Towards Bedmap Himalayas: development of an airborne ice-sounding radar for glacier thickness surveys in High Mountain Asia

APPENDIX

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This appendix contains engineering reports commissioned to inform the design of the Bedmap Himalayas helicopter-borne radar airframe.

1. Helicopter-borne Dipole Radar - Structural Aspects: This report was commissioned for investigating the structural engineering aspects of the antenna airframe, with a focus on the aerodynamic loads that might arise during survey operations.

2. Aerodynamic Stability of a Dipole Radar Concept: This report was commissioned for investigating the aerodynamic behaviour of our antenna-airframe concept, with a focus on potential instabilities in flight.

3. Dipole Radar Combined Summary Report: A brief summary of the above engineering reports.

Helicopter-borne Dipole Radar - Structural Aspects

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Abstract

This report is for Dr Hamish Pritchard of the British Antarctic Survey. It describes the structural aspects of the proposal for a 39 metre dipole radar to be hung below a helicopter.

1 Introduction

The author was commissioned, via Cambridge University Technical Services, to provide advice on the structural engineering aspects of a large but lightweight structure which is intended to be hung beneath a helicopter. Its purpose is to carry a dipole radar which will be used, in-flight, to measure glacier thicknesses in remote mountainous regions, possibly in Central Asia. Although the helicopter may travel comparatively slowly whilst taking radar measurements, it may need to fly very quickly when travelling from its airbase to the remote region being studied. The structure will thus need to be able to withstand sizeable aerodynamic loads, as well as its own weight. The aerodynamic aspects of the proposal are covered in a separate report by Dr Will Graham.

2 About the author

Allan McRobie is Reader in Structural Engineering at Cambridge University Engineering Department, where he has worked for around 25 years. Prior to this he was a structural design engineer, mostly in Australia, where he was responsible for the design of very many millions of pounds worth of major civil engineering infrastructure. At Cambridge his research interests include structural dynamics, wind-induced structural vibration and structural analysis and stability. For many years, he has given courses on structural stability, structural dynamics and on the design of lightweight structures.

3 Preliminaries

At the start of this project, the author was given an outline of a potential design to be investigated. This is shown in Fig. 1. This consists of 13 identical lightweight tubes connected to form a horizontal 39 m pole. The pole is suspended from a central yoke 10 m above the pole centreline, by means of 14 cables in a fan-like arrangement, much like a cable-stayed bridge. The proposed structural system weighs around 160 kg, and there will be additional weights at the centre due to the electronic measuring equipment there (Peli Cases with electronics) with an estimated weight of 50kg. The total suspended weight is thus approximately 210kg.

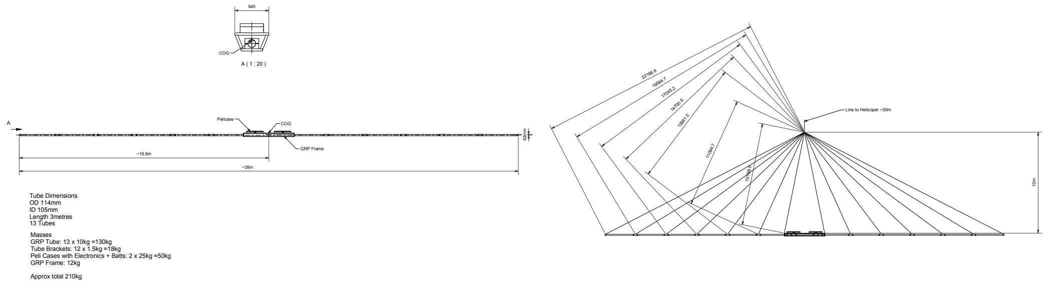

Figure 1: Outline design by BAS

4 Structural modelling

Given the comparatively simple structural geometry, it was at first envisaged that the structural analysis would be undertaken using a standard computational Finite Element Analysis (FEA) package such as ABAQUS. However, this was soon discarded for two main reasons.

Firstly, the structure is highly statically indeterminate. That is, the load paths through the structure will depend heavily on the relative stiffnesses of the cables in tension compared to the main pole in bending. Whilst a finite element program can readily calculate such relative stiffnesses, the subtly is that statical indeterminacy means there are an infinite number of possible equilibrium solutions. These correspond to states of self-stress in the structure. A naive FEA would assume that there is no self-stress initially, and then the dead loads would be applied, with the poles and cables sharing the load according to their relative stiffnesses. However, even for cable-stayed bridges, it is common to "tune" the cables to specific pretensions in order to ensure that in the deadweight-only configuration, the deck (or here, the pole) is essentially straight and horizontal. It is not a straight-forward procedure to determine this state of initial self-stress and to simulate it correctly in a finite element package. In contrast, it is much simpler to arrange this in the real, physical system: if part of the pole sags, say, then one simply shortens the nearest cable(s) until everything is level. It is also much easier to arrange this in a simple non-computer model. The horizontal pole is likely to be very flexible in bending, and under its weight would sag several metres at the tip. In comparison, the cables are likely to deflect significantly less under the tensions caused by carrying the pole weight. To a good first approximation, the pole will thus be horizontal under dead load (and can be arranged to be perfectly horizontal by tuning). The pole, being straight, has no bending, and so the vertical load components are carried solely by the cables. The resulting equilibrium system is so simple that one does not need a computer model to determine the internal forces.

Secondly, the extremely slender nature of the structure, with the pole acting in compression, means that stability against buckling will be a major design consideration. Stability is a higher order effect than simple equilibrium, and finite element programs can require very delicate adjustments in order to correctly model all the nuances. Essentially, the analyst needs to have a very good idea beforehand of what the actual buckling behaviour will be, in order to ensure that the Finite Element model has computed the correct solution.

It was thus decided that a full Finite Element Analysis was not the appropriate methodology at this preliminary design phase, and that all potential structural issues should be understood and calculated from first principles. It was also decided that the testing of a full-scale prototype would give far more confidence that the structural behaviour was understood, than would numerous elaborate computations or theoretical hand calculations. A nonlinear Finite Element Analysis should however be used at a later design stage to analyse the behaviour of the final design that emerges, allowing a three-way comparison between experiment, computation and fundamental theory. If all three agree then there would be confidence that the various subtleties of the behaviours are being understood.

5 Preliminary analysis - Self-weight buckling of initial design

First, we assume that the cables are pretensioned under dead load such that the pole is essentially horizontal for its full length across all cable attachment points. Since the beam is essentially straight it has no curvature and thus carries no bending moment. All vertical load at each cable attachment point is thus carried by the vertical component of the cable tension. The coexistent horizontal component of the cable tension thus puts a pure axial compression into the pole.

The axial compression thus grows along the pole, being small at the tip and high in the centre. If all cables were at 45 degrees, then to a good approximation, the axial load in one half of the horizontal antenna would be very close to the axial load that would be created by self-weight if the pole (i.e half the antenna) were stood vertical. The self-weight buckling of a pole thus gives a ready indication of the likelihood that the horizontally-suspended pole will buckle under self-weight.

The classical formula for the buckling of a vertical cantilever under its own weight is described by Greenhill's formula (from 1881). This gives the critical length L_{cr} beyond which buckling will occur as

$$L_{cr} = \left(7.8373\frac{EI}{\rho g A}\right)^{1/3} \tag{1}$$

where E and ρ are the Young's Modulus and density of the pole material, A and I are the area and the second moment of area of the pole cross-section, and g is the acceleration due to gravity.

The pole of the initial design is made of Glass-Fibre Reinforced Plastic (GFRP) tubes. GFRP has a Young's Modulus typically in the range 15-28 GPa (assume 15) and density in the range 1750-1970 kg/m^3 (assume 1800) (CUED Databook). The uniform tube has external and internal diameters of 114 and 105mm respectively, giving a wall thickness t = 4.5mm and radius r = 54.75mm. The area A and second moment of area I are given by $2\pi r t$ and $\pi r^3 t$ respectively. Inserting these into Greenhill's formula gives a critical length for self-weight buckling of the GFRP tubes as 21.5 metres. This compares with 19.5m actual length. However, the outer cables are inclined at considerably less that 45 degrees, thus the horizontal axial force in the horizontally-suspended pole will be greater than in the free-standing vertical configuration. The critical length in the horizontally-suspended configuration will thus be less than 21.5m. We conclude that the preliminary design, when suspended horizontally, will either buckle or be on the very point of buckling. This thus allows no additional margin to deal with loads due to aerodynamic drag from the forward motion and the downdraft, nor any inertial forces due to accelerations during any pendulum-like motions. The initial design was thus considered to be inadequate.

6 Redesign: a lighter alternative

The initial design contained a long slender pole under the axial compressive forces induced, via the inclined cables, by its own self-weight. Analysis suggested that this would buckle. It is known from the general structural literature that the optimal shape to obtain the maximum height whilst avoiding self-weight buckling is to have a non-uniformly tapered section, thicker at the base and thinner at the top. Indeed this accords with everyday experience, for everything from tree trunks to fishing rods. A new design was thus considered which tapered out towards it tips. The forces out in the tip regions have the greatest effect on inducing buckling, and these are thus reduced in that vicinity. The centre of the structure (the root of the cantilever) has the greatest influence on resisting buckling, and thus the dimensions are increased in that vicinity. Note that Greenhill's formula contains an I/A term, and the critical length for a uniform tube is thus independent of wall thickness t: a thicker wall adds strength but it also adds weight. The two effects cancel, such that nothing is gained by thickening the wall.

Rather than designing a bespoke tapered tube, which could be costly to fabricate, it was pointed out by Dan Ashurst (BAS) that there exist comparatively cheap GFRP antennae intended for use as guyed masts for amateur radio enthusiasts. These are made by Spiderbeam and extend to 26m.

`https://www.spiderbeam.com/product_info.php?`
`info=p233_Spiderbeam%2026m%20fiberglass%20pole.html`
`&XTCsid=4813f78651884f6fa0e701f258521659`

Two such masts were purchased by BAS - at a cost of around 500 pounds each - and made available to the author for testing. The poles come as a telescopic set of 15 poles nestling inside one another, each section being around 1.95m in length. By discarding the three slender tip sections, the remainder extend to a combined length measured at 19.27m, giving a structure of total length 38.54m, just 0.5m short of the 39m desired.

7 Static tests on a full-scale prototype

A full-scale static load test was performed on 17 July 2017 at Laundry Farm, Barton Rd, Cambridge, using the two Spiderbeam masts back-to-back and suspended by ten cables from a 15m cherry-picker belonging to Cambridge University Estates Management.

The antenna mast arrives with a set of large, customized jubilee clips, with rubber pads and shrink-wrap. These are then attached adjacent to the overlapping joints between mast sections in order to prevent the mast de-telescoping under compressive loads. These were attached as per the manufacturer's instructions.

The two masts were coupled merely by inserting their bases into a snug-fitting aluminium alloy tube of approximately 700mm length, this having been found serendipitously in the BAS workshops. Since there would be no tensile forces at this connection under dead load, there was no need to design a more sophisticated connection.

The cables were kite strings made of Dacron, each of approximately 1mm diameter.

`http://www.emmakites.com/index.php`
`?main_page=product_info&cPath=336_427&products_id=1196`

This has a breaking load of 300lb or approximately 120kg. Remarkably, the weight of the whole structure can thus be carried by a single piece of this line, with a Factor of Safety of around 3. Alternative kite strings are also available, and of different materials, which may be stiffer, stronger or more robust. However the 300lb Dacron was considered suitable for the purposes of the prototype tests. The mast manufacturer, Spiderbeam, also sell 1mm and 2mm Kevlar rope, which may be a future option.

The day before the test, the structure was laid out on the field, and all cables were connected via simple knots (bowlines, etc.) to the mast and to a central carabiner located 10 metres offset from the pole centre.

The evening before the test, the newly-arrived mast data was fed into a buckling calculation (see later) which predicted that the mast with a 10m suspension point would be on the very point of buckling, having a Factor of Safety of almost exactly 1. Further calculations revealed the now-obvious result that the Factor of Safety against buckling scales exactly with the height of the yoke: the higher the yoke, the steeper the inclination of the cables and the lower the horizontal component of the cable force that is causing the mast to buckle.

On the morning of the test the cables were thus re-jigged to suit a 15m suspension point. This was limited by the maximum height the cherry picker could reach, and should give a Factor of Safety on mast buckling of 1.5.

At the first attempt at lifting, the mast did not leave the ground. This was due to a combination of the cherry-picker not being at its full possible height, combined with some extension of the Dacron cables. The cables were thus re-jigged again to meet at a yoke height of around 13m. This time the lift was successful, and the mast was raised some 300mm off the ground for its full length, where it remained stable for several hours in an essentially straight configuration (Figure 2).

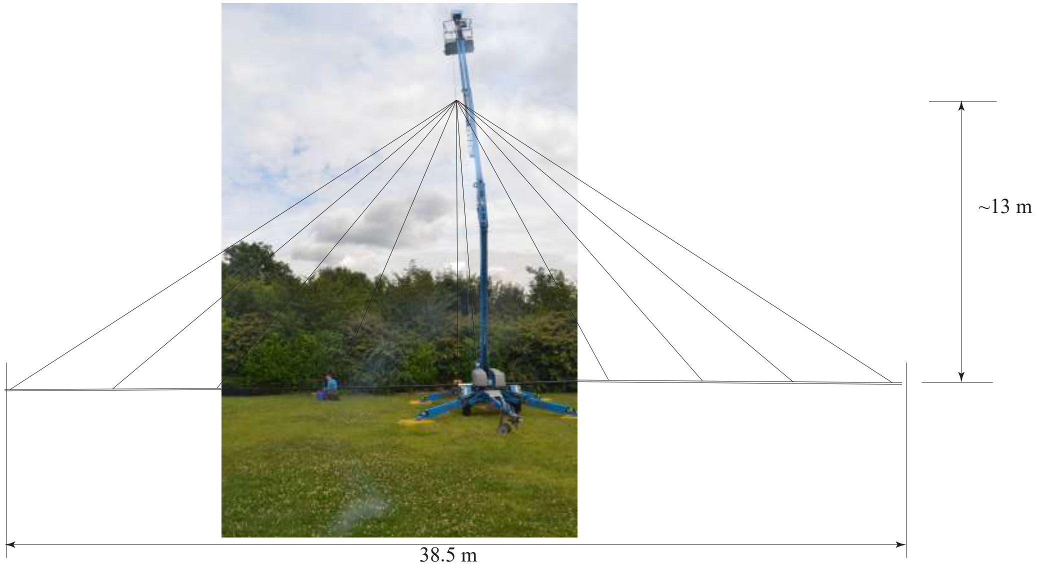

Figure 2: The full-scale static test of the back-to-back Spiderbeams

Simple observations confirmed that the pendulum mode had a natural period of around 6 seconds. Structural natural frequencies appeared to be considerably higher, perhaps close to 1Hz. No detailed measurement of frequencies was possible before Estates Management asked that the experiments be terminated.

In summary, the test revealed that the simple back-to-back arrangement of poles is stable under dead load. Further analysis and testing is now required to determine how it will function under aerodynamic loads.

8 Structural stability theory

There is no closed form analytical solution for the buckling of a tapered column under varying dead load. The author thus wrote a Matlab script to calculate the Rayleigh-Ritz estimate of buckling load, using the section dimensions of the actual mast and the compressive axial forces due the horizontal component of the cable tensions. Since the Rayleigh-Ritz method gives only an upper bound on the buckling load, the deflected shape was assumed to be a high order polynomial with variable coefficients which were then sent to an optimisation loop to minimise the upper bound. The program was checked against known solutions, such as Greenhill's formula, and found to agree closely.

The Rayleigh-Ritz program is thus capable of giving highly accurate estimates of the buckling loads of that particular conceptual model - a tapered beam under varying axial loads. However, the conceptual model is not an exact representation of the more complicated reality. As the beam buckles, the cable tensions will change, which in turn will alter the axial forces in the beam. The full complexity could, in theory, be captured by a carefully-honed nonlinear finite element model, but this would take some considerable time to create and verify.

However, although the conceptual model solved does not capture the fully-nonlinear behaviour, this inability is mitigated by the known post-buckling behaviour of flexible slender rods. The classical problem of the post-buckling of flexible struts is known as The Elastica, and was solved analytically for simple cases by Euler some several hundred years ago. The upshot is that the buckling of slender struts is governed by a *stable symmetric bifurcation* and thus it has post-buckling strength and stiffness. That is, the word "buckling" may conjure images of immediate catastrophic collapse, but if there is stable post-buckling, it means that once the critical load is exceeded the structure will begin to experience very large displacements. Provided the material can withstand the large curvatures involved, it will not necessarily collapse. This accords with intuition: a slender vertical fishing rod carrying a large load at its tip will not remain vertical but will simply bend into a significantly bowed configuration.

The GFRP antenna falls very much into the ambit of the Elastica, thus one expects that if the critical loads for buckling are exceeded, all that will be observed is that the displacements become large.

Figure 3 shows a simple home experiment with an 8m model, consisting of two lightweight GFRP poles back-to-back. The poles are supported by a fan of cables from a low suspension point, and loaded with additional weights to create axial forces far in excess of the buckling load of the pole. It can be seen that the poles simply deform into an exaggerated bowed configuration.

9 Yoke height

Given that the Factor of Safety against buckling scales with the yoke height (the yoke being the point at which the fan of cables meet) it is preferable to have a high yoke height. Initial considerations suggest that the antenna poles will be some 40m below the helicopter. A yoke height of around 30m then seems to be a reasonable choice, at least as far as the structural stability is concerned. This would give a Factor of Safety of 3 on self-weight buckling, and this large safety margin on static loads will be required to have some chance of resisting all the additional dynamic and aerodynamic loads that the mast will be subject to when in flight.

It is worth pointing out here that the outline design initially proposed by BAS of a uniform GFRP tube of 114mm external diameter could also now work merely by raising the yoke height. The initial design had a yoke height of 10m and a Factor of Safety against static self-weight buckling of 1. Again, then, by raising the yoke height to 30m, this could be increased to a FoS of 3, to give a margin for resisting dynamic and aerodynamic loads.

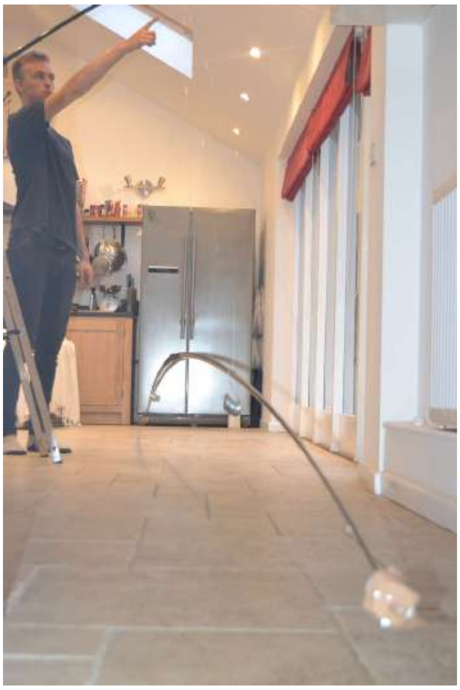

Figure 3: Post-buckling behaviour of an 8m pole under enhanced self-weight.

At this stage, then, we have two viable designs going forward, the BAS original and the much lighter back-to-back Spiderbeams.

10 Quasistatic aerodynamic forces

To provide aerodynamic stability when the structure is suspended beneath the helicopter in flight, it is presently proposed to have a small cone at the tail of the pole, with a diameter of around 0.5m. This provides a drag force in the flow direction. For a drag coefficient of 0.75, and at a velocity of 30m/s (68mph, 108 km/hr), this leads to a tail force of around 85N. This creates a moment about the yoke, and will cause the whole assembly to swing backwards until there is an equilibrating moment due to weight acting through the now-offset centre of mass.

Considering only the tail drag (85N) and the light Spiderbeam mast-plus-equipment weight (40+ 50 = 90kg = 900N), this would lead to the structure tilting to only around 5 degrees.

The internal structural forces may readily be recalculated in this inclined configuration. The main consequence is that the compressive forces along the Spiderbeams are reduced by the presence of the tension now coming from the tail cone drag. These additional tensile forces decrease approximately linearly as one heads from tail to tip. From a buckling perspective, then, the stability has thus been improved. Rather remarkably, one concludes at this stage that the structure seems to be less likely to buckle when in flight than when hanging stationary.

However, there are further aerodynamic effects to be considered which are not so beneficial. Firstly, there are the cables. Travelling at 30m/s, the drag on each cable would apply a horizontal force of the order 10N to the beam at the cable attachment point. This is around twice the compressive force in the tip end section of the beam under static dead loads. Such a 10N increment needs to be added at every cable attachment point. The total for 10 cables is thus around 100N, which is comparable to the drag from the tail cone.

There are two obvious consequences. The first is that the beneficial effects of the tensions arising from the tail have now been removed by the additional compressions that arise from the cable drags in the leading half of the beam. The second is that the angle of inclination of the mast is approximately doubled, and so would be around 10 degrees.

However, this increase in the inclination angle leads to a new effect: the beam itself now has a frontal area facing the wind. Although the beam is narrow, it is long, and seen head-on, it has a projected height of around 7m. Taking an average width of around 75mm, this is a projected area of around 0.5m^2, which is over twice the tail cone area. We must therefore at least double the inclination angle again, to around 20 degrees. Clearly this exposes more frontal area of the mast, and the process must be iterated. In addition, there will be lift forces normal to the beam line at the nose and tail induced by the mast now having an angle of attack. Even ignoring these temporarily, it is clear at this stage that the angle of inclination of the mast during rapid flight could be at a very steep angle.

The inclination can be estimated directly without iteration, by simple moment equilibrium about the yoke. This calculation suggests the Spiderbeams plus 50kg of kit (total weight 90kg) would have an inclination angle of around 60 degrees at 30m/s (68mph). This clearly large. In that configuration, the tail of the mast is now approaching the same height as the helicopter.

This situation can be considerably improved by adding weight at the centre of the beam. The relationship between inclination angle and weight is nonlinear, but can be readily solved, leading to the estimate that doubling the total weight to 180kg leads to an inclination angle of around 25 degrees at 30m/s (68mph). This appears to be a more reasonable attitude. Moreover, it is all now somewhat similar to the weight of the original BAS outline design, which was 210kg.

Further effects that will need to be considered at the detailed design stage are the quasistatic lift forces due to the angle of attack. Also, under static loads, the support cables are not highly stressed, and thus any transverse drag will cause them to exhibit substantial bowing, and the cable geometry can no longer be assumed to be a linear fan.

11 Summary and Conclusion

The structural investigation has focussed on two designs, the original BAS outline design (but with a higher yoke height) and the much lighter Spiderbeam design which is then made artificially heavier by the incorporation of a large central weight. The analysis here suggests that both systems are structurally feasible.

Each requires a yoke height of around 30m, this being 3/4 of the way to the attachment point on the helicopter 40 m above the beam (at rest).

The BAS design has the advantages of higher stiffness and weight, both of which are beneficial with respect to resisting dynamic loads.

The Spiderbeam design has the advantages of low cost (particularly when compared to the costs of helicopter rental) and easier transportation to site via its ability to telescope down to a small packing volume.

Aerodynamic Stability
of a Dipole Radar Concept

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July 2017

Abstract

The British Antarctic Survey wishes to design a dipole radar, to be deployed from a helicopter. The aerodynamic stability of this configuration is uncertain, and has therefore been investigated using the standard modelling techniques of aircraft stability theory. Two significant oscillatory motions have been identified: yawing of the radar about the hanging axis, and side-to-side displacements perpendicular to the flight direction. Over the range of parameters investigated, both motions are predicted to decay over time. Their frequencies vary depending on configuration parameters and flight speed; it is recommended that they should be kept apart via suitable design and operation specifications. Other uncertainties, associated with modelling limitations, should be addressed through full-scale flight tests of prototypes.

1 Introduction

This report describes a theoretical investigation into the aerodynamic properties of a proposed dipole radar. The radar consists of a long, thin rod, and is to be suspended from a helicopter. To prevent excessive rod deflections, it will be joined to the hanging line via a multi-cable yoke.

The steady aerodynamic and inertia forces associated with the radar will be small in the context of the helicopter's lifting capacity. However, the aerodynamic stability of the radar is of concern. This is the topic of the current investigation. As the radar design is yet to be finalised, the work has focussed on general characterisation of the unsteady behaviour, and its qualitative dependence on the design parameters.

The report consists of three main sections. In the first, a brief overview of the theoretical approach and its limitations is given. Next, the results of the model are presented. Finally, the implications of the results for design development are summarised via a set of recommendations.

2 Scope of Modelling

2.1 Theory

The theoretical formulation is a modified version of that developed for an earlier study [1]. It models the response of the radar to small (strictly speaking, infinitesimal) perturbations from its equilibrium orientation. Classification as 'stable' means that the subsequent motion is predicted to decay with time.

The restriction to small perturbations is necessary to develop a tractable model. It means that large-scale instability of a 'stable' configuration cannot be ruled out, given sufficient perturbations. However, this approach has a long, well-established and successful history in the field of aircraft design.

2.2 Geometry

Fore-aft symmetry has been assumed. Yoke cables have been taken to remain straight, i.e. unaffected by aerodynamic forces. In contrast, the hanging cable deflection under these forces *is* estimated. (This is a consequence of the evolution of the calculation from the earlier study. However, the apparent inconsistency in treatment can be partially justified by the longer hanging cable envisaged in the nominal design.)

The calculation is only able to consider a radar cylinder of constant diameter. This is consistent with the nominal design, but not with subsequent developments towards a telescoping device. An ad hoc approach for the latter is described below.

A 45° tail cone, with base diameter 0.35m, has been assumed. Its aerodynamic properties have been estimated from previous measurements on a similar device [1]. Its contributions to mass and moment of inertia have not been considered.

2.3 Cable dynamics

As for the steady deflections, the hanging cable is modelled with higher fidelity than the yoke cables. The latter are constrained to move as rigid bodies, whereas the former can exhibit 'string' modes.

3 Results

3.1 General features

Figure 1 shows the calculation results for the nominal configuration [2]. Here the 39m-long radar weighs 210kg, and is suspended 40m below the helicopter. The yoke cables are anchored at the joins between the 3m tube sections, and are gathered 10m above the radar (i.e. 30m below the helicopter). Flight speeds from 10–250km/h are considered.

The plot shows real and imaginary parts of the complex (radian) frequency for the two slowest motions predicted: ‘weathercocking’ (oscillations in yaw) and penduluming. It is arranged so the decay rate corresponds to the x coordinate, with increasingly negative values corresponding to faster decay.

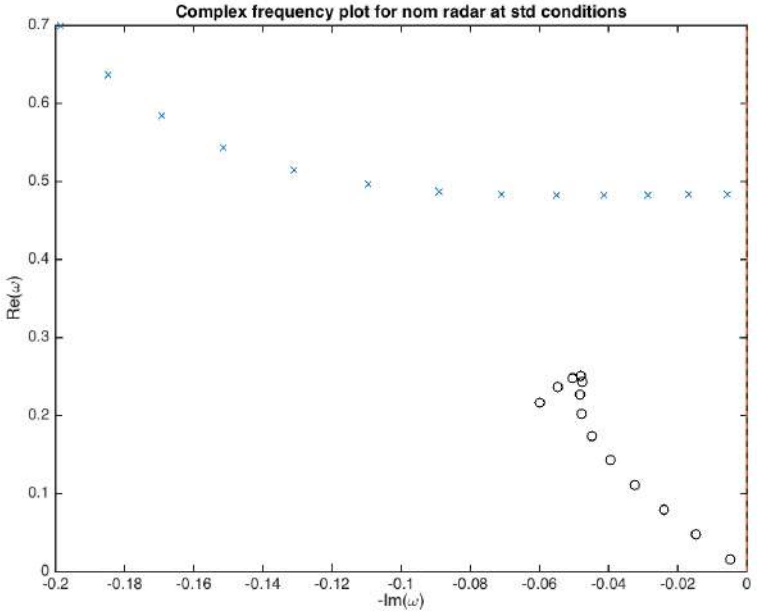

Figure 1. Complex-frequency plot for the weathercock (circles) and pendulum (crosses) motions of the nominal configuration at speeds from 10 to 250km/h.

At lower speeds (to right on plot), the pendulum motion has frequency very close to the theoretical value for a point mass on a 40m cable ($\sqrt{g/40} = 0.5\,\text{rad/s}$), while the weathercocking motion is extremely slow (the predicted period is about 400s). As the speed increases, the pendulum motion gains damping (higher decay rate), but retains an effectively constant frequency, while both properties rise for weathercocking. This is due to the greater aerodynamic effectiveness of the tail, which is responsible for restoring moments in yaw. At the highest speeds, evidence of interaction between the two modes of motion can be seen in the departures of their paths from their previous directions.

The lowest and highest speeds for this test case have also been investigated using a simplified calculation, which neglects hanging-cable dynamics. (It assumes the hanging cable has no mass or drag, in which case it must be straight.) This gives an indirect check on the assumptions made for the yoke cables; if the hanging cable can be successfully approximated as straight in this application, the same should apply for the yoke cables. The complex frequencies for both the simplified and the full calculations are given below, in Table 1. They are in excellent agreement, showing that hanging cable dynamics (and drag) are not significant.

Speed	**Weathercock**		**Pendulum**	
	Simplified	Full	Simplified	Full
10km/h	$(15.8 + 4.9i)\times10^{-3}$	$(15.8 + 4.9i)\times10^{-3}$	0.483+0.005i	0.483+0.006i
250km/h	0.214+0.058i	0.217+0.060i	0.693+0.197i	0.700+0.199i

Table 1. Comparison of simplified and full calculation results for complex frequencies.

3.2 Qualitative trends

The stability calculation has been run for various parameter ranges supplied by BAS [3]. In all cases, the radar was predicted to be stable to small perturbations.

When the mode frequencies are well separated, their damping increases with speed. The frequency of the pendulum motion remains approximately constant at its point-mass value, while that of the weathercock motion increases.

The weathercock frequency is also affected by the radar design parameters. Here the clearest, and most easily modified, influences are the moment of inertia and the tail; the greater the moment of inertia and the smaller the tail, the slower the oscillation. The number of yoke cables also has a noticeable effect; they contribute additional damping via their aerodynamic drag. Operationally, the overall hanging length is important via its impact on the pendulum frequency.

Finally, the form of mode interaction seen in Figure 1 is not universal. Various topologies have been observed; this feature seems to depend on specific design and operational details.

3.3 Telescoping design

The stability calculation has also been run for a design based on the telescopic rods used in the ground structural test. The parameters altered were the cylinder mass and yaw inertia, and the yoke arrangement. (The two central cable attachments were specified slightly further out than in the test, at the ends of the peli-case frame.) The yoke rope properties were left unchanged. As the calculation assumes a constant-diameter cylinder, configurations with maximum (102mm) and minimum (31mm) values were considered.

Figure 2 shows the complex frequency plot (cf. Fig. 1) for the minimum-diameter configuration. The key change is the increase in weathercock mode frequency. This is due to a significant reduction in yaw inertia compared to the nominal design. As a result, the weathercock frequency increases beyond that of the pendulum at the highest speeds. When their frequencies are comparable, their interaction leads to a reversal in the pendulum-mode speed/damping relationship. While no loss in stability is predicted, it is likely that this behaviour would be undesirable in practice.

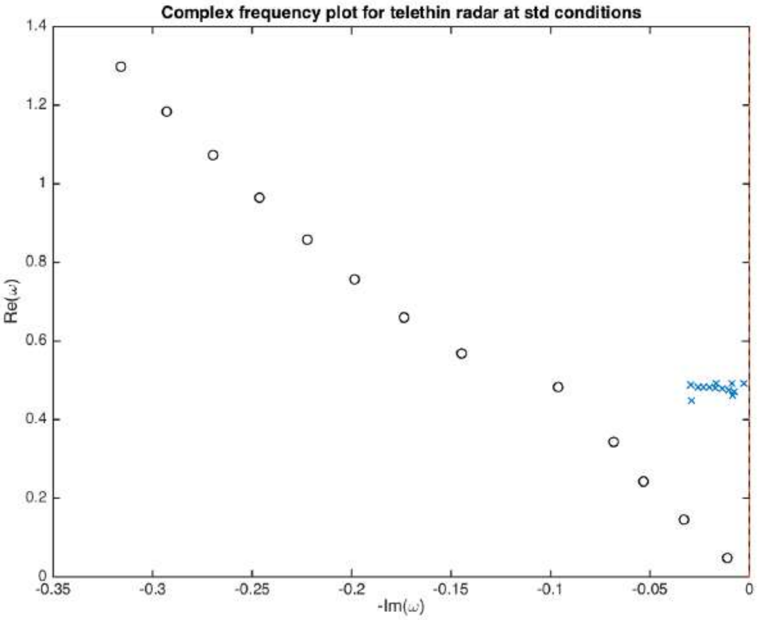

Figure 2. Complex-frequency plot for the weathercock (circles) and pendulum (crosses) motions of the minimum-diameter telescopic-inertia-matching configuration at speeds from 10 to 250km/h.

The maximum-diameter case is nominally more benign, in that the larger cylinder has greater aerodynamic damping, so the decay rate of the pendulum oscillation is predicted to be higher. However, this leads to a stronger interaction with the weathercock mode, to the extent that the two are no longer easily distinguishable at some speeds. Again, while this feature is not categorically pathological, it is probably best avoided if possible.

4 Recommendations

4.1 Mode interactions

As suggested above, these should ideally be avoided. This implies keeping the weathercock frequency well below that of the pendulum over the working speed range. The weathercock frequency is controllable in the design via the yaw inertia of the radar and the authority of the stabilising tail. In operation, its increase can be limited by reducing the maximum flight speed; equally, the pendulum frequency can be raised by shortening the hanging distance.

4.2 Yoke design

From an aerodynamic viewpoint, it appears that an increase in the number of yoke cables is beneficial; mode decay rates are predicted to increase.

4.3 Parameter uncertainty

Although sufficient historical data exists to estimate the aerodynamic parameters in the calculation with reasonable accuracy, they have not been measured for this specific configuration. Hence development testing should allow for some departure from the predicted behaviour. In particular, it would be prudent to investigate a range of potential tail-cone designs, if possible.

4.4 Flutter

'Flutter' refers to self-excited oscillations arising from interactions between aerodynamics and structural dynamics. The stability theory used here makes a rigid-body assumption for the radar; as a result it implicitly precludes flutter. In general, stiffer structures are less likely to flutter. A more specific minimum recommendation is that the rigid-body weathercocking and penduluming motions should have frequencies well below that of the first structural mode. This may impose a more stringent requirement on the structural design than buckling constraints alone.

5 Conclusions

This report has presented an investigation into the aerodynamic stability of a dipole-radar concept. Under the standard assumptions of aircraft-stability theory, the radar has been found to exhibit two significant modes of oscillation: 'weathercocking' (yaw fluctuations) and 'penduluming' (side-to-side motions). Both are predicted to be stable to small perturbations for all cases calculated to date.

In the parameter range of interest, the pendulum motion is normally of higher frequency than the weathercocking. However, the latter increases in frequency with flight speed. If it approaches the pendulum frequency, the modes interact, with case-dependent consequences. Although none are predicted to result in instability, it is recommended that the radar be designed to avoid them.

Finally, given the inevitable limitations of the theoretical model, there remains uncertainty over the behaviour of the radar in practice. Key concerns are: aerodynamic parameter uncertainty; large-scale perturbations; and the potential for flutter. Full-scale flight testing will be a prerequisite, and subsequent design modifications may well be necessary.

References

[1] 'Investigation of EM-bird stability in an open-jet facility', W R Graham. Fourth project report, November 2015.

[2] Private communication, D Ashurst, May 2017.

[3] Private communication, H Pritchard, June 2017.

Dipole Radar Combined Summary Report

Issues

1. Buckling

Buckling is a worry because of the compressive forces imposed by the yoke cables. For a given construction material, buckling safety is improved by increasing the factor I/A (see structures report), and by increasing yoke height.

2. Flutter

The likelihood of flutter depends on the natural vibration characteristics of the structure, and on flight speed. Higher natural frequencies (achievable as per buckling alleviation) are beneficial, as are lower flight speeds. Hence flutter margin may set the maximum flight speed.

3. Rigid-body mode interactions

The modes of concern are weathercocking and penduluming. Weathercocking frequency increases with flight speed, while the pendulum frequency is essentially fixed by overall hanging length. Hence the recommendation that interaction be avoided is another possible constraint on flight speed. The weathercocking frequency can also be reduced by increasing moment of inertia or decreasing tail size. Equally, the pendulum frequency can be increased by decreasing overall hanging length.

4. Attitude

At higher flight speeds, the radar may depart significantly from a horizontal orientation. The chief concern here is increased compression (and hence buckling risk); the associated deviation of the hanging cable from the vertical might also be undesirable for operational safety. This issue is mitigated by: reducing flight speed; increasing radar weight; decreasing tail size; decreasing yoke height; minimising yoke cable diameter.

Suggested design approach

The optimum design approach is not obvious, given the conflicts in some of the mitigation measures identified above. However, the following methodology is expected to produce a workable solution.

- Improve the buckling parameter I/A. For the spiderbeams, this would involve bundling; for the bespoke design it would mean wider tubes. In each case, it would also have the benefits of increasing weight (attitude issue) and moment of inertia (mode-interaction issue).
- Specify the yoke height on the basis of buckling and flutter requirements.
- Specify the hanging length to be as short as possible, subject to practical considerations and yoke height.
- Set tail size to maintain mode separation at maximum flight speed.
- Add central weight to maintain near-horizontal attitude at maximum flight speed.

Note that maximum flight speed is a free parameter, for which an initial value would need to be estimated. This could then be refined by successive design iterations.

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