**Table S1:** Beryllium-10 exposure dates from the 11 moraines treated in this study, plus best-fit model parameter estimates.

| Sample | Apparent age (ka) | 1σ measure. uncertainty (ka) | Summary statistic | Value | Degradation best fit Parameter | Best value | Search range | Inheritance best fit Parameter | Best value | Search range |
|---|---|---|---|---|---|---|---|---|---|---|
| G-I moraines (Kelly et al., 2008; recalculated by M. Kelly with uplift correction but no erosion correction) | | | | | | | | | | |
| MKG-93 | 0.832 | 0.041 | Number | 4 | Age (ka) | 0.755 | 0-1 ka | Age (ka) | 0.305 | 0-1 ka |
| MKG-94 | 0.432 | 0.075 | Skew | 0.0181 | Slope (deg) | 26.40 | 15-40 deg | Preexp. (ka) | 1.89 | 0-3 ka |
| MKG-95 | 0.723 | 0.049 | Chi2_R | 52.04 | log10(Diff., m2/yr) | 0.229 | -3 to 1 | Depth (m) | 2.13 | 0-3 m |
| MKG-97 | 0.333 | 0.024 | Mean (ka) | 0.580 | KS statistic | 0.252 | | KS statistic | 0.213 | |
| | | | Median (ka) | 0.578 | | | | | | |
| | | | Oldest (ka) | 0.832 | | | | | | |
| | | | Youngest (ka) | 0.333 | | | | | | |
| G-II moraines (Kelly et al., 2008; recalculated by M. Kelly with uplift correction but no erosion correction) | | | | | | | | | | |
| MKG-32 | 11.68 | 0.30 | Number | 15 | Age (ka) | 15.02 | 0-60 ka | Age (ka) | 11.14 | 5-55 ka |
| MKG-33 | 13.46 | 0.31 | Skew | 3.26 | Slope (deg) | 33.62 | 15-40 deg | Preexp. (ka) | 16.57 | 0-150 ka |
| MKG-35 | 15.27 | 0.33 | Chi2_R | 208.41 | log10(Diff., m2/yr) | -2.377 | -3 to 1 | Depth (m) | 2.51 | 0-5 m |
| MKG-36 | 12.66 | 0.42 | Mean (ka) | 15.77 | KS statistic | 0.258 | | KS statistic | 0.100 | |
| MKG-89 | 11.87 | 0.35 | Median (ka) | 12.77 | | | | | | |
| MKG-90 | 14.97 | 0.38 | Oldest (ka) | 52.55 | | | | | | |
| MKG-91 | 19.14 | 0.47 | Youngest (ka) | 11.19 | | | | | | |
| MKG-92 | 12.27 | 0.33 | | | | | | | | |
| MKG-98 | 12.97 | 0.44 | | | | | | | | |
| MKG-99 | 52.55 | 0.92 | | | | | | | | |
| MKG-100 | 12.77 | 0.57 | | | | | | | | |
| MKG-101 | 11.26 | 0.46 | | | | | | | | |
| MKG-102 | 12.99 | 0.31 | | | | | | | | |
| MKG-103 | 11.51 | 0.29 | | | | | | | | |
| MKG-104 | 11.19 | 0.28 | | | | | | | | |
| G-III moraines (Kelly et al., 2008; recalculated by M. Kelly with uplift correction but no erosion correction) | | | | | | | | | | |
| MKG-11 | 14.17 | 0.48 | Number | 12 | Age (ka) | 27.15 | 5-70 ka | Age (ka) | 10.62 | 5-70 ka |
| MKG-12 | 41.34 | 0.96 | Skew | 1.89 | Slope (deg) | 16.59 | 15-40 deg | Preexp. (ka) | 59.15 | 0-200 ka |
| MKG-13 | 19.77 | 0.59 | Chi2_R | 502.37 | log10(Diff., m2/yr) | -0.7414 | -3 to 1 | Depth (m) | 1.74 | 0-5 m |
| MKG-14 | 21.04 | 0.49 | Mean (ka) | 24.65 | KS statistic | 0.167 | | KS statistic | 0.107 | |
| MKG-15 | 12.54 | 0.29 | Median (ka) | 20.41 | | | | | | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| MKG-20 | 14.03 | 0.36 | | Oldest (ka) | 66.73 | | | | | | |
| MKG-21 | 25.51 | 1.08 | | Youngest (ka) | 12.54 | | | | | | |
| MKG-22 | 14.63 | 0.34 | | | | | | | | | |
| MKG-24 | 23.21 | 0.80 | | | | | | | | | |
| MKG-25 | 66.73 | 1.60 | | | | | | | | | |
| MKG-26 | 26.31 | 0.51 | | | | | | | | | |
| MKG-30 | 16.49 | 0.39 | | | | | | | | | |
| G-IV moraines (Kelly et al., 2008; recalculated by M. Kelly with uplift correction but no erosion correction) | | | | | | | | | | | |
| MKG-07 | 28.52 | 0.66 | | Number | 7 | Age (ka) | 50.71 | 20-120 ka | | Age (ka) | 24.69 | 20-120 ka |
| MKG-08 | 43.09 | 1.02 | | Skew | 1.59 | Slope (deg) | 25.28 | 15-40 deg | | Preexp. (ka) | 271.45 | 0-300 ka |
| MKG-09 | 27.74 | 0.94 | | Chi2_R | 334.11 | log10(Diff., m2/yr) | -1.254 | -3 to 1 | | Depth (m) | 4.08 | 0-5 m |
| MKG-16 | 101.35 | 2.23 | | Mean (ka) | 45.82 | KS statistic | 0.208 | | | KS statistic | 0.188 | |
| MKG-19 | 50.28 | 0.85 | | Median (ka) | 41.17 | | | | | | | |
| MKG-27 | 28.58 | 1.05 | | Oldest (ka) | 101.35 | | | | | | | |
| MKG-28 | 41.17 | 0.96 | | Youngest (ka) | 27.74 | | | | | | | |
| Yellowstone (Munroe et al., 2006; recalculated by Laabs et al., 2009) | | | | | | | | | | | |
| YS-3 | 17.20 | 0.80 | | Number | 7 | Age (ka) | 17.90 | 5-25 ka | | Age (ka) | 12.70 | 5-25 ka |
| YS-6 | 13.00 | 0.50 | | Skew | -0.49 | Slope (deg) | 19.01 | 15-40 deg | | Preexp. (ka) | 22.20 | 0-50 ka |
| YS-7 | 14.90 | 0.60 | | Chi2_R | 21.27 | log10(Diff., m2/yr) | 0.3108 | -3 to 1 | | Depth (m) | 2.01 | 0-5 m |
| YS-8 | 18.70 | 0.70 | | Mean (ka) | 15.77 | KS statistic | 0.156 | | | KS statistic | 0.242 | |
| YS-9 | 17.20 | 0.70 | | Median (ka) | 17.20 | | | | | | | |
| YS-10 | 17.60 | 0.70 | | Oldest (ka) | 18.70 | | | | | | | |
| YS-11 | 11.80 | 0.50 | | Youngest (ka) | 11.80 | | | | | | | |
| Lake Fork, proximal ridge (Munroe et al., 2006; recalculated by Laabs et al., 2009) | | | | | | | | | | | |
| LFR-1 | 16.70 | 0.70 | | Number | 7 | | | | | | |
| LFR-3 | 17.60 | 1.00 | | Skew | 0.90 | | | | | | |
| LFR-4 | 16.10 | 0.80 | | Chi2_R | 0.29 | | | | | | |
| LFR-5 | 16.30 | 1.50 | | Mean (ka) | 16.63 | | | | | | |
| LFR-6 | 16.20 | 0.90 | | Median (ka) | 16.70 | | | | | | |
| LFR-7 | 16.80 | 0.80 | | Oldest (ka) | 17.60 | | | | | | |
| LFR-9 | 16.70 | 0.80 | | Youngest (ka) | 16.10 | | | | | | |
| Lake Fork, distal ridge (Munroe et al., 2006; recalculated by Laabs et al., 2009) | | | | | | | | | | | |
| LF-RK-5 | 18.90 | 1.20 | | Number | 7 | Age (ka) | 19.09 | 5-25 ka | | Age (ka) | 16.30 | 5-25 ka |
| LF04-1 | 16.40 | 0.80 | | Skew | -1.49 | Slope (deg) | 37.34 | 15-40 deg | | Preexp. | 24.76 | 0-50 ka |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | (ka) | | |
| LF04-2 | 17.20 | 0.70 | Chi2_R | 27.96 | log10(Diff., m2/yr) | -2.897 | -3 to 1 | Depth (m) | 4.17 | 0-5 m |
| LF04-3 | 10.70 | 0.50 | Mean (ka) | 16.91 | KS statistic | 0.173 | | KS statistic | 0.224 | |
| LF04-4 | 19.60 | 1.00 | Median (ka) | 17.50 | | | | | | |
| LF04-5A | 18.10 | 0.80 | Oldest (ka) | 19.60 | | | | | | |
| LF04-5B | 17.50 | 0.80 | Youngest (ka) | 10.70 | | | | | | |
| Manikala M1 (Chevalier et al., 2005a; recalculated by Chevalier et al., 2011) | | | | | | | | | | |
| WG1-11 | 36.77 | 3.20 | Number | 9 | Age (ka) | 39.77 | 15-50 ka | Age (ka) | 33.03 | 15-50 ka |
| WG1-12 | 35.82 | 3.11 | Skew | -1.05 | Slope (deg) | 36.24 | 15-40 deg | Preexp. (ka) | 57.15 | 0-100 ka |
| WG1-13 | 39.69 | 3.53 | Chi2_R | 16.54 | log10(Diff., m2/yr) | -0.8723 | -3 to 1 | Depth (m) | 3.95 | 0-5 m |
| WG1-14 | 20.04 | 1.77 | Mean (ka) | 33.65 | KS statistic | 0.146 | | KS statistic | 0.222 | |
| WG1-15 | 42.01 | 3.73 | Median (ka) | 36.66 | | | | | | |
| WG1-16 | 19.87 | 1.76 | Oldest (ka) | 42.01 | | | | | | |
| WG1-17 | 36.66 | 3.31 | Youngest (ka) | 19.87 | | | | | | |
| WG1-18 | 38.04 | 3.39 | | | | | | | | |
| WG1-19 | 33.97 | 2.99 | | | | | | | | |
| Manikala M2W (Chevalier et al., 2005a; recalculated by Chevalier et al., 2011) | | | | | | | | | | |
| WG1-20 | 143.48 | 12.68 | Number | 8 | Age (ka) | 143.6 | 50-200 ka | Age (ka) | 100.5 | 50-200 ka |
| WG1-21 | 127.23 | 11.37 | Skew | 0.23 | Slope (deg) | 39.99 | 15-40 deg | Preexp. (ka) | 77.85 | 0-150 ka |
| WG1-22 | 95.80 | 8.45 | Chi2_R | 3.90 | log10(Diff., m2/yr) | -1.747 | -3 to 1 | Depth (m) | 0.62 | 0-5 m |
| WG1-23 | 130.51 | 11.43 | Mean (ka) | 126.74 | KS statistic | 0.213 | | KS statistic | 0.175 | |
| WG1-24 | 163.34 | 14.53 | Median (ka) | 126.53 | | | | | | |
| WG1-25 | 125.83 | 11.07 | Oldest (ka) | 163.34 | | | | | | |
| WG1-26 | 105.00 | 9.20 | Youngest (ka) | 95.80 | | | | | | |
| WG1-27 | 122.75 | 10.92 | | | | | | | | |
| Manikala M2E (Chevalier et al., 2005a; recalculated by Chevalier et al., 2011) | | | | | | | | | | |
| WG1-1 | 157.35 | 14.07 | Number | 10 | Age (ka) | 227.4 | 100-300 ka | Age (ka) | 110.5 | 100-300 ka |
| WG1-2 | 191.36 | 16.99 | Skew | 0.72 | Slope (deg) | 26.24 | 15-40 deg | Preexp. (ka) | 294.0 | 0-400 ka |
| WG1-3 | 282.25 | 25.76 | Chi2_R | 12.85 | log10(Diff., m2/yr) | -2.657 | -3 to 1 | Depth (m) | 1.19 | 0-5 m |
| WG1-4 | 226.43 | 20.61 | Mean (ka) | 186.87 | KS statistic | 0.200 | | KS statistic | 0.120 | |
| WG1-5 | 161.72 | 14.38 | Median (ka) | 167.99 | | | | | | |
| WG1-6 | 129.24 | 11.78 | Oldest (ka) | 287.02 | | | | | | |
| WG1-7 | 287.02 | 26.12 | Youngest | 125.49 | | | | | | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | (ka) | | | | | | | |
| WG1-8 | 133.59 | 11.86 | | | | | | | | | |
| WG1-9 | 125.49 | 10.98 | | | | | | | | | |
| WG1-10 | 174.26 | 15.42 | | | | | | | | | |
| | | | | | | | | | | | |
| Waiho Loop (Barrows et al., 2007; recalculated using the Balco calculator and the Putnam et al., 2009, calibration) | | | | | | | | | | | |
| WH-01B | 12.82 | 0.48 | | Number | 8 | | Age (ka) | 12.98 | 5-15 ka | Age (ka) | 7.40 | 5-15 ka |
| WH-02 | 12.03 | 0.81 | | Skew | -0.85 | | Slope (deg) | 39.37 | 15-40 deg | Preexp. (ka) | 9.54 | 0-30 ka |
| WH-03 | 12.32 | 0.44 | | Chi2_R | 26.36 | | log10(Diff., m2/yr) | -0.6563 | -3 to 1 | Depth (m) | 0.37 | 0-5 m |
| WH-04B | 11.11 | 0.55 | | Mean (ka) | 10.82 | | KS statistic | 0.125 | | KS statistic | 0.177 | |
| WH-05 | 10.60 | 0.48 | | Median (ka) | 11.57 | | | | | | | |
| WH-08A | 7.95 | 1.06 | | Oldest (ka) | 13.56 | | | | | | | |
| WH-09 | 6.15 | 0.39 | | Youngest (ka) | 6.15 | | | | | | | |
| WH-10 | 13.56 | 1.86 | | | | | | | | | | |

**Table S2:** Parameter estimates from fitting models to synthetic data sets.

**Perfect model experiments**

Moraine degradation

| *n* | 5 | 5 | 5 | 10 | 15 | 20 | 25 | (true vals) | (ranges) |
|---|---|---|---|---|---|---|---|---|---|
| Age (ka) | 20.04 | 20.07 | 20.08 | 20.02 | 20.01 | 20.05 | 20.09 | 20 | 5-25 |
| Slope (deg) | 39.09 | 35.83 | 26.40 | 36.44 | 17.56 | 33.55 | 19.98 | 34 | 15-40 |
| log10(Diffusivity) | -2.161 | -0.9989 | -0.6730 | -0.9822 | -1.344 | -1.986 | -1.464 | -2 | -3 to 1 |
| KS statistic | 0.1002 | 0.1150 | 0.1152 | 0.07520 | 0.03420 | 0.02600 | 0.02120 | -- | -- |

Inheritance

| *n* | 5 | 5 | 5 | 10 | 15 | 20 | 25 | (true vals) | (ranges) |
|---|---|---|---|---|---|---|---|---|---|
| Age (ka) | 19.08 | 19.91 | 19.98 | 20.03 | 19.88 | 20.05 | 20.11 | 20 | 0-120 |
| Max. pre. time (ka) | 80.45 | 106.5 | 101.0 | 99.8 | 96.6 | 101.1 | 104.9 | 100 | 0-150 |
| Max. pre. depth (m) | 1.62 | 2.10 | 1.98 | 2.02 | 1.93 | 2.01 | 2.11 | 2 | 0-5 |
| KS statistic | 0.1306 | 0.1052 | 0.1056 | 0.0534 | 0.040333 | 0.02940 | 0.02680 | -- | -- |

**Fitting the wrong model to the data**

Fitting the inheritance model to data sets biased by degradation

| *n* | 5 | 5 | 5 | 10 | 15 | 20 | 25 | (true vals) | (ranges) |
|---|---|---|---|---|---|---|---|---|---|
| Age (ka) | 11.54 | 10.10 | 11.70 | 9.52 | 10.11 | 9.59 | 8.91 | 20 | 0-25 |
| Max. pre. time (ka) | 14.22 | 17.22 | 12.06 | 14.87 | 13.86 | 15.44 | 17.10 | -- | 0-50 |
| Max. pre. depth (m) | 0.61 | 0.56 | 0.39 | 0.28 | 0.29 | 0.38 | 0.43 | -- | 0-3 |
| KS statistic | 0.2000 | 0.2000 | 0.2000 | 0.1590 | 0.1433 | 0.1516 | 0.1512 | -- | -- |

Fitting the degradation model to data sets biased by inheritance

| *n* | 5 | 5 | 5 | 10 | 15 | 20 | 25 | (true vals) | (ranges) |
|---|---|---|---|---|---|---|---|---|---|
| Age (ka) | 43.17 | 43.34 | 43.59 | 46.24 | 47.46 | 48.36 | 48.63 | 20 | 0-120 |
| Slope (deg) | 21.73 | 27.48 | 19.37 | 16.55 | 18.81 | 24.20 | 23.30 | -- | 15-40 |
| log10(Diffusivity) | -1.605 | -1.755 | -1.419 | -1.214 | -1.350 | -1.497 | -1.460 | -- | -3 to 1 |
| KS statistic | 0.2000 | 0.2000 | 0.2000 | 0.2000 | 0.2000 | 0.2024 | 0.2042 | -- | -- |

**Supplement: Matlab code**

To accompany Applegate et al., Estimating moraine ages from cosmogenic exposure dates: Matching geomorphic process models to exposure dates from single landforms. For publication in Quaternary Research.

This file contains most of the MATLAB code used to generate the results shown in the accompanying manuscript. We did not write the Differential Evolution code, so it is not part of this package.

The model files are

-- the model files (bb_deg.m and bb_inh.m, for degradation and inheritance, respectively), and

-- two files called by these models (m_diffusion.m and data_model_comparison.m).

We also included Matlab scripts for calculating the weighted mean (wtdmean.m) and the reduced chi-squared statistic (chi2r.m) for different data sets.

To fit a model to a particular data set, open the data_model_comparison.m file and uncomment the data set you wish to fit (be sure that all other data sets are commented out!). Now, call the bb_deg.m or bb_inh.m functions with different parameter combinations until you find a minimum value for the KS statistic, which is the output from each run of bb_deg.m or bb_inh.m. The syntax is

```
KS_stat = bb_deg([moraine_age initial_slope log10_diffusivity])
```

or

```
KS_stat = bb_inh([moraine_age preexposure_time preexposure_depth])
```

The bb_deg.m and bb_inh.m codes are based on our earlier Geoscientific Model Development paper. You can find more complete documentation online at http://www.geosci-model-dev.net/3/293/2010/gmd-3-293-2010.html (accessed 7 June 2011).

```
function ksstat = bb_deg(FVr_temp)

% bb_deg.m
%
% Evaluates probability distributions of cosmogenic exposure dates on a
% degrading moraine.  Uses an analytical solution for moraine degradation
% developed by Dr. Nathan Urban.  Code written by Patrick Applegate.

% This code was written carefully and has been checked for obvious errors.
% However, no warranty of any kind is implied.  The code may not even run
% on your system.  The output from the code should not be trusted without
% testing.
%
% Please give proper credit if using this code in research and teaching.
% Derivative works based on this code should include a reference to the
% original paper.

% Clear all variables, commands, and figures.  Set figures to dock
% automatically.
% clear all
% close all
% clc
% set(0,'DefaultFigureWindowStyle','docked')

moraine_age = FVr_temp(1);
initial_slope = FVr_temp(2);
k = 10^ (FVr_temp(3));

% Define parameters that will be tuned during model inversion.
% moraine_age = 11.6;       % ka (10^ 3 yr); true age of moraine
initial_height = 50;        % m; initial height of moraine
% initial_slope = 34;       % degrees; initial moraine slope angle (Hallet
%                           % and Putkonen (1994) assume 31 degrees;
%                           % Putkonen and Swanson (2003) use 34 degrees)
% k = 0.5* 10^ -2;          % sq. m/ yr; topographic diffusion coefficient

% Define other geomorphic parameters that are not part of the inversion.
erosion_rate = 0.0;         % mm/ ka; erosion rate of exposed boulders
boulder_height = 1.0;       % m; observed height of boulders when sampled
rho_rock = 2.6;             % g/ cm^ 3; density of boulders
rho_till = 2.0;             % g/ cm^ 3; density of till matrix

% Define nuclide production parameters.
P_spall = 4.97;             % atoms/ g/ yr; surface production rate due to
                            % spallation (get this from the CRONUS online
                            % calculator described in Balco et al., 2008;
                            % assumed to be constant over the lifetime of
                            % the moraine)
P_mu = 0.133;               % atoms/ g/ yr; surface production rate due to
                            % muons (also get this from the CRONUS
                            % calculator)
decay_const = 4.62* 10^ -7; % yr^ -1; nuclear decay constant of nuclide of
                            % interest (4.62* 10^ -7 for 10Be, following
                            % Balco et al., 2008, and refs therein)
P_slhl = [5 0.09 ...        % atoms/ g/ yr; sea level, high-latitude
```

```
    0.02 0.02];             % production rates of different cosmic ray
                            % flux components, following Granger and
                            % Muzikar (2001); for 10Be, about
                            % [5 0.09 0.02 0.02]
att_length = [160 738 ...   % g/ sq. cm; effective attenuation lengths of
    2688 4360];             % exponential components of Granger and Muzikar
                            % (2001) production-as-a-function-of-depth
                            % parameterization; for 10Be and 26Al, about
                            % [160 738 2688 4360]

% Define model parameters.
num_boulders = 5* 10^ 3;    % number of randomly generated synthetic
                            % boulders (at least 10^ 3; bigger numbers
                            % yield more consistent results, but the model
                            % will take more time to run)
time_step = 100;            % yr; time step during post-depositional
                            % period (25 yr works well; small values
                            % increase the accuracy of the calculation,
                            % but also cause the code to run more
                            % slowly)
bin_width = 1;              % ka; width of bins in naive age histogram

% Turn plotting on and off.
plots = 0;                  % If 1, plots the moraine profile, height of
                            % the moraine's crest as a function of time,
                            % and a histogram of the modeled exposure
                            % dates.  If 0, none of these plots are
                            % produced.
comparison_plot = 1;        % If 1, produces a plot showing the match
                            % between a cumulative distribution function of
                            % the modeled exposure dates and an observed
                            % data set.  If 0, this plot will not be
                            % produced.  The observed exposure dates are
                            % stored in the file data_model_comparison.m .

% Convert all quantities to consistent units.  All lengths should be in
% meters, slopes should be dimensionless, times should be in years, and
% masses should be in grams.
moraine_age = moraine_age* 10^ 3;               % yr
initial_slope = tand(initial_slope);            % d'less
erosion_rate = erosion_rate* 10^ -6;            % m/ yr
rho_till = rho_till* 100^ 3;                    % g/ cu. m
rho_rock = rho_rock* 100^ 3;                    % g/ cu. m
att_length = att_length* 100^ 2;                % g/ sq. m

% Scale production rates to site.
P_surf(1) = P_spall; % atoms/ g/ yr
P_surf(2: 4) = P_slhl(2: 4)* P_mu/ sum(P_slhl(2: 4));

% Determine length scales for nuclide production.
L_till = att_length/ rho_till; % m
L_rock = att_length/ rho_rock; % m

% Determine height of moraine crest as a function of time and initial
% and final moraine profiles.
```

```
[times, crest_height, distances, initial_profile, ...
    final_profile] = m_diffusion(initial_height, initial_slope, k, ...
    moraine_age, time_step);

% Establish the initial depth for each boulder.
final_height = min(crest_height); % m
max_depth = initial_height- final_height- boulder_height; % m
% initial_depth = max_depth* rand(1, num_boulders); % m
initial_depth = max_depth* lhsdesign(num_boulders, 1, 'criterion', ...
    'maximin'); %, 'smooth', 'off');

% Determine the thickness of the erodible shell on each boulder.  This
% thickness depends on the time that each boulder's upper surface is
% higher than the crest of the moraine, and on the erosion rate.
shell_thick = zeros(1, num_boulders); % m
if erosion_rate > 0; % don't do these steps if erosion is nil
    for count1 = 1: 1: num_boulders;
        boulder_top = initial_height- initial_depth(count1); % m
        yn = 0;
        count2 = 1;
        while yn == 0;
            if crest_height(count2) <= boulder_top;
                exposure_time = moraine_age- times(count2); % yr
                shell_thick(count1) = exposure_time* erosion_rate;
                yn = 1;
            end
            count2 = count2+ 1;
        end
    end
    initial_shell_thick = shell_thick; % m
end

% Step through time, tracking the nuclide concentration in each boulder.
boulder_conc = zeros(1, num_boulders); % atoms/ g
% num_exposed = zeros(1, numel(times));
for count1 = 2: 1: numel(times);
%     disp(['Calculating time step #', num2str(count1- 1), ' of ',...
%         num2str(numel(times)- 1), '... '])
    % Increment concentrations for nuclear decay.
    boulder_conc = boulder_conc.* exp(-decay_const* time_step);
    % Step through the list of boulders.
    for count2 = 1: 1: num_boulders;
        depth = crest_height(count1)- ...
            (initial_height- initial_depth(count2)); % m
        % If the boulder is at the surface,
        if depth <= 0;
            P_sample = P_surf.* exp(-shell_thick(count2)./ L_rock);
            shell_thick(count2) = shell_thick(count2)- ...
                erosion_rate* time_step;
            % num_exposed(count1) = num_exposed(count1)+ 1;
        % Otherwise,
        else
            P_till = P_surf.* exp(-depth./ L_till);
            P_sample = P_till.* exp(-shell_thick(count2)./ L_rock);
        end
```

```
        % Increment the concentration in the boulder by the production rate
        % during this time step.
        boulder_conc(count2) = boulder_conc(count2)+ ...
            sum(P_sample)* time_step;
    end
end

% Calculate the apparent exposure time for each boulder.
naive_age = -decay_const^ -1* ...
    log(1- ((boulder_conc.* decay_const)./ (P_spall+ P_mu))); % yr

% For ease of plotting, convert variables with a time dimension to ka
% (10^3 yr).
times = times/ 10^ 3;
naive_age = naive_age/ 10^ 3;
moraine_age = moraine_age/ 10^ 3;

if plots == 1;
    % Plot the initial (dotted) and final (solid) moraine profiles.
    figure
    plot(distances, initial_profile, 'k--', 'LineWidth', 1.5)
    axis square
    hold on
    plot(distances, final_profile, 'k', 'LineWidth', 1.5)
    xlabel('Distance from moraine crest (m)', 'FontSize', 16, ...
        'FontWeight', 'bold')
    ylabel('Height (m)', 'FontSize', 16, ...
        'FontWeight', 'bold')
    h_leg = legend('Initial profile', 'Final profile');
    legend('boxoff')
    set(h_leg, 'FontSize', 14)
    set(gca, 'FontSize', 14)
    set(gca, 'LineWidth', 1)
%    set(gca, 'Box', 'off')

    % Plot moraine height as a function of time.
    figure
    plot(times, crest_height, 'k', 'LineWidth', 1.5)
    axis square
    xlabel('Elapsed time (ka)', 'FontSize', 16, 'FontWeight', 'bold')
    ylabel('Crest height (m)', 'FontSize', 16, ...
        'FontWeight', 'bold')
    set(gca, 'FontSize', 14)
    set(gca, 'LineWidth', 1)
%    set(gca, 'Box', 'off')

    % Histogram the apparent ages given by the modeled boulders.
    figure
    nbins = ceil((max(naive_age)- min(naive_age))/ bin_width);
    hist(naive_age, nbins)
    axis square
    hold on
    xlabel('Apparent age (ka)', 'FontSize', 16, 'FontWeight', 'bold')
    ylabel('Number of boulders', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize', 14)
```

```
    set(gca, 'LineWidth', 1)
    set(gca, 'XTickMode', 'auto')
%    set(gca, 'Box', 'off')
    h = findobj(gca, 'Type', 'patch');
    set(h, 'FaceColor', 'b', 'EdgeColor', 'k')
    ylimits = get(gca, 'YLim');
    plot([moraine_age moraine_age], ylimits, 'k--', 'LineWidth', 1.5)
end

% Compare the modeled exposure dates to an observed data set.  If the
% plotting variable comparison_plot is 1, produce a plot showing the
% agreement between the model and observations in cdf-space.
ksstat = data_model_comparison(naive_age, moraine_age, comparison_plot);

% disp('.')
% beep
```

```
function ksstat = bb_inh(FVr_temp)

% bb_inh.m
%
% Evaluates probability distributions of cosmogenic exposure dates for
% boulders that contain inherited nuclides.  Code written by Patrick
% Applegate.

% This code was written carefully and has been checked for obvious errors.
% However, no warranty of any kind is implied.  The code may not even run
% on your system.  The output from the code should not be trusted without
% testing.
%
% Please give proper credit if using this code in research and teaching.
% Derivative works based on this code should include a reference to the
% original paper.

% Clear all variables, commands, and figures.  Set figures to dock
% automatically.
% clear all
% close all
% clc
% set(0,'DefaultFigureWindowStyle','docked')

moraine_age = FVr_temp(1);
max_pre_time = FVr_temp(2);
max_pre_depth = FVr_temp(3);

% Define parameters that will be tuned during model inversion.
% moraine_age = 10.0;       % ka (10^ 3 yr); true age of moraine
% max_pre_time = 52;        % ka; maximum time that any individual boulder
%                           % had to acquire inherited nuclides
% max_pre_depth = 1.9;      % m; maximum depth of sample point on any
%                           % boulder during predepositional exposure time

% Define other geomorphic parameters that are not part of the inversion.
pre_slope = 0;              % degrees; slope of surface from which boulders
                            % are derived
erosion_rate = 0.0;         % mm/ ka; erosion rate of boulders on moraine
rho_rock = 2.6;             % g/ cm^ 3; density of boulders
rho_over = 2.0;             % g/ cm^ 3; density of material overlying
                            % boulders during predepositional exposure time

% Define nuclide production parameters.
P_spall = 4.97;             % atoms/ g/ yr; surface production rate due to
                            % spallation (get this from the CRONUS online
                            % calculator described in Balco et al., 2008;
                            % assumed to be constant over the lifetime of
                            % the moraine)
P_mu = 0.133;               % atoms/ g/ yr; surface production rate due to
                            % muons (also get this from the CRONUS
                            % calculator)
decay_const = 4.62* 10^ -7; % yr^ -1; nuclear decay constant of nuclide of
                            % interest (4.62* 10^ -7 for 10Be, following
                            % Balco et al., 2008, and refs therein)
```

```
P_slhl = [4.97 0.09 ...      % atoms/ g/ yr; sea level, high-latitude
    0.02 0.02];              % production rates of different cosmic ray
                             % flux components, following Granger and
                             % Muzikar (2001); for 10Be, about
                             % [4.97 0.09 0.02 0.02]
att_length = [160 738 ...    % g/ sq. cm; effective attenuation lengths of
    2688 4360];              % exponential components of Granger and Muzikar
                             % (2001) production-as-a-function-of-depth
                             % parameterization; for 10Be and 26Al, about
                             % [160 738 2688 4360]

% Define model parameters.
num_boulders = 5* 10^ 3;     % number of randomly generated synthetic
                             % boulders (at least 10^ 3; bigger numbers
                             % yield more consistent results, but the model
                             % will take more time to run)
bin_width = 10;              % ka; width of bins in naive age histogram


% Turn plotting on and off.
plots = 0;                   % If 1, plots the moraine profile, height of
                             % the moraine's crest as a function of time,
                             % and a histogram of the modeled exposure
                             % dates.  If 0, none of these plots are
                             % produced.
comparison_plot = 1;         % If 1, produces a plot showing the match
                             % between a cumulative distribution function of
                             % the modeled exposure dates and an observed
                             % data set.  If 0, this plot will not be
                             % produced.  The observed exposure dates are
                             % stored in the file data_model_comparison.m .

% Convert all quantities to consistent units.  All lengths should be in
% meters, times should be in years, and masses should be in grams.  Note
% that pre_slope should remain in degrees -- do not reduce this angle to
% its slope equivalent.
moraine_age = moraine_age* 10^ 3;                % yr
max_pre_time = max_pre_time* 10^ 3;              % yr
erosion_rate = erosion_rate* 10^ -6;             % m/ yr
rho_over = rho_over* 100^ 3;                     % g/ cu. m
rho_rock = rho_rock* 100^ 3;                     % g/ cu. m
att_length = att_length* 100^ 2;                % g/ sq. m

% Scale production rates to site.
P_surf(1) = P_spall; % atoms/ g/ yr
P_surf(2: 4) = P_slhl(2: 4)* P_mu/ sum(P_slhl(2: 4));

% Determine length scales for nuclide production.
L_over = att_length/ rho_over; % m
L_rock = att_length/ rho_rock; % m

% Determine predepositional exposure time for each boulder, and the depth
% of the sample point on each boulder during that time.
% pre_time = max_pre_time* rand(1, num_boulders);
% pre_depth = max_pre_depth* rand(1, num_boulders);
```

```
lh = lhsdesign(num_boulders, 2, 'criterion', 'maximin');
pre_time = max_pre_time* lh(:, 1);
pre_depth = max_pre_depth* lh(:, 2);

% Calculate the final nuclide concentration in each boulder.  The
% production rate parameterization here follows Dunne et al. (1999), using
% the four-component production rate scheme described by Granger and
% Muzikar (2001).
boulder_conc = zeros(1, num_boulders);
for count1 = 1: 1: num_boulders;
    P_pre = sum(P_surf.* (1- 3.6* 10^ -6* pre_slope^ 2.64).* ...
        exp((-pre_depth(count1)./ L_over).* (1+ pre_slope^ 2/ 5000)));
    C_pre = (P_pre/ decay_const)* ...
        (1- exp(-decay_const* pre_time(count1)));
%     boulder_conc(count1) = C_pre* exp(-decay_const* moraine_age)+ ...
%         (P_spall+ P_mu)/ (decay_const+ erosion_rate* L_over(1))* ...
%         (1- exp(-decay_const* moraine_age));
    boulder_conc(count1) = C_pre* exp(-decay_const* moraine_age)+ ...
        (P_spall+ P_mu)/ (decay_const+ erosion_rate/ L_over(1))* ...
        (1- exp(-moraine_age* (decay_const+ erosion_rate/ L_over(1))));
end

% Calculate the apparent exposure time for each boulder.
naive_age = -decay_const^ -1* ...
    log(1- ((boulder_conc.* decay_const)./ (P_spall+ P_mu))); % yr

% For ease of plotting, convert variables with a time dimension to ka
% (10^3 yr).
naive_age = naive_age/ 10^ 3;
moraine_age = moraine_age/ 10^ 3;
pre_time = pre_time/ 10^ 3;

if plots == 1;
    % Histogram the apparent ages given by the modeled boulders.
    figure
%     bins = floor(min(naive_age)): bin_width: ceil(max(naive_age));
%     bar(bins, histc(naive_age, bins), 'histc')
    nbins = ceil((max(naive_age)- min(naive_age))/ bin_width);
    hist(naive_age, nbins)
    axis square
    xlabel('Apparent age (ka)', 'FontSize', 16, 'FontWeight', 'bold')
    ylabel('Number of boulders', 'FontSize', 16, 'FontWeight', 'bold')
    set(gca, 'FontSize', 14)
    set(gca, 'LineWidth', 1)
    set(gca, 'XTickMode', 'auto')
%     set(gca, 'Box', 'off')
    h = findobj(gca, 'Type', 'patch');
    set(h, 'FaceColor', 'b', 'EdgeColor', 'k')
    hold on
    ylimits = get(gca, 'YLim');
    plot([moraine_age moraine_age], ylimits, 'k--', 'LineWidth', 1.5)
end

% Compare the modeled exposure dates to an observed data set.  If the
% plotting variable comparison_plot is 1, produce a plot showing the
```

```
% agreement between the model and observations in cdf-space.
ksstat = data_model_comparison(naive_age, moraine_age, comparison_plot);

% disp('.')
% beep
```

```
function [times, crest_height, distances, initial_profile, ...
    final_profile] = m_diffusion(initial_height, initial_slope, k, ...
    moraine_age, time_step)

% m_diffusion.m
%
% Calculates the height of a moraine's crest as a function of time, plus
% the final topographic profile of the moraine.  Assumes a "sawtooth"
% initial profile.  Based on an analytical solution developed by Dr. Nathan
% Urban.
%
% Syntax: [times, crest_height, distances, initial_profile, ...
%    final_profile] = m_diffusion(initial_height, initial_slope, k, ...
%    moraine_age, time_step)
% times, vector of elapsed time values (yr)
% crest_height, height of moraine crest as a function of the values in the
%   vector times (m)
% distances, vector of distance from the moraine crest (m)
% initial_profile, height of moraine as a function of the values in the
%   vector distances (m)
% final_profile, height of moraine as a function of the values in the
%   vector distances (m)
% initial_height, initial height of the moraine (m)
% initial_slope, initial slope of the moraine sides (d'less)
% k, topographic diffusion coefficient (sq. m/ yr)
% moraine_age, assumed age of moraine (yr)
% time_step, interval between calculations of moraine height (yr)

% Set model variables.
length_step = 2;            % m; space step (1-2 m is best)
length_factor = 1.5;        % profile length factor (at least 2; not
                            % important, except for large, old moraines)

% Establish plotting variables times and distances.
L = initial_height/ initial_slope; % m; half-width of the moraine's base
times = 0: time_step: moraine_age; % yr
distances = 0: length_step: (length_factor* L); % m

% Calculate height of moraine as a function of time.
crest_height = zeros(1, numel(times)); % m
h0 = initial_height; % m
for count1 = 1: 1: numel(times);
    t = times(count1);
    crest_height(count1) = (h0/ L)* ((2* sqrt(k* t)/ sqrt(pi))* ...
        (exp(-L^ 2/ (4* k* t))- 1)+ ...
        L* erf(L/ (2* sqrt(k* t))));
end

% Calculate initial moraine profile as a function of distance from the
% moraine's crest.
initial_profile = zeros(1, numel(distances)); % m
for count1 = 1: 1: numel(distances);
    initial_profile(count1) = initial_height- (count1- 1)* ...
        length_step* initial_slope;
    if initial_profile(count1) < 0;
```

```
        initial_profile(count1) = 0;
    end
end

% Calculate final moraine profile as a function of distance from the
% moraine's crest.
final_profile = zeros(1, numel(distances)); % m
h0 = initial_height; % m
t = moraine_age; % yr
for count1 = 1: 1: numel(distances);
    x = distances(count1); % m
    z1 = exp(-(L+ x)^ 2/ (4* k* t))- ...
        2* exp(-x^ 2/ (4* k* t))+ ...
        exp(-(L- x)^ 2/ (4* k* t));
    z2 = (L+ x)* erf((L+ x)/ (2* sqrt(k* t)))- ...
        2* x* erf(x/ (2* sqrt(k* t)))+ ...
        (L- x)* erf((L- x)/ (2* sqrt(k* t)));
    final_profile(count1) = (h0/ (2* L))* ...
        ((2* sqrt(k* t)/ sqrt(pi))* z1+ z2);
end
```

```
function ksstat = data_model_comparison(naive_age, moraine_age, ...
    comparison_plot)

% The data set to compare goes in the variable Be_dates.  Column 1 of
% Be_dates must contain the central estimate of the sample exposure time;
% column 2 should contain the 1-sigma uncertainties of these exposure time
% estimates.  Be_dates should be in ka, not years.

% Perfect model experiment test data set.  Generated by the degradation
% model with parameters moraine_age = 20 ka, initial_height = 50 m,
% initial_slope = 34 deg., k = 10^-2 m^2/ yr, time_step = 100 yr.  Selected
% from a modeled population of 10^5 synthetic boulders.  Uncertainties are
% 3%.
% load perfect_model_expts/deg_test_5
% load perfect_model_expts/deg_test_10
% load perfect_model_expts/deg_test_15
% load perfect_model_expts/deg_test_20
% load perfect_model_expts/deg_test_25

% Perfect model experiment test data set.  Generated by the inheritance
% model with parameters moraine age = 20 ka, max predepositional exposure
% time = 100 ka, max predepositional burial depth = 2.0 m.  Selected
% from a modeled population of 10^5 synthetic boulders.  Uncertainties are
% 3%.
% load perfect_model_expts/inh_test_5
% load perfect_model_expts/inh_test_10
% load perfect_model_expts/inh_test_15
% load perfect_model_expts/inh_test_20
% load perfect_model_expts/inh_test_25

% 10Be dates from the Manikala M1 moraine (Chevalier et al., 2005), as
% recalculated by Chevalier et al. (2011) with the Lal/Stone
% time-independent scaling model
% Be_dates = [36.77, 3.20;
%             35.82, 3.11;
%             39.69, 3.53;
%             20.04, 1.77;
%             42.01, 3.73;
%             19.87, 1.76;
%             36.66, 3.31;
%             38.04, 3.39;
%             33.97, 2.99];

% 10Be dates from the Manikala M2W moraine (Chevalier et al., 2005), as
% recalculated by Chevalier et al. (2011) with the Lal/Stone
% time-independent scaling model
% Be_dates = [143.48, 12.68;
%             127.23, 11.37;
%             95.80, 8.45;
%             130.51, 11.43;
%             163.34, 14.53;
%             125.83, 11.07;
%             105.00, 9.20;
%             122.75, 10.92];
```

```
% 10Be dates from the Manikala M2E moraine (Chevalier et al., 2005), as
% recalculated by Chevalier et al. (2011) with the Lal/Stone
% time-independent scaling model
% Be_dates = [157.35, 14.07;
%             191.36, 16.99;
%             282.25, 25.76;
%             226.43, 20.61;
%             161.72, 14.38;
%             129.24, 11.78;
%             287.02, 26.12;
%             133.59, 11.86;
%             125.49, 10.98;
%             174.26, 15.42];

% 10Be dates from Laabs et al. (2009), outer Lake Fork moraine, Lal/
% Stone scaling scheme
% Be_dates = [18.90, 1.20;
%             16.40, 0.80;
%             17.20, 0.70;
%             10.70, 0.50;
%             19.60, 1.00;
%             18.10, 0.80;
%             17.50, 0.80];

% 10Be dates from Laabs et al. (2009), Yellowstone moraine, Lal/
% Stone scaling scheme
% Be_dates = [17.20, 0.80;
%             13.00, 0.50;
%             14.90, 0.60;
%             18.70, 0.70;
%             17.20, 0.70;
%             17.60, 0.70;
%             11.80, 0.50];

% 10Be dates from the Waiho Loop moraine, Barrows et al. (2007),
% recalculated using the CRONUS online calculator and the Putnam et al.
% (2010) New Zealand calibration
% Be_dates = [12.82, 0.48;
%             12.03, 0.81;
%             12.32, 0.44;
%             11.11, 0.55;
%             10.60, 0.48;
%             7.95, 1.06;
%             6.15, 0.39;
%             13.56, 1.86];

% 10Be dates from Kelly et al.'s G-I moraines, Lal/ Stone scaling scheme
% with uplift correction (these numbers supplied by M. Kelly)
% Be_dates = [0.832, 0.041;
%             0.432, 0.075;
%             0.723, 0.049;
%             0.333, 0.024];

% 10Be dates from Kelly et al.'s G-II moraines, Lal/ Stone scaling scheme
% with uplift correction (these numbers supplied by M. Kelly)
```

```
% Be_dates = [11.68, 0.30;
%             13.46, 0.31;
%             15.27, 0.33;
%             12.66, 0.42;
%             11.87, 0.35;
%             14.97, 0.38;
%             19.14, 0.47;
%             12.27, 0.33;
%             12.97, 0.44;
%             52.55, 0.92;
%             12.77, 0.57;
%             11.26, 0.46;
%             12.99, 0.31;
%             11.51, 0.29;
%             11.19, 0.28];

% 10Be dates from Kelly et al.'s G-III moraines, Lal/ Stone scaling scheme
% with uplift correction (these numbers supplied by M. Kelly)
% Be_dates = [14.17, 0.48;
%             41.34, 0.96;
%             19.77, 0.59;
%             21.04, 0.49;
%             12.54, 0.29;
%             14.03, 0.36;
%             25.51, 1.08;
%             14.63, 0.34;
%             23.21, 0.80;
%             66.73, 1.60;
%             26.31, 0.51;
%             16.49, 0.39];

% 10Be dates from Kelly et al.'s G-IV moraines, Lal/ Stone scaling scheme
% with uplift correction (these numbers supplied by M. Kelly)
Be_dates = [28.52, 0.66;
            43.09, 1.02;
            27.74, 0.94;
            101.35, 2.23;
            50.28, 0.85;
            28.58, 1.05;
            41.17, 0.96];

% Sort the dates.
Be_dates = sortrows(Be_dates, 1);

% If desired, create a plot showing the agreement between the data and the
% model.
if comparison_plot == 1;

    close

    % Plot cumulative density function of apparent exposure dates.
    figure
    cdfplot(naive_age)
    set(findobj('Type', 'line'), 'LineWidth', 1.25)
    % axis square
```

```
    hold on

    % Plot observed exposure dates with error bars.
    n = numel(Be_dates(:, 1));
    y = (0.5* 1/ n: 1/ n: 1- 0.5* 1/ n);
    for count1 = 1: 1: n;
        plot(Be_dates(count1, 1), y(count1), 'r.', 'MarkerSize', 12);
        x = [(Be_dates(count1, 1)- Be_dates(count1, 2)), ...
            (Be_dates(count1, 1)+ Be_dates(count1, 2))];
        plot(x, [y(count1) y(count1)], 'r-', 'LineWidth', 1)
    end

    % Plot a vertical, dashed line indicating the assumed true age of the
    % moraine.
    x2 = [moraine_age moraine_age];
    y2 = [0 1.1];
    plot (x2, y2, 'k--', 'LineWidth', 1)

    % Label the plot.
    xlabel('Apparent age (ka)', 'FontSize', 16, 'FontWeight', 'bold')
    ylabel('Cumulative density', 'FontSize', 16, 'FontWeight', 'bold')
    title(' ')
    range = [min(Be_dates(:, 1)) max(Be_dates(:, 1))];
    axis([(range(1)- 0.5* (range(2)- range(1))) ...
    (range(2)+ 0.5* (range(2)- range(1))) 0 1])
    set(gca, 'FontSize', 14)
    % set(gca, 'LineWidth', 1)
    set(gca, 'YTick', [0; 0.25; 0.5; 0.75; 1.0])
    set(gca, 'YTickLabel', {'0.00'; '0.25'; '0.50'; '0.75'; '1.00'})
    set(gca, 'Box', 'off')
    grid off

    pause(1)
end

% Calculate the value of the Kolmogorov-Smirnov test criterion for the data
% set and this run of the model.  The best-fit model run minimizes this
% criterion.
[h, p, ksstat] = kstest2(naive_age, Be_dates(:, 1));

% Enforce the additional criterion that the range of the modeled
% distribution must include all the observed exposure dates.
% if or((min(Be_dates(:, 1))<= min(naive_age)), ...
%         (max(Be_dates(:, 1)>= max(naive_age))))
%     ksstat = 2* ksstat;
% end
```

```
function output = chi2r(mu, sigma)

% Calculates the reduced chi-squared value for a data set with central
% estimates mu and uncertainties sigma.  The number of values in mu and
% sigma must be the same.

mean_date = mean(mu);

for count1 = 1: 1: numel(mu);
    x(count1) = (mu(count1)- mean_date)^ 2/ sigma(count1)^ 2;
end

output = sum(x)/ (numel(mu)- 1);
```

```
function [muprime, sigmaprime] = wtdmean(x, sigma)

% WTDMEAN Weighted mean.
% Calculates the weighted mean, and the uncertainty of that mean, for a set
% of data points having unequal uncertainties.
%
% Syntax: [muprime, sigmaprime] = wtdmean(means, stdevs)
% muprime, weighted mean
% sigmaprime, uncertainty of weighted mean
% x, vector of measurements
% sigma, vector of uncertainties of measurements
%
% N. B., means(1) corresponds to sigma(1), and so forth; numel(means) must
% equal numel(sigma).

for count1 = 1: 1: numel(x);
    numerator(count1) = x(count1)/ sigma(count1)^2;
    denominator(count1) = 1/ sigma(count1)^2;
end

muprime = sum(numerator)/ sum(denominator);
sigmaprime = sqrt(1/ sum(denominator));
```