# Supplementary Information: Proofs of Propositions

Notes: Italicized statements in the proofs correspond to conditions in the propositions. refers to agent biases that follow from the group’s optimal selections of and/or .

**Proof of Proposition 1** In the third stage, a captured agent seeks to maximize . Differentiating this quantity with respect to yields , so that . In the second stage, the solutions for and for a group not facing a binding rent-seeking cap follow immediately from taking the first derivatives of (1) with respect to and . The regulatory conditions described in the main text assure that these solutions identify a unique global maximum. If the group does face a binding rent-seeking cap, the Lagrangian it maximizes is , and the Kuhn-Tucker conditions are

*,*

and .

The third of these equations implies that either or . If , then reduces to the marginal benefit of rent-seeking in (3), and (with the solution to (3)), in contradiction to the rent-seeking cap. Instead, , and , which is consistent with the marginal benefit of quality exceeding the marginal cost. Since , substitution into the equation for implies that the group’s choice of satisfies (2) at . *■*

**Proof of Proposition 2** (a) When the rent-seeking capbinds, sets in equilibrium and
selects based on the first-order condition Equation (2). Differentiating (2) and rearranging yields

|  |  |  |
| --- | --- | --- |
|  |  | (SI.1) |

The assumptions on the second derivatives with respect to or and ensure that

|  |  |  |
| --- | --- | --- |
|  |  | (SI.2) |

so if the numerator in (SI.1) is positive. Since , *provided that is not so negative that* .

(b) The statement is true if . Substitution and algebra yield

The numerator’s first term is negative, so if , *provided that is not so positive, nor so great that . ■*

**Proof of Proposition 3** (a) If does not bind, ’s first-order conditions are Equations (2) and (3). Differentiating these equations with respect to and solving yields

|  |  |  |
| --- | --- | --- |
|  |  | (SI.3) |

where is the Hessian of (1). This Hessian is expressible in terms of other Hessians. First,

The first matrix on the right-hand side is positive definite, as is , so is also positive definite. Also, define

which is negative semidefinite based on ’s concavity. Then , and is negative definite. Then , so if the numerator in (SI.3) is positive. The first term is positive, so *provided that is not so positive, nor so great that* .

(b) To show , the first step is to derive by differentiating (2) and (3) with respect to :

|  |  |  |
| --- | --- | --- |
|  |  | (SI.4) |

Then , and (SI.3) and (SI.4) can be substituted to yield

Positive definiteness of implies that . Substituting and rearranging yields:

which is negative. Next, increases if , or equivalently, . The result is

The right-hand side is positive: the first term is nonnegative and the second is strictly positive due to positive definiteness of and concavity of . ■

**Proof of Proposition 4** (a) Differentiating (2) and (3) with respect to *a* implies

while differentiating these equations with respect to yields

|  |  |  |
| --- | --- | --- |
|  | . | (SI.5) |

Therefore, whenever , when .

(b) The statement follows if . Then (SI.5) implies . Proposition 3(b) implies that , making the second term negative; and that , making the third term less than . Then . ■

**Proof of Proposition 5** Preliminary to proving this proposition is establishing the following Lemma:

**Lemma.** *When*  *continues to bind strictly, (i) increasing and (ii) when he is more than halfway captured, decreasing increase and .*

*Proof.* For (i), if stays strictly binding, differentiating (2) with respect to and solving yields

|  |  |  |
| --- | --- | --- |
|  |  | (SI.6) |

which is positive since and . Next, ’s final ideal point increases if , which by (SI.6) is equivalent to . Substituting from (SI.2) yields based on the assumptions on and .

For (ii), differentiating (2) with respect to and solving yields

|  |  |  |
| --- | --- | --- |
|  |  | (SI.7) |

For a more than halfway captured agent, , so the derivative is negative, which means that . To show that increases, it first helps to derive from (SI.6) and (SI.7) that . This equation and imply that . Since , the second term is negative. Also, has been shown, so . Then the third term is less than , so . □

Let the triple represent a combination of the three institutional designs, the equilibrium quality, and . (Note that and here may be constrained by ). Also, let denote the solution towhen and . Based on (4), the statement for when the condition in Proposition 2(a) always holds follows if and for any triple with other than . The first condition holds because , the maximum possible. The second condition follows if . Since always binds, the first inequality in this chain follows from (SI.7), which implies that quality is maximized when is halfway captured; the second from the Lemma; and the third from everywhere by assumption. At least one inequality holds strictly as long as .

 For the case when the condition in Proposition 2(a) always fails, let denote the solution towhen and is not binding. The corresponding statement follows if and for any triple with other than or an equivalent triple in which the rent-seeking cap is not binding. Again, the first condition holds because , the maximum possible. Meanwhile, the second condition follows if . The first inequality in this chain follows from Proposition 4(a), the second from Proposition 3(a), and the third from the proof of Proposition 2(a) since by assumption. At least one inequality holds strictly as long as is not or an equivalent triple . ■