Online Appendix

Why the West Became Wild: Informal Governance with Incomplete Networks*

By Jennifer M. Larson

World Politics doi: 10.1017/S0043887117000181

Contents

*Current version: June 20, 2017.

1 Extension: Model Setup with Varying Punishment Length

Here I reintroduce the model, extended to allow finite punishment that lasts longer than a single period. The extension requires 1) accounting for the fact that as the punishment phase unfolds, more and more people may be learning about the offense as gossip keeps spreading, and 2) making sure that players keep passing gossip, and keep updating appraisals, based on judgments made in a window of time at least as long as the punishment phase. The latter could be accomplished in many ways. Here, I assume that players assign any piece of gossip a period of relevance, that that period is as long as the length of the punishment phase, and gossip and negative judgment expire after this period.

Society is comprised of a finite set of players $N = \{1, \ldots, n\}$ with n even. In every time period $t = 1 \ldots \infty$, every player is matched with one other player uniformly at random to play a stage game. Call $\mu(i,t)$ player i's assigned match in time t. Then $Prob\{\mu(i,t) = j\} = \frac{1}{n-1}$ for all $i \neq j \in N$. Matching is independent across time periods.

In the stage game, each matched pair of players plays one round of prisoner's dilemma. Players i and j taking actions from action set $A = \{c, d\}$ earn payoffs

$$\begin{array}{cc} & \begin{array}{cc} c & d \end{array} \\ \begin{array}{c} c \\ d \end{array} & \begin{pmatrix} 1,1 & -\beta,\alpha \\ \alpha,-\beta & 0,0 \end{pmatrix} \end{array}$$

where $\alpha > 1, \beta > 0$ and $\frac{\alpha-\beta}{2} < 1$. Players discount future payoffs with common discount factor $\delta < 1$. Stage games are private; no one except the two parties to a match observe their actions.

Player i's private history at time t, h_i^t, is the set of all her matches, the actions of both i and her match, and the time of the match in all stage games strictly prior to t. The set of all possible private histories at t is $\mathcal{H}^t$.

Players are interconnected in an exogenously determined social network g defined by the pair (N, g^N) with $n \times n$ adjacency matrix g^N. Links in the network are unweighted and undirected. The network reveals some private information, specified below, to others according to a transmission process τ. g and τ are common knowledge among the players.

This game can be said to have *network information processing* in that it has the following information structure: players form a *judgment* about their match's action in the stage game.

A network transmission process reveals players' judgements to others based on their positions in the network. Players use all judgements revealed to them to form an *appraisal* of each other player. Strategies are a mapping from private histories and appraisals into actions. Specifically:

1. All players have an *appraisal* of every other player: i's appraisal of any $j \neq i$ before playing the stage game in time t is $z_{i,j,t}$. The set of i's appraisals of all other players in N by t is $Z_{i,t}$. The set of all possible sets of appraisals in t is $\mathcal{Z}^t$.

2. All players play the stage game and form a *judgment* about their match's action in t. i forms judgment $J_{i,j,t}$ about any match j's action in t as a function of i's private history and j's action in t, given the network transmission process and strategy profile σ: $J_{i,j,t} = f(h_i^t, a_{j,t} | \tau, \sigma)$.

3. After the stage game in t, the network transmission process (τ) reveals to each player a (possibly empty) subset of all judgments that have been made by any player up to and including t. The set of judgments that are revealed to i at the end of round t is $J_{\rightarrow i,t}$.

4. Players' appraisals in $t+1$ are a function of their past appraisals and all judgments revealed to them in t. $Z_{i,t+1} = h(Z_{i,t}, J_{\rightarrow i,t})$.

5. A player's strategy is a mapping from her private history, her appraisals of others, and her match into an action, $\mathcal{H}^t \times \mathcal{Z}^t \times N \rightarrow A$.

Specifying a game with network information processing requires specifying f, the function which determines players' judgments of actions, h, the function which determines players' appraisals of other players, and τ, the process by which the network transmits information from some players to others.

Here, let an appraisal be a positive integer, $z_{i,j,t} \in \mathbb{Z}^+$. Let judgment be an ordered 4-tuple, $J_{i,j,t} = (J^1_{i,j,t}, J^2_{i,j,t}, J^3_{i,j,t}, J^4_{i,j,t})$ where the first element is binary, $J^1_{i,j,t} \in \{0, 1\}$, the second element is the identity of the person forming judgment, i, the third element is the identity of the person being judged, j, and the fourth is the time of the judgment, t. Call a judgment with $J^1 = 1$ a *positive judgment*, and a judgment with $J^1 = 0$ a *negative judgment*. Judgments are formed by referencing a strategy profile: a player judges the action of a match to be negative when that action is not in compliance with the strategy.

Consider the following strategy profile in which a subset of players ($COOP$) participate in in-group policing and condition punishment which lasts for T rounds on their appraisals of others, while the rest ($CHEAT$) perpetually play d:[1]

Definition 1 (**Network In-Group Policing with Cheaters** σ^{CHEAT})**.** *For all players $i \in COOP$: When matched with a player $j \in COOP$: play c in the first round. In round t, play c if $z_{i,j,t} > T$ and d if $z_{i,j,t} \leqslant T$. When matched with a player $j \in CHEAT$: Always play d. For all players $i \in CHEAT$: Always play d.*

For convenience, say that when i's appraisal of j is such that $z_{i,j,t} > T$, i appraises j to be *in good standing*, and when $z_{i,j,t} \leqslant T$, i appraises j to be *in bad standing*. Players using the strategies in σ^{CHEAT} as a member of $COOP$ play c with all others in $COOP$ whom they appraise to be in good standing, and play d to punish those in $COOP$ whom they appraise to be in bad standing. This strategy profile implies that punishment takes the form of capitulation to those appraised to be in good standing; a player i appraised by his match j be in bad standing plays c while j plays d unless both appraise each other to be in bad standing, in which case they both play d. Players always play d with those in $CHEAT$, and all players in $CHEAT$ play d with everyone.

Since this is a game of network information processing, appraisals are determined by judgments that players form which are then transmitted through the network according to τ. Players form judgments of the actions taken by their matches based on whether the action complies with σ^{CHEAT}. If i's match j in t deviates from σ^{CHEAT} in t, i judges j's action negatively: $J_{i,j,t} = \{0, i, j, t\}$. If j does not deviate, i judges j's action positively: $J_{i,j,t} = \{1, i, j, t\}$. A player can always tell if her match deviated from σ^{CHEAT} in the round with her.[2] She can distinguish her match playing d out of punishment from d out of defection, because she knows her private history h_i^t and so knows if she deserved punishment, and she

[1] In-group policing strategies like those considered here capture enforcement schemes used by real groups well. Cooperation supported by the strategies played by $COOP$ has other desirable properties as well, which is perhaps why these strategies seem to be favored by real groups. Carrying out punishment is renegotiation-proof– punishers gain from punishing. Additionally, finite punishments are desirable in environments prone to errors or mistakes since they destroy minimal value off the equilibrium path and give groups the chance to return to the efficient outcome, which may have been particularly important in frontier life where drunken mishaps were common (McGrath, 1987, p. 75). Most importantly for the account here, they also resemble punishments that settlers opted to use to enforce communal norms. Accounts of misbehavior cite fines and other concessions that the offenders were pressured to pay for a finite period of time.

[2] Because not all players know about every round in the game, even if they could observe what actions other pairs take, they could not discern whether all play in the game is in compliance with σ^{CHEAT}; specifically, distinguishing d the defection from d the due punishment requires extensive knowledge of histories. However, a player can always tell if *her match* is deviating from σ^{CHEAT}.

knows g and τ, and so knows if her match would know this.[3]

The network transmission process τ governs how judgments are revealed to others in the network g. Fix τ to be the following process: when i forms a negative judgment about j in t, $J_{i,j,t}$ is revealed (i.e., is sent as gossip) to all network neighbors of i in radius r by $t+1$, to all of their neighbors in radius r by $t+2$, to all of their neighbors in $t+3$, and so on until $t+T$.[4] Call all players within radius r of i on network g, including i himself, i's $r-neighborhood$, written $N_i^r(g)$.[5] This network transmission process implies that the judgments revealed to i in t, $J_{\rightarrow i,t}$, are the negative judgments formed in t by anyone in i's $r-neighborhood$, by anyone formed in $t-1$ in i's $2r-neighborhood$, and so on:

$$J_{\rightarrow i,t} = \bigcup_{l=0}^{T-1} \left\{ J_{k,j,t-l} | k \in N_i^{(l+1)r} \right\}.$$

Players begin by presuming that everyone is in good standing. For convenience, represent this by setting $z_{i,j,1} = T+1$ for all $i, j \in N$. Player i's appraisal of j records the number of rounds that have elapsed since the most recent instance i is aware of that j was judged negatively. Specifically, for $t > 1$,

$$z_{i,j,t+1} = \min\{z_{i,j,t} + 1, t - (J^4_{\rightarrow i,j,t} - 1)\}$$

That is, a player i's appraisal of j (which, recall, records the number of rounds since the most recent negative judgment of j that i is aware of), is the smaller of the old appraisal plus one, which updates the number of rounds since j's most recent negatively judged action that i knew about, or the number of rounds since j's most recent defection according to any

[3]Player i's match $\mu(i,t) = j$ can deviate when playing i by playing d when j appraised i to be in good standing, by playing d when i appraised j to be in bad standing (failing to accept punishment), and by playing c when j appraised i to be in bad standing. The simplest statement of judgments is to say i judges all to be negative. However, realistically, i would not mind j neglecting to punish i and so may not judge this deviation negatively. Since punishment is capitulation, allowing the punisher to play d while the punished plays c, punishers always prefer to punish. This incentive holds even if no one would punish a player for not punishing. Therefore, modifying the setup to say that i only judges j's deviation to be negative when it entails j playing d against an i whom j appraises to be in good standing, does not change any of the results.

[4]That only negative gossip spreads in this way is meant to realistically capture how information about one another spreads through groups. Changing τ to also transmit positive judgments would not change the results.

[5]Player i's **r-neighborhood** in network g is the set of all j such that the shortest path from i to j is less than or equal to r, including i himself. That is, $N_i^r(g) = \{j \in N | \ell(i,j) \leqslant r\} \cup i$.

messages reaching i in t.

This form implies that if i has heard nothing negative about j in the last T periods (from himself or others), he will regard j to be in good standing in this period. This form builds in forgiveness after T rounds.

In short, each time period proceeds as follows: at the beginning of t, nature randomly matches players. Each matched pair plays one round of prisoner's dilemma, after which each player forms a judgment about each other's action. Judgments that are negative spread r degrees through the network. Players' appraisals of other players are updated based on the negative judgments received, ending t.

If everyone in $COOP$ complies with σ^{CHEAT}, all interactions among $COOP$ can be cooperative. A player's incentive to comply hinges on the extent of punishment she expects for deviating. Since she will be punished by anyone who hears the gossip that she deviated, her expected punishment is a function of the number of others who hear the gossip.

As play continues, more and more people may know about an offense. It turns out that the binding case of a defection is a second defection in a row, which extends punishment by a single period. Consequently, the binding conditions for cooperation depend on the size of players' $rT - neighborhoods$. Gossip is sent by the victim of a deviation, so deviations in rounds with players who have small $rT - neighborhoods$ are more profitable than deviations in rounds with players with larger $rT - neighborhoods$.

1.1 Results with Varying Punishment Length

Here I reproduce the results from the article text, accounting for varying finite punishment length. Note that all results in the article follow by setting $T = 1$.

For σ^{CHEAT} to be sequentially rational, it must be that for a division of N into $CHEAT$ and $COOP$, no individual has an incentive to deviate from her prescribed strategy in any history of play. This holds under the following conditions:

Result 1 (**Partial Cooperation).** *σ^{CHEAT} with partition of N $\{COOP, CHEAT\}$ is sequentially rational if and only if, given r, T, g_N, for all $i \in COOP$:*

$$\delta^T \geqslant \frac{(n-1)(\alpha-1)}{\#(N_i^{rT} \cap COOP)(\beta+1)}$$

and

$$\delta^T \geqslant \frac{(n-1)\beta}{\#(N_i^{rT} \cap COOP)(\beta+1)}.$$

In fact, we can use the conditions in Result 1 to specify when full cooperation ($COOP = N$) is impossible:

Result 2 (When Full Cooperation is Impossible). *There exists no equilibrium with* $COOP = N$, $CHEAT = \varnothing$ *if, given* r, T, *and* g_N,

$$\delta^T < \min_{i \in N} \left\{ \frac{(n-1)(\alpha-1)}{\#N_i^{Tr}(\beta+1)} \right\}$$

or

$$\delta^T < \min_{i \in N} \left\{ \frac{(n-1)\beta}{\#N_i^{Tr}(\beta+1)} \right\}.$$

Since supporting $COOP = N$ can be impossible, what is the most cooperative feasible equilibrium be for a given set of parameter values? In other words, what is the largest set of cooperators (or equivalently the smallest set of cheaters) that can be supported in equilibrium? The following result characterizes the maximally cooperative feasible equilibrium.

Result 3 (Maximally Cooperative Equilibrium). *An equilibrium with set of cooperators* $COOP$ *is the maximally cooperative partial cooperation equilibrium possible when, given* r, T, g^N,

$$\delta^T \geqslant \min_{i \in COOP} \left\{ \frac{(n-1)(\alpha-1)}{\#(N_i^{Tr} \cap COOP)(\beta+1)} \right\} \tag{1}$$

and

$$\delta^T \geqslant \min_{i \in COOP} \left\{ \frac{(n-1)\beta}{\#(N_i^{Tr} \cap COOP)(\beta+1)} \right\}, \tag{2}$$

and, for any other set of cooperators $COOP'$ *such that* $\#COOP' > \#COOP$, *either*

$$\delta^T < \min_{i \in COOP'} \left\{ \frac{(n-1)(\alpha-1)}{\#(N_i^{Tr} \cap COOP')(\beta+1)} \right\} \tag{3}$$

or

$$\delta^T < \min_{i \in COOP'} \left\{ \frac{(n-1)\beta}{\#(N_i^{Tr} \cap COOP')(\beta+1)} \right\}. \tag{4}$$

Result 4 (**The Cheating Periphery**). *In a maximally cooperative equilibrium, there exists a threshold*

$$x^\star := \min_{j \in COOP} \left\{ \#(N_j^{Tr} \cap COOP) \right\}$$

such that

$$\textit{if } \#N_i^{Tr} < x^\star, \textit{ then } i \in CHEAT.$$

Result 5 (**Peripheral Become the Cheaters**). *In an equilibrium with $CHEAT$ and $COOP$, if a new equilibrium with $CHEAT'$ and $COOP'$ is such that $\#CHEAT' > \#CHEAT$, the the most peripheral in the subnetwork induced by $COOP$ (have the smallest $N^{Tr} \cap COOP$) will be in $CHEAT'$.*

The following corollary helps to make sense of the consequences of changes in n:

Result 6 (**The Mixed Consequences of Changes in Group Size**). *Changes in group size have ambiguous effects on the maximum extent of cooperation in equilibrium. The direction of change depends on how the network changes as a result of the change in group composition. For an original group N of size n with network g and $COOP$ cooperators in a maximally cooperative equilibrium, consider a change in group size resulting in new group N' of new size n' and attendant new network g'. The change in population strictly decreases (increases) the maximum extent of cooperation in equilibrium if, for*

$$P^\star := \frac{x^\star}{n-1} = \min_{j \in COOP} \left\{ \frac{\#(N_j^{Tr}(g) \cap COOP)}{n-1} \right\},$$

$$\frac{\#COOP}{n-1} > (<) \max_{NEW \in \mathcal{P}(A')} \left\{ \frac{\# \left\{ i \in A' | \#(N_i^{Tr}(g') \cap NEW)/(n'-1) \geqslant P^\star \right\}}{n'-1} \right\}$$

where $\mathcal{P}(N')$ is the power set of N'.

2 Conditions for Sequential Equilibrium and Proof

These conditions are derived and proven for an arbitrary, finite length of the punishment phase T (introduced in Section 1 above). The conditions for the simple version of the model in the article with punishment that lasts only a single round can be found by setting $T = 1$.

σ^{CHEAT} with partition of N $\{COOP, CHEAT\}$ is sequentially rational if and only if, given r, p, T, and g, for all $i \in COOP$:

$$\delta^T \geqslant \frac{(n-1)(\alpha-1)}{\#(N_i^{rT} \cap COOP)(\beta+1)}$$

and

$$\delta^T \geqslant \frac{(n-1)\beta}{\#(N_i^{rT} \cap COOP)(\beta+1)}.$$

Proof. To establish sequential rationality, I will show that for any history and at any information set, all players prefer to comply with σ^{CHEAT} given the conditions above. First consider individuals in $COOP$ (the players playing in-group policing with each other). Players' strategies respond to negative judgments sent to them via the network transmission process τ. Specifically, they implement punishment in response to messages sent from victims of deviations that occurred within the last T periods.

Players in $CHEAT$ are trivially playing a best response in all interactions: everyone plays unconditional d with them, so they never have an incentive to play anything other than d. Strategies instruct players in $COOP$ to respond to their appraisals, z, of others in $COOP$ (which are formed as a function of all negative judgments that have reached a player by t). There are thus eight ways for an individual i in $COOP$ to deviate from her strategy:

When player i is appraised to be in good standing by everyone in N ($z_{j,i,t} \geqslant T \forall j \in N$):

i Play d with a j when $z_{i,j,t} \geqslant T$ (defect against someone appraised to be in good standing)

ii Play c with a j when $z_{i,j,t} < T$ (fail to punish someone in bad standing)

iii Play c with $j \in CHEAT$

When player i is appraised to be in bad standing by at least someone in N $\exists k \in N$ such that $z_{k,i,t} < T$):

iv Play d with j when $z_{i,j,t} \geqslant T$ and $z_{j,i,t} < T$ (fail to capitulate in punishment) v Play d with j when $z_{i,j,t} \geqslant T$ and $z_{i,j,t} \geqslant T$ (fail to cooperate with someone expecting cooperation)

vi Play c with j when $z_{i,j,t} < T$ and $z_{j,i,t} < T$ (fail to punish)

vii Play c with j when $z_{i,j,t} < T$ and $z_{j,i,t} \geqslant T$ (fail to cooperate with someone expecting cooperation)

viii Play c with $j \in CHEAT$

Assessing compliance with σ^{CHEAT} requires a lot of accounting detail for intermediate rounds that ends up falling out of the binding conditions. Recall from the article text that N_i^k is the k-neighborhood of player i in a network, which is the set of individuals reachable in paths of length k or shorter. The assumptions about message transmission and rate r result in a message sent from i reaching individuals N_i^{rl} in l rounds. The complement of this set (excluding i) will be denoted $\overline{N}_i^{rl}$. The number of individuals in these sets are $\#N_i^{rl}$ and $\#\overline{N}_i^{rl}$.

When a player assesses future costs and benefits of a decision made in t at a hypothesized history of play, an intermediate range of future periods are ambiguous to the player. Specifically, i cannot know how many others about whom he will receive messages for the next few rounds.[6] Let $\star$ denote terms that are the result of a player's guess. $G_i^{l\star}$ is player i's guess of the set of individuals about whom i will not have received a negative judgment by period l (will be thought to be in Good standing), and likewise $B_i^{l\star}$ is i's guess of the set of individuals about whom i *will* have received a negative judgment by l (will be known to be in Bad standing). When a player knows this information rather than guesses it, it will be displayed without the $\star$. The binding conditions are independent of these guesses, precluding the need to precisely specify beliefs.

2.1 Deviations via (i)

First consider deviations according to (i), in which a player in good standing plays d in a round with an in-group player j about whom i appraises to be in good standing. The expected number of future punishers depends on j's network position. Expected payoffs for the next $T-1$ periods depend on guesses $G^\star$ and $B^\star$; after that, regardless of history,

[6] Though he does know this number is bounded by his rT-neighborhood, the maximum set of people i could ever receive messages about.

σ^{CHEAT} ensures that $G^\star = COOP$ and $B^\star = \varnothing$. Complying with σ^{CHEAT} would yield:

$$1 + \sum_{l=1}^{T-1} \delta^l \left[\frac{1}{n-1} \left[\#G_i^{l\star} + \#B_i^{l\star}\alpha \right] \right] + \delta^T \left[\frac{1}{n-1} \left[\#COOP \right] \right] + \sum_{l=T+1}^{\infty} \delta^l \left[\frac{1}{n-1} \left[\#COOP \right] \right]$$

whereas deviating would yield:

$$\alpha + \sum_{l=1}^{T-1} \delta^l \left[\frac{1}{n-1} \left[\#(N_j^{rl} \cap G_i^{l\star})(-\beta) + \#(N_j^{rl} \cap B_i^{l\star})(0) + \#(\overline{N}_j^{rl} \cap G_i^{l\star}) + \#(\overline{N}_j^{rl} \cap B_i^{l\star})\alpha \right] \right]$$
$$+\delta^T \left[\frac{1}{n-1} \left[\#(N_j^{rl} \cap COOP)(-\beta) + \#(\overline{N}_j^{rl} \cap COOP) \right] \right] + \sum_{l=T+1}^{\infty} \delta^l \left[\frac{1}{n-1} \left[\#COOP \right] \right].$$

Complying in a round with j is thus weakly preferred to deviating when:

$$\alpha - 1 \leqslant \sum_{l=1}^{T-1} \frac{\delta^l(1)}{n-1} \left[\#(N_j^{rl} \cap G_i^{l\star})(1+\beta) + \#(N_j^{rl} \cap B_i^{l\star})\alpha \right] + \frac{\delta^T(1)}{n-1} \left[\#(N_j^{rT} \cap COOP)(1+\beta) \right].$$

The sum in the condition contains terms for all but the final round of expected punishment and depends in each round on the number of individuals reached by j's gossip about whom i doesn't expect to have received a message by then, and the number of individuals reached by j's gossip about whom i does expect to have received a message by then. By the end of the punishment phase, everyone in $COOP$ will be cooperative in compliance with σ^{CHEAT}, so the net cost of punishment in that round is only determined by which of these are reached by j's message by then.

2.2 Deviations via (ii)

Deviations according to (ii), in which a player i in good standing plays c rather than d with a player whom i appraises to be in bad standing, are trivially not preferred. Complying with σ^{CHEAT} yields α; deviating earns 1, strictly less by assumption.

2.3 Deviations via (iii)

Deviations via (iii) are trivially not preferred. Since all $j \in CHEAT$ always play d, playing anything other than d when paired with such a j is not a best response.

2.4 Deviations via (iv)

Consider an i whom j appraises to be bad standing contemplating deviating by playing d against a j whom i appraises to be in good standing. This deviation is similar in form to (i), with one important difference: i, being in bad standing, already expected some amount of punishment. His assessment of this deviation then depends on the net additional cost from punishment. This amount depends both on how recently his most recent past deviation was, as well as the identity of his past victim.

Call t^d the number of periods ago i most recently defected, and call k the victim of i's offense in that round. This means in t, i faces $T - t^d$ more rounds of punishment from that offense. A new offense today extends the punishment to $t + T$. Moreover, defecting on someone new j in t who is far away from his past victim k can increase the amount of punishment i expects even in the next $T - t^d$ rounds.

i's expected punishment from his defection against k that occurred t^d rounds ago depends on the network position of k. [7] From the round under consideration until $T - t^d$ rounds into the future, i expects punishment for this offense which is a function of N_k. Specifically, his expected punishment l rounds from now depends on

$$N_k^{r(t^d+l)}.$$

His net expected punishment from defecting on j now then also depends on j's network position. For the next $T - t^d$ rounds, his expected punishment depends on all those who receive either a message from k and/or a message from j. That is, his expected punishment l rounds from now until $T - t^d$ depends on:

$$N_k^{r(t^d+l)} \cup N_j^{rl}$$

[7]This implicitly assumes that k is a member of the in-group. The same condition results if we assume k is a member of the out-group and then use i's network position as the relevant consideration for the cost of his past defection.

After that and until T rounds from now, his punishment depends on the message sent from j, N_j^{rl}. i then prefers to comply rather than deviate via (iv) against j so long as:

$$\beta \leqslant \sum_{l=1}^{T-t^d} \frac{\delta^l(1)}{n-1}\left[NW_G^{l\star}(-\beta) + \overline{NW}_G^{l\star} + \overline{NW}_B^{l\star}\alpha\right]$$
$$+ \sum_{l=T-t^d+1}^{T-1} \frac{\delta^l(1)}{n-1}\left[\#(N_j^{rl} \cap G_i^{l\star})(1+\beta) + \#(N_j^{rl} \cap B_i^{l\star})\alpha\right]$$
$$+\frac{\delta^T(1)}{n-1}\left[\#(N_j^{rT} \cap COOP)(1+\beta)\right]$$

where

$$\begin{aligned}
NW_G^{l\star} &= \#(N_j^{rl} \cap N_k^{r(t^d+l)} \cap G_i^{l\star}) - \#(N_j^{rl} \cap G_i^{l\star}),\\
\overline{NW}_G^{l\star} &= \#\overline{(N_j^{rl} \cap N_k^{r(t^d+l)}} \cap G_i^{l\star}) - \#\overline{(N_j^{rl} \cap G_i^{l\star})}\\
\overline{NW}_B^{l\star} &= \#\overline{(N_j^{rl} \cap N_k^{r(t^d+l)}} \cap B_i^{l\star}) - \#\overline{(N_j^{rl} \cap B_i^{l\star})}.
\end{aligned}$$

This condition can be further simplified.[8] First, note that the condition is more difficult to satisfy as the right hand side decreases. The only term dependent on the identity of the first victim k is the first sum. This first sum takes its maximum when the neighborhoods around j and k are distinct; conversely, when new victim j is in the t^d-neighborhood of k, the condition is hardest to satisfy.

Consequently, one of the set of binding cases is the scenario in which i is contemplating defecting for a second time against the same person, i.e. when $j = k$. In that case, $N_j^{rl} \cap N_k^{r(t^d+l)} = N_j^{rl}$, and since intersection is associative, $NW_G^{l\star} = \overline{NW}_G^{l\star} = \overline{NW}_B^{l\star} = 0$. That is, in the binding case, an additional defection adds no cost to the first $T - t^d$ rounds of expected punishment. The binding condition becomes:

$$\beta \leqslant \sum_{l=T-t^d+1}^{T-1} \frac{\delta^l(1)}{n-1}\left[\#(N_j^{rl} \cap G_i^{l\star})(1+\beta) + \#(N_j^{rl} \cap B_i^{l\star})\alpha\right]$$
$$+\frac{\delta^T(1)}{n-1}\left[\#(N_j^{rT} \cap COOP)(1+\beta)\right].$$

[8] The current form makes use of the following set operation: $\#(N_k \cap G) - \#((N_k \cup N_j) \cap G) = \#(N_k \cap N_j \cap G)) - \#(N_j \cap G)$.

Furthermore, since the terms of the sum are all positive, the right hand side is minimized when the sum includes fewer periods. This means the binding case is one in which the second defection on k occurs immediately following the first, i.e. in which the old defection just occurred in the last period so that $t^d = 1$. In this binding case, the first $T-1$ rounds of punishment would be the same, and the condition reduces further to

$$\beta \leqslant \frac{\delta^T(1)}{n-1}\left[\#(N_j^{rT} \cap COOP)(1+\beta)\right].$$

So long as the person considering defecting a second time against a player in back-to-back rounds can be disincentivized from doing so, no one wants to defect a second time against anyone at any time.

2.5 Deviations via (v)

Deviations via (v), in which a player i plays d against a j whom i appraises to be in good standing when j appraises i to be in good standing (even though someone else in the network does not) differ from (iv) only in the immediate gain. Expected net future punishment is the same. In this case, complying yields 1 immediately while deviating earns α, so the condition becomes:

$$\alpha - 1 \leqslant \frac{\delta^T(1)}{n-1}\left[\#(N_j^{rT} \cap COOP)(1+\beta)\right].$$

2.6 Deviations via (vi) and (vii)

As with deviations via (ii), deviating by playing c against someone whom a player appraises to be in bad standing is trivially not preferred. In (vi) complying yields 0 while deviating yields $-\beta$; in (vii) complying yields α while deviating yields 1.

2.7 Deviations via (viii)

Finally, and for the same reason as (iii), playing anything other than d against $j \in CHEAT$ is trivially not preferred, regardless of appraisals.

2.8 Combining Conditions

Satisfying the condition to prevent deviations via (v) for all potential opponents j implies satisfying the conditions to prevent deviations via (i). Rearranging the remaining conditions to place the discount factor on the left hand side, we have the conditions presented above. If both conditions are satisfied, no player has an incentive to deviate from σ^{CHEAT} in any history, and the binding player is minimally enticed to comply with σ^{CHEAT} if both conditions are satisfied. Since the conditions for sequential rationality are independent of beliefs, any consistent beliefs trivially extend the behavior to sequential equilibrium.

□

3 Proof of Proposition 1

The proof of this Proposition follows immediately from the proof of the conditions for sequential rationality presented in the last section, with $T = 1$ for the simple version of the model presented in the paper. The first two conditions are stated in terms of the binding player. The second two ensure that the set $COOP$ is maximal; that for any possible larger set of in-group policers, at least one in this larger set would find deviating from σ^{CHEAT} too profitable to comply.

4 Intuition for Corollaries

This intuition assumes that the length of the punishment phase can vary. The intuition is identical for a single round of punishment as in the article text. The conditions are the same, with T set to 1.

4.1 Corollary 1

Corollary 1 follows from the conditions for sequential rationality when $COOP$ is set to N, that is when the set of players playing in-group policing is the whole group. In that case $N_i^{rT} \cap COOP = N_i^{rT} \cap N = N_i^{rT}$. In other words, the binding case for full cooperation is the extent to which player are peripheral in the full network g.

4.2 Corollary 2

Corollary 2 identifies that in a maximally cooperative equilibrium, there exists a threshold such that players who are at least this peripheral in the whole network cheat. This threshold can be defined as the extent to which the most peripheral player within $COOP$ is peripheral among those in $COOP$. Clearly in a maximally cooperative equilibrium, there is a threshold which separates the most peripheral player in $COOP$ among $COOP$ from any other player $i \notin COOP$ among $COOP \cup i$: $\#(N_i^{rT} \cap (COOP \cup i)) < \min_{j \in COOP} \#(N_j^{rT} \cap COOP)$. This establishes that those who would be most peripheral when considered among the set of cooperators cheat. The stronger result stems from the observation that for any player i with $N_i^{rT} < \min_{j \in COOP} \#(N_j^{rT} \cap COOP)$, it will necessarily be the case that $\#(N_i^{rT} \cap (COOP \cup i)) < \min_{j \in COOP} \#(N_j^{rT} \cap COOP)$. Any player peripheral enough in the whole network will be too peripheral in any subnetwork among a set of in-group policers to cooperate in equilibrium.

The condition is sufficient but not necessary because there can exist a player $i \notin COOP$ for whom $\#(N_i^{rT} \cap (COOP \cup i)) > \min_{j \in COOP} \#(N_j^{rT} \cap COOP)$. To see this, consider an example in terms of out-group offenses (similar logic holds for in-group offenses). Take the network depicted in Figure 1, let $r = 2, T = 1$, and set δ such that each player needs 5 others to learn about an out-group offense to dissuade him from cheating. This means that in a maximally cooperative equilibrium, all cooperators must be such that $\#(N_i^2 \cap COOP) \geqslant 5$. Note that $N_8^2 = 6$, so player 8 is a candidate for $COOP$– enough would know about his out-group offense to punish him. However, of these 6 other players reachable, not enough could themselves be enticed to play $COOP$. For instance, players $10 and 12$ are too peripheral: news spreads to only two others in $rT = 2$ steps. Consequently, $10 \notin COOP$ and $12 \notin COOP$. This means that $\#(N_8^2 \cap COOP)$ can be at most 4, which means $8 \notin COOP$, despite sitting in one of the most central positions in the whole network. The same logic reveals that none of $7, \ldots 12$ can be contained in a maximally cooperative equilibrium for these parameter values. Hence, if a player's centrality is too heavily dependent on connections to peripheral players, that player cannot be enticed to be cooperative.

This corollary highlights the importance of factoring in incentives generated by networks. Considering individuals separately can mask important ways that incentives interact to produce an aggregate outcome. Moreover, this result and the others illustrate why heterogeneity in network position is important. If the network were assumed to be complete (as is often implicitly assumed in non-network models), then there would be no maximally cooperative equilibrium in which only a strict subset of the group plays as $COOP$. If anyone could be

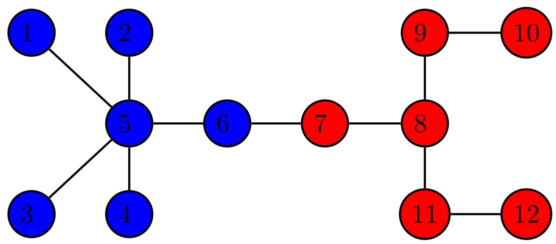

Figure 1: Let $r = 2$, $T = 1$, and set δ such that players need 5 punishers to be enticed to cooperate. The maximally cooperative equilibrium entails $COOP = \{1, \ldots, 6\}$ and $CHEAT = \{7, \ldots, 12\}$, even though $\#N_8^2 = 6 > min_{j \in COOP}\{\#(N_j^2 \cap COOP)\} = 5$.

enticed to be cooperative through in-group policing, everyone could; if anyone could not, no one could. Here, heterogeneity in network position admits the possibility of equilibria in which a strict subset cooperate via $COOP$ and a non-empty subset perpetually defect.

4.3 Corollary 3

All else equal, a maximally cooperative equilibrium with $\#CHEAT' > \#CHEAT$ implies a lower threshold $x^{\star\prime} < x^{\star}$, which implies $\min_{j \in COOP'} \left\{\#(N_j^{Tr} \cap COOP')\right\}$ $< \min_{j \in COOP} \left\{\#(N_j^{Tr} \cap COOP)\right\}$. Intuitively, since the most peripheral member in in-group policing binds, adding a more peripheral member to the set of cooperators changes the ease of enforcing cooperation at the margin and raises the minimum discount factor for which the conditions hold.

4.4 Corollary 4

Since the equilibrium conditions bind for the most peripheral individual, marginal changes to the set $COOP$ in maximally cooperative occur at the peripheral end. More cooperators mean more peripheral players are included in $COOP$; fewer cooperators mean more peripheral players are included in $CHEAT$.

4.5 Corollary 5

Because the peripheral individuals bind, changes in group size that do not change the ratio of the reach of the most peripheral to the total group size do not change equilibrium outcomes. This is because in the binding case, the expected number of players playing the in-group policing strategy who learn about an offense in T rounds determines the conditional probability of future punishment in the random matches. If the addition or subtraction of group members results in the peripheral player expecting higher future punishment (because a larger proportion of his group will know about an offense), then the population change improves prospects for cooperation. If the change results in peripheral players expecting lower future punishment because a smaller proportion of his group will know abut an offense, then enforcing cooperation becomes more difficult. Other changes have no effect.

5 Additional Analyses

5.1 Strategic Lying

No victim of misbehavior gains from not reporting the offense. The worrisome case is a message which claims someone misbehaved when they did not. i can only gain from a lie that j misbehaved if in the future i can defect against j and some people who would have punished i for this offense instead mistake it for punishment. However, the only people who would punish i for an offense against j are those whom j's message reaches, and these are the same people who would be reached by a message if i defects against j in the future. Best case scenario for i some of these people first received his message, and so are confused by or ignore j's report of i's misbehavior. But (a) in even moderately large groups, the expected future opportunity to defect against any particular person is quite unlikely, and (b) a modification which has players react strongly to conflicting messages could disincentivize dishonesty in the rare case that it could be on net profitable. Assuming honesty avoids this complication.

References

McGrath, R.D. 1987. *Gunfighters, highwaymen, and vigilantes: Violence on the frontier*. Univ of California Pr.