartesian product of sets, and the operator $\setminus$ produces the relative complement of the right-side set in the left-side set. By moving from $\mathbf{u}(\sigma_1, \sigma_2)$ to $\mathbf{u}(\sigma_1', \sigma_2')$, at least one player can increase its payoff without decreasing the other player's payoff. The negation of the Pareto superiority is the Pareto inferiority. The Pareto inferior region to $\mathbf{u}(\sigma_1, \sigma_2)$, denoted by $\Phi_{\mathrm{I}}(\sigma_1, \sigma_2)$, is expressed as

$$\Phi_{\mathrm{I}}(\sigma_1, \sigma_2) := \{[0, \infty) \times [0, \infty)\} \setminus \Phi_{\mathrm{S}}(\sigma_1, \sigma_2).$$

Figure S1 shows the Pareto superior and inferior regions to $\mathbf{u}(\sigma_1, \sigma_2)$. Let $\mathcal{P}(\sigma_1, \sigma_2)$ be the proposition that the profile (σ_1, σ_2) is a Pareto equilibrium. The vector $\mathbf{u}(\sigma_1, \sigma_2)$ is a Pareto equilibrium if all the other vectors are Pareto inferior to $\mathbf{u}(\sigma_1, \sigma_2)$, that is

$$\mathcal{P}(\sigma_1, \sigma_2) \leftrightarrow \forall (\sigma_1', \sigma_2') \in \Sigma^2,\ \mathbf{u}(\sigma_1', \sigma_2') \in \Phi_{\mathrm{I}}(\sigma_1, \sigma_2).$$

Since $U_1(\mathrm{T}, \mathrm{F}) < U_1(\mathrm{T}, \mathrm{T})$ and $U_2(\mathrm{F}, \mathrm{T}) < U_2(\mathrm{T}, \mathrm{T})$, the free-rider vectors $\mathbf{u}(\mathrm{T}, \mathrm{F})$ and $\mathbf{u}(\mathrm{F}, \mathrm{T})$ are Pareto inferior to the all-T vector $\mathbf{u}(\mathrm{T}, \mathrm{T})$. Hence, the all-T vector is a Pareto equilibrium if the all-F vector $\mathbf{u}(\mathrm{F}, \mathrm{F})$ is Pareto inferior to the all-T vector. Since the two players have the same payoff at the all-T or all-F profile,

$$\mathbf{u}(\mathrm{F}, \mathrm{F}) \in \Phi_{\mathrm{I}}(\mathrm{T}, \mathrm{T}) \leftrightarrow U_1(\mathrm{F}, \mathrm{F}) \leq U_1(\mathrm{T}, \mathrm{T}).$$

Solving this inequality with respect to P gives the interval of the all-T Pareto equilibrium, that is

$$\mathcal{P}(\mathrm{T}, \mathrm{T}) \leftrightarrow P \in [P_L^*, 1],\ P_L^* := \frac{\beta - 1}{\beta - \alpha_1 \alpha_2^2} \in (0, 1).$$

Note that $P_L^* < P_L$.

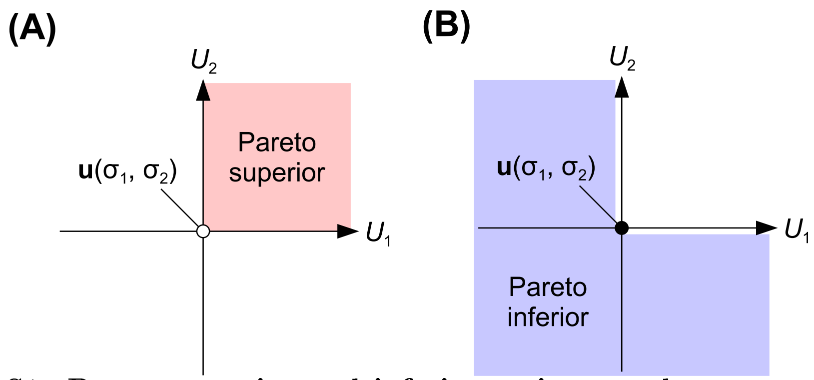

Figure S1: Pareto superior and inferior regions to the expected payoff vector at the profile (σ_1, σ_2). (A) Pareto superior region to the vector $\mathbf{u}(\sigma_1, \sigma_2)$. If the players move from $\mathbf{u}(\sigma_1, \sigma_2)$ to the Pareto superior region, at least one player can increase its payoff without decreasing the other player's payoff. **(B)** Pareto inferior region to the vector $\mathbf{u}(\sigma_1, \sigma_2)$. In the Pareto inferior region, at least one player's payoff decreases compared to $\mathbf{u}(\sigma_1, \sigma_2)$.

Since $U_2(\mathrm{F}, \mathrm{F}) < U_2(\mathrm{T}, \mathrm{F})$ and $U_1(\mathrm{F}, \mathrm{F}) < U_1(\mathrm{F}, \mathrm{T})$, the all-F vector is Pareto inferior to the free-rider vectors. The free-rider vectors are Pareto inferior to each other due to the symmetry as follows:

$$U_i(\mathrm{T}, \mathrm{F}) \leq U_i(\mathrm{F}, \mathrm{T}) \leftrightarrow U_j(\mathrm{T}, \mathrm{F}) \geq U_j(\mathrm{F}, \mathrm{T}).$$

If a player increases its payoff by moving from one free-rider profile to another, the other player's payoff decreases. The free-rider vectors are Pareto equilibria if the all-T vector is Pareto inferior to the free-rider vectors. Since $U_1(\mathrm{T}, \mathrm{F}) < U_1(\mathrm{T}, \mathrm{T})$ and $U_2(\mathrm{F}, \mathrm{T}) < U_2(\mathrm{T}, \mathrm{T})$,

$$\mathbf{u}(\mathrm{T}, \mathrm{T}) \in \Phi_{\mathrm{I}}(\mathrm{T}, \mathrm{F}) \leftrightarrow U_2(\mathrm{T}, \mathrm{T}) < U_2(\mathrm{T}, \mathrm{F}),$$
$$\mathbf{u}(\mathrm{T}, \mathrm{T}) \in \Phi_{\mathrm{I}}(\mathrm{F}, \mathrm{T}) \leftrightarrow U_1(\mathrm{T}, \mathrm{T}) < U_1(\mathrm{F}, \mathrm{T}).$$

The two inequalities are equivalent, and hence the interval of the free-rider Pareto equilibria is

$$\mathcal{P}(\mathrm{T}, \mathrm{F}) \leftrightarrow \mathcal{P}(\mathrm{F}, \mathrm{T}) \leftrightarrow P \in [0, P_R) \cap [0, 1].$$

The all-T vector is Pareto inferior to the all-F vector if $U_i(\mathrm{T}, \mathrm{T}) \leq U_i(\mathrm{F}, \mathrm{F})$. When this inequality holds, the free-rider vectors are also Pareto inferior to the all-F vector because $U_1(\mathrm{T}, \mathrm{F}) < U_1(\mathrm{T}, \mathrm{T})$ and $U_2(\mathrm{F}, \mathrm{T}) < U_2(\mathrm{T}, \mathrm{T})$. Hence, the interval of the all-F Pareto equilibrium is

$$\mathcal{P}(\mathrm{F}, \mathrm{F}) \leftrightarrow P \in [0, P_L^*].$$

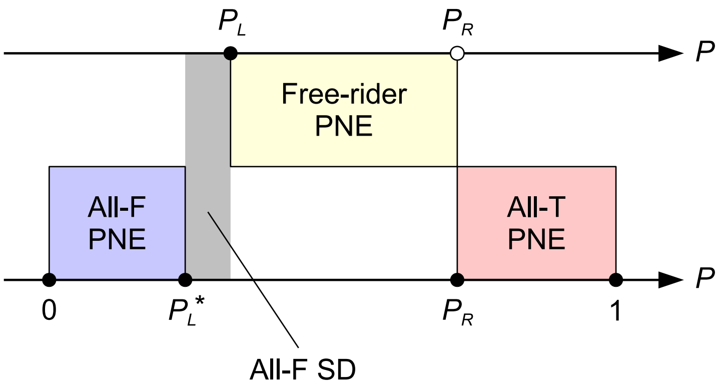

Figure S2: Distribution of Pareto efficient Nash equilibria (PNE) in the ITN use game. At the PNE, the Nash equilibrium and the Pareto equilibrium hold simultaneously. If P lies in the gap between P_L^* and P_L, the players are attracted to the all-F social dilemma (SD) at which the all-F profile is a Nash equilibrium but is not a Pareto equilibrium.

3 Distribution of Pareto efficient Nash equilibria

The Pareto efficient Nash equilibrium (PNE) is the set of profiles at which the Nash equilibrium and the Pareto equilibrium are achieved simultaneously. At the PNE, each player's best response to the other player is also desirable for the public welfare, that is, for the expected payoff vector. For each profile, the interval of the PNE is obtained by taking the intersection of the intervals of the two equilibria. Let $\mathcal{S}(\sigma_1, \sigma_2)$ be the proposition that the profile (σ_1, σ_2) is a PNE. The PNE is defined as

$$\mathcal{S}(\sigma_1, \sigma_2) \leftrightarrow \mathcal{N}(\sigma_1, \sigma_2) \wedge \mathcal{P}(\sigma_1, \sigma_2).$$

The intervals of the all-T, free-rider, and all-F PNE are written as follows:

$$\begin{aligned} \mathcal{S}(\mathrm{T}, \mathrm{T}) &\leftrightarrow P \in [P_R, 1] \cap [P_L^*, 1] = [P_R, 1], \\ \mathcal{S}(\mathrm{T}, \mathrm{F}) \leftrightarrow \mathcal{S}(\mathrm{F}, \mathrm{T}) &\leftrightarrow P \in [P_L, P_R] \cap [0, P_R) \cap [0, 1] = [P_L, P_R) \cap [0, 1], \\ \mathcal{S}(\mathrm{F}, \mathrm{F}) &\leftrightarrow P \in [0, P_L] \cap [0, P_L^*] = [0, P_L^*]. \end{aligned}$$

The all-T Nash equilibrium is Pareto efficient everywhere. The free-rider Nash equilibria are also Pareto efficient almost everywhere. The all-F Nash equilibrium is Pareto efficient except for the interval $(P_L^*, P_L]$. This gap is covered by the all-F SD defined as

$$\mathcal{N}(\mathrm{F}, \mathrm{F}) \wedge \neg\mathcal{P}(\mathrm{F}, \mathrm{F}),$$

where the operator $\neg$ denotes the negation of a proposition.

Figure S2 shows the distribution of PNE in the ITN use game. If $P_R \leq 1$, the domain $[0, 1]$ is covered by the four solutions: the all-F, free-rider, all-T PNE, and the all-F SD. If $P_R > 1$, the all-T PNE is crowded out of the domain $[0, 1]$. The union of the all-F PNE and the all-F SD is equivalent to the all-F Nash equilibrium. Figure 3 in the main manuscript illustrates the PNE for different parameter combinations.