

Supplementary material on "Dynamic Analysis of the UVMS: Effect of Disturbances, Coupling, and Joint-Flexibility on End-Effector Positioning"

Appendix A: Equations of motion of the UVMS considering flexible joints

The equations of motion of the UVMS considering flexible joints are given by (30):

$$M_v \dot{\xi} + C_v \xi + D_v \xi + G_v = F_v, \quad (\text{A1})$$

where the mass matrix M_v is given as follows:

$$M_v = \begin{bmatrix} m_{1,1} & \cdots & m_{1,10} \\ \vdots & \ddots & \vdots \\ m_{10,1} & \cdots & m_{10,10} \end{bmatrix}, \quad (\text{A2})$$

where all the non zero elements of M_v are given as follows:

$$\begin{aligned} m_{1,1} &= m_{a1} + m_{mau} + m_v + m_{vau} + (m_1 + m_2), \\ m_{5,1} &= z_g m_v, m_{6,1} = -y_g m_v, \\ m_{2,2} &= m_{mav} + m_v + m_{vav} + (m_1 + m_2), \\ m_{4,2} &= -z_g m_v, \\ m_{6,2} &= x_g m_v, \\ m_{3,3} &= m_{maw} + m_v + m_{vaw} + (m_1 + m_2), \\ m_{4,3} &= y_g m_v, \\ m_{5,3} &= -x_g m_v, \\ m_{2,4} &= -z_g m_v, \\ m_{3,4} &= y_g m_v, \\ m_{4,4} &= J_{ax} + J_{xx}, \\ m_{5,4} &= -J_{xy}, \\ m_{6,4} &= -J_{xz}, \\ m_{1,5} &= z_g m_v, \\ m_{3,5} &= -x_g m_v, \\ m_{4,5} &= -J_{xy}, \\ m_{5,5} &= J_{ay} + J_{yy}, \\ m_{6,5} &= -J_{yz}, \\ m_{1,6} &= -y_g m_v, \\ m_{2,6} &= x_g m_v, \\ m_{4,6} &= -J_{xz}, \\ m_{5,6} &= -J_{yz}, \\ m_{6,6} &= J_{az} + J_{zz}, \\ m_{7,7} &= J_{1,a} + J_{2,a} + l_{c1}^2 m_{1,au} + 2l_1 l_{c2} m_{2,au} \cos(\alpha_2(t)) + l_{c2}^2 m_{2,au} + l_1^2 m_{2,au} + J_{a2} + l_1^2 m_{a2} \\ &+ m_1 l_{c1}^2 + 2l_1 m_2 l_{c2} \cos(\alpha_2(t)) + m_2 l_{c2}^2 + J_1 + J_2 + l_1^2 m_2, \end{aligned}$$

$$\begin{aligned}
m_{8,7} &= J_{2,a} + l_1 l_{c2} m_{2,au} \cos(\alpha_2(t)) + l_{c2}^2 m_{2,au} + l_1 m_2 l_{c2} \cos(\alpha_2(t)) + m_2 l_{c2}^2 + J_2, \\
m_{7,8} &= J_{2,a} + l_1 l_{c2} m_{2,au} \cos(\alpha_2(t)) + l_{c2}^2 m_{2,au} + l_1 m_2 l_{c2} \cos(\alpha_2(t)) + m_2 l_{c2}^2 + J_2, \\
m_{8,8} &= J_{2,a} + l_{c2}^2 m_{2,au} + m_2 l_{c2}^2 + J_2, \\
m_{9,9} &= J_{a1}, \\
m_{10,10} &= J_{a2}.
\end{aligned}$$

The matrix C_v is given as follows:

$$C_v = \begin{bmatrix} c_{1,1} & \cdots & c_{1,10} \\ \vdots & \ddots & \vdots \\ c_{10,1} & \cdots & c_{10,10} \end{bmatrix}, \quad (\text{A3})$$

where all the non-zero elements of C_v are given below. It is to be noted that, the derivatives with respect to time are given as ()' and for a compact representation of the matrix components, the cosine and sine terms are represented as follows:

$$C_\phi = \cos(\phi(t)), S_\phi = \sin(\phi(t)), C_\theta = \cos(\theta(t)), S_\theta = \sin(\theta(t)), C_\psi = \cos(\psi(t)), S_\psi = \sin(\psi(t)),$$

$$\begin{aligned}
c_{1,1} = & -p(t)C_\phi J_{xz}S_\psi C_\theta^2 + C_\phi m_v S_\psi x_g v(t)C_\theta^2 + C_\phi C_\psi m_v y_g v(t)C_\theta^2 - C_\psi m_v S_\phi y_g w(t)C_\theta^2 \\
& + C_\psi m_{a1} S_\phi S_\psi \theta'(t)C_\theta^2 + C_\psi(m_1 + m_2)S_\phi S_\psi \theta'(t)C_\theta^2 + C_\psi m_{mau} S_\phi S_\psi \theta'(t)C_\theta^2 + C_\psi m_v S_\phi S_\psi \theta'(t)C_\theta^2 \\
& + C_\psi m_{vau} S_\phi S_\psi \theta'(t)C_\theta^2 - C_\psi m_{a1} S_\psi \psi'(t)C_\theta^2 - C_\psi(m_1 + m_2)S_\psi \psi'(t)C_\theta^2 - C_\psi m_{mau} S_\psi \psi'(t)C_\theta^2 \\
& - C_\psi m_v S_\psi \psi'(t)C_\theta^2 - C_\psi m_{vau} S_\psi \psi'(t)C_\theta^2 - p(t)J_{xz}S_\theta S_\phi C_\theta + p(t)J_{xy}S_\phi S_\psi C_\theta \\
& + m_v S_\theta S_\phi x_g v(t)C_\theta + C_\psi m_v S_\phi z_g v(t)C_\theta + m_v S_\phi S_\psi x_g w(t)C_\theta - m_v S_\theta S_\psi y_g w(t)C_\theta \\
& + C_\phi C_\psi m_v z_g w(t)C_\theta - C_\psi^2 m_{a1} S_\theta \theta'(t)C_\theta - C_\phi C_\psi m_{a1} S_\theta \theta'(t)C_\theta - C_\psi^2(m_1 + m_2)S_\theta \theta'(t)C_\theta \\
& - C_\phi C_\psi(m_1 + m_2)S_\theta \theta'(t)C_\theta - C_\psi^2 m_{mau} S_\theta \theta'(t)C_\theta - C_\phi C_\psi m_{mau} S_\theta \theta'(t)C_\theta - C_\psi^2 m_v S_\theta \theta'(t)C_\theta \\
& - C_\phi C_\psi m_v S_\theta \theta'(t)C_\theta - C_\psi^2 m_{vau} S_\theta \theta'(t)C_\theta - C_\phi C_\psi m_{vau} S_\theta \theta'(t)C_\theta + m_{a1} S_\phi S_\psi^2 \phi'(t)C_\theta \\
& + (m_1 + m_2)S_\phi S_\psi^2 \phi'(t)C_\theta + m_{mau} S_\phi S_\psi^2 \phi'(t)C_\theta + m_v S_\phi S_\psi^2 \phi'(t)C_\theta + m_{vau} S_\phi S_\psi^2 \phi'(t)C_\theta \\
& + C_\phi C_\psi m_{a1} S_\theta S_\psi \phi'(t)C_\theta + C_\phi C_\psi(m_1 + m_2)S_\theta S_\psi \phi'(t)C_\theta + C_\phi C_\psi m_{mau} S_\theta S_\psi \phi'(t)C_\theta \\
& + C_\phi C_\psi m_{vau} S_\theta S_\psi \phi'(t)C_\theta - m_{a1} S_\theta S_\phi S_\psi^2 \psi'(t)C_\theta - (m_1 + m_2)S_\theta S_\phi S_\psi^2 \psi'(t)C_\theta - m_{mau} S_\theta S_\phi S_\psi^2 \psi'(t)C_\theta \\
& - m_v S_\theta S_\phi S_\psi^2 \psi'(t)C_\theta - m_{vau} S_\theta S_\phi S_\psi^2 \psi'(t)C_\theta - C_\phi C_\psi m_{a1} S_\psi \psi'(t)C_\theta - C_\phi C_\psi(m_1 + m_2)S_\psi \psi'(t)C_\theta \\
& - C_\phi C_\psi m_{mau} S_\theta \psi'(t)C_\theta - C_\phi C_\psi m_v S_\psi \psi'(t)C_\theta - C_\phi C_\psi m_{vau} S_\psi \psi'(t)C_\theta - p(t)C_\phi J_{xy} S_\theta \\
& + r(t)(J_{yz}(C_\theta S_\phi S_\psi - C_\phi S_\theta) + C_\theta J_{az}(S_\theta S_\phi + C_\phi C_\phi S_\psi) + C_\phi C_\psi m_v S_\theta S_\psi \phi'(t)C_\theta \\
& + C_\theta J_{zz}(S_\theta S_\phi + C_\theta C_\phi S_\psi)) + q(t)(-C_\theta J_{yz}(S_\theta S_\phi + C_\theta C_\phi S_\psi) + J_{ay}(C_\phi S_\theta - C_\theta S_\phi S_\psi) + J_{yy}(C_\phi S_\theta \\
& - C_\theta S_\phi S_\psi)) - m_v S_\theta^2 y_g v(t) - C_\phi m_v S_\theta x_g w(t) + C_\psi m_{a1} S_\theta^2 S_\phi \phi'(t) + C_\phi m_{mau} S_\theta^2 S_\psi \psi'(t) \\
& + C_\psi(m_1 + m_2)S_\theta^2 S_\phi \phi'(t) + C_\psi m_{mau} S_\theta^2 S_\phi \phi'(t) + C_\psi m_v S_\theta^2 S_\phi \phi'(t) + C_\psi m_{vau} S_\theta^2 S_\phi \phi'(t) \\
& - C_\phi m_{a1} S_\theta S_\psi \phi'(t) - C_\phi(m_1 + m_2)S_\theta S_\psi \phi'(t) - C_\phi m_{mau} S_\theta S_\psi \phi'(t) - C_\phi m_v S_\theta S_\psi \phi'(t) \\
& - C_\phi m_{vau} S_\theta S_\psi \phi'(t) - C_\psi m_{a1} S_\theta S_\phi \psi'(t) - C_\psi(m_1 + m_2)S_\theta S_\phi \psi'(t) - C_\psi m_{mau} S_\theta S_\phi \psi'(t) \\
& - C_\psi m_v S_\theta S_\phi \psi'(t) - C_\psi m_{vau} S_\theta S_\phi \psi'(t) + C_\phi m_{a1} S_\theta^2 S_\psi \psi'(t) + C_\phi(m_1 + m_2)S_\theta^2 S_\psi \psi'(t) \\
& + C_\phi m_v S_\theta^2 S_\psi \psi'(t) + C_\phi m_{vau} S_\theta^2 S_\psi \psi'(t),
\end{aligned}$$

$$\begin{aligned}
c_{2,1} = & p(t)J_{xz}S_\phi^2C_\theta^2 - m_vS_\phi^2x_gv(t)C_\theta^2 + C_\phi C_\psi m_{a1}S_\phi\theta'(t)C_\theta^2 + C_\phi C_\psi(m_1 + m_2)S_\phi\theta'(t)C_\theta^2 \\
& + C_\phi C_\psi m_{mau}S_\phi\theta'(t)C_\theta^2 + C_\phi C_\psi m_vS_\phi\theta'(t)C_\theta^2 + C_\phi C_\psi m_{vau}S_\phi\theta'(t)C_\theta^2 - p(t)C_\phi^2C_\psi J_{xz}C_\theta \\
& + p(t)C_\phi J_{xy}S_\phi C_\theta - p(t)C_\phi J_{xz}S_\theta S_\phi S_\psi C_\theta + C_\phi^2C_\psi m_vx_gv(t)C_\theta + C_\phi m_vS_\theta S_\phi S_\psi x_gv(t)C_\theta \\
& + C_\phi C_\psi m_vS_\theta S_\phi y_gv(t)C_\theta + m_vS_\theta S_\phi y_gv(t)C_\theta - C_\phi^2m_vS_\psi y_gv(t)C_\theta + C_\phi m_vS_\phi x_gw(t)C_\theta \\
& - C_\phi m_vS_\theta S_\phi^2y_gw(t)C_\theta + C_\phi m_vS_\phi S_\psi y_gw(t)C_\theta + C_\phi C_\psi^2m_{a1}S_\phi\theta'(t)C_\theta + C_\phi C_\psi^2(m_1 + m_2)S_\phi\theta'(t)C_\theta \\
& + C_\phi C_\psi^2m_{mau}S_\phi\theta'(t)C_\theta + C_\phi C_\psi^2m_vS_\phi\theta'(t)C_\theta + C_\phi C_\psi^2m_{vau}S_\phi\theta'(t)C_\theta + C_\psi m_{a1}S_\theta S_\phi^2S_\psi\theta'(t)C_\theta \\
& + C_\psi(m_1 + m_2)S_\theta S_\phi^2S_\psi\theta'(t)C_\theta + C_\psi m_{mau}S_\theta S_\phi^2S_\psi\theta'(t)C_\theta + C_\psi m_vS_\theta S_\phi^2S_\psi\theta'(t)C_\theta \\
& - C_\psi m_{a1}S_\theta S_\phi^2\phi'(t)C_\theta - C_\psi(m_1 + m_2)S_\theta S_\phi^2\phi'(t)C_\theta - C_\psi m_{mau}S_\theta S_\phi^2\phi'(t)C_\theta - C_\psi m_vS_\theta S_\phi^2\phi'(t)C_\theta \\
& - C_\psi m_{vau}S_\theta S_\phi^2\phi'(t)C_\theta + C_\phi m_{a1}S_\phi S_\psi\phi'(t)C_\theta + C_\phi(m_1 + m_2)S_\phi S_\psi\phi'(t)C_\theta + C_\phi m_{mau}S_\phi S_\psi\phi'(t)C_\theta \\
& + C_\phi m_vS_\phi S_\psi\phi'(t)C_\theta + C_\phi m_{vau}S_\phi S_\psi\phi'(t)C_\theta + C_\psi m_{a1}S_\phi^2\psi'(t)C_\theta + C_\psi(m_1 + m_2)S_\phi^2\psi'(t)C_\theta \\
& + C_\psi m_{mau}S_\phi^2\psi'(t)C_\theta + C_\psi m_vS_\phi^2\psi'(t)C_\theta + C_\psi m_{vau}S_\phi^2\psi'(t)C_\theta + C_\phi m_{a1}S_\psi^2\psi'(t)C_\theta \\
& + C_\phi(m_1 + m_2)S_\psi^2\psi'(t)C_\theta + C_\phi m_{mau}S_\psi^2\psi'(t)C_\theta + C_\phi m_vS_\psi^2\psi'(t)C_\theta + C_\phi m_{vau}S_\psi^2\psi'(t)C_\theta \\
& - C_\phi m_{a1}S_\theta S_\phi S_\psi\psi'(t)C_\theta - C_\psi m_{a1}S_\theta S_\phi S_\psi\psi'(t)C_\theta - C_\phi(m_1 + m_2)S_\theta S_\phi S_\psi\psi'(t)C_\theta \\
& - C_\phi m_{mau}S_\theta S_\phi S_\psi\psi'(t)C_\theta - C_\psi m_{mau}S_\theta S_\phi S_\psi\psi'(t)C_\theta - C_\phi m_vS_\theta S_\phi S_\psi\psi'(t)C_\theta - C_\psi m_vS_\theta S_\phi S_\psi\psi'(t)C_\theta \\
& - C_\phi m_{vau}S_\theta S_\phi S_\psi\psi'(t)C_\theta - C_\psi m_{vau}S_\theta S_\phi S_\psi\psi'(t)C_\theta + p(t)C_\phi C_\psi J_{xy}S_\phi + q(t)(C_\theta J_{yz}(-C_\psi C_\phi^2 \\
& - S_\theta S_\phi S_\psi C_\phi + C_\theta S_\phi^2) - J_{ay}S_\phi(C_\theta C_\phi - C_\psi(m_1 + m_2)S_\theta S_\phi S_\psi\psi'(t)C_\theta + C_\psi m_{vau}S_\theta S_\phi^2S_\psi\theta'(t)C_\theta \\
& + C_\psi C_\phi + S_\theta S_\phi S_\psi) - J_{yy}S_\phi(C_\theta C_\phi + C_\psi C_\phi + S_\theta S_\phi S_\psi)) + r(t)(C_\theta J_{az}(C_\psi C_\phi^2 + S_\theta S_\phi S_\psi C_\phi - C_\theta S_\phi^2) \\
& + C_\theta J_{zz}(C_\psi C_\phi^2 + S_\theta S_\phi S_\psi C_\phi - C_\theta S_\phi^2) + J_{yz}S_\phi(C_\theta C_\phi + C_\psi C_\phi + S_\theta S_\phi S_\psi)) + p(t)J_{xy}S_\theta S_\phi^2S_\psi \\
& + C_\psi m_vS_\theta S_\phi^2z_gv(t) - C_\phi m_vS_\phi S_\psi z_gv(t) + C_\phi C_\psi m_vS_\phi x_gw(t) + m_vS_\theta S_\phi^2S_\psi x_gw(t) - C_\phi C_\psi m_vS_\theta y_gw(t) \\
& - m_vS_\theta^2S_\phi S_\psi y_gw(t) + C_\phi C_\psi m_vS_\theta S_\phi z_gw(t) - C_\phi^2m_vS_\psi z_gw(t) - C_\psi^2m_{a1}S_\theta^2S_\phi\theta'(t) \\
& - C_\psi^2(m_1 + m_2)S_\theta^2S_\phi\theta'(t) - C_\psi^2m_{mau}S_\theta^2S_\phi\theta'(t) - C_\psi^2m_vS_\theta^2S_\phi\theta'(t) - C_\psi^2m_{vau}S_\theta^2S_\phi\theta'(t) \\
& + C_\phi C_\psi m_{a1}S_\theta S_\psi\theta'(t) + C_\phi C_\psi(m_1 + m_2)S_\theta S_\psi\theta'(t) + C_\phi C_\psi m_{mau}S_\theta S_\psi\theta'(t) + C_\phi C_\psi m_vS_\theta S_\psi\theta'(t) \\
& + C_\phi C_\psi m_{vau}S_\theta S_\psi\theta'(t) + m_{a1}S_\theta S_\phi^2S_\psi^2\phi'(t) + (m_1 + m_2)S_\theta S_\phi^2S_\psi^2\phi'(t) + m_{mau}S_\theta S_\phi^2S_\psi^2\phi'(t) \\
& + m_vS_\theta S_\phi^2S_\psi\phi'(t) + m_{vau}S_\theta S_\phi^2S_\psi\phi'(t) + C_\phi^2C_\psi^2m_{a1}S_\theta\phi'(t) + C_\phi^2C_\psi^2(m_1 + m_2)S_\theta\phi'(t) \\
& + C_\phi^2C_\psi^2m_{mau}S_\theta\phi'(t) + C_\phi^2C_\psi^2m_vS_\theta\phi'(t) + C_\phi^2C_\psi^2m_{vau}S_\theta\phi'(t) + C_\phi C_\psi m_{a1}S_\theta^2S_\phi S_\psi\phi'(t) \\
& + C_\phi C_\psi(m_1 + m_2)S_\theta^2S_\phi S_\psi\phi'(t) + C_\phi C_\psi m_{mau}S_\theta S_\psi\phi'(t) + C_\phi C_\psi m_vS_\theta S_\psi\phi'(t) + C_\phi C_\psi m_{vau}S_\theta S_\psi\phi'(t) \\
& + C_\phi C_\psi m_{vau}S_\phi S_\psi\phi'(t) - m_{a1}S_\theta^2S_\phi^2S_\psi^2\psi'(t) - (m_1 + m_2)S_\theta^2S_\phi^2S_\psi^2\psi'(t) - m_{mau}S_\theta^2S_\phi^2S_\psi^2\psi'(t) \\
& - m_vS_\theta^2S_\phi^2S_\psi^2\psi'(t) - m_{vau}S_\theta^2S_\phi^2S_\psi^2\psi'(t) - C_\phi^2C_\psi^2m_{a1}\psi'(t) - C_\phi^2C_\psi^2(m_1 + m_2)\psi'(t) \\
& - C_\phi^2C_\psi^2m_{mau}\psi'(t) - C_\phi^2C_\psi^2m_v\psi'(t) - C_\phi^2C_\psi^2m_{vau}\psi'(t) - 2C_\phi C_\psi m_{a1}S_\theta S_\phi S_\psi\psi'(t) \\
& - 2C_\phi C_\psi(m_1 + m_2)S_\theta S_\phi S_\psi\psi'(t) - 2C_\phi C_\psi m_{mau}S_\theta S_\phi S_\psi\psi'(t) - 2C_\phi C_\psi m_vS_\theta S_\phi S_\psi\psi'(t) \\
& - 2C_\phi C_\psi m_{vau}S_\theta S_\phi S_\psi\psi'(t) + C_\phi C_\psi m_{vau}S_\theta^2S_\phi S_\psi\phi'(t),
\end{aligned}$$

$$\begin{aligned}
c_{3,1} = & p(t)C_\phi J_{xz}S_\phi C_\theta^2 - C_\phi m_v S_\phi x_g v(t)C_\theta^2 + C_\phi^2 C_\psi m_{a1} \theta'(t)C_\theta^2 + C_\phi^2 C_\psi (m_1 + m_2) \theta'(t)C_\theta^2 \\
& + C_\phi^2 C_\psi m_{mau} \theta'(t)C_\theta^2 + C_\phi^2 C_\psi m_v \theta'(t)C_\theta^2 + C_\phi^2 C_\psi m_{vau} \theta'(t)C_\theta^2 + p(t)C_\phi^2 J_{xy} C_\theta \\
& + p(t)C_\phi C_\psi J_{xz} S_\phi C_\theta - p(t)C_\phi^2 J_{xz} S_\theta S_\psi C_\theta - C_\phi C_\psi m_v S_\phi x_g v(t)C_\theta + C_\phi^2 m_v S_\theta S_\psi x_g v(t)C_\theta \\
& + C_\phi m_v S_\theta y_g v(t)C_\theta + C_\phi^2 C_\psi m_v S_\theta y_g v(t)C_\theta + C_\phi m_v S_\phi S_\psi y_g v(t)C_\theta + C_\phi^2 m_v x_g w(t)C_\theta \\
& - C_\phi C_\psi m_v S_\theta S_\phi y_g w(t)C_\theta - m_v S_\phi^2 S_\psi y_g w(t)C_\theta - C_\psi^2 m_{a1} S_\phi^2 \theta'(t)C_\theta - C_\psi^2 (m_1 + m_2) S_\phi^2 \theta'(t)C_\theta \\
& - C_\psi^2 m_{mau} S_\phi^2 \theta'(t)C_\theta - C_\psi^2 m_v S_\phi^2 \theta'(t)C_\theta - C_\psi^2 m_{vau} S_\phi^2 \theta'(t)C_\theta + C_\phi C_\psi m_{a1} S_\theta S_\phi S_\psi \theta'(t)C_\theta \\
& + C_\phi C_\psi (m_1 + m_2) S_\theta S_\phi S_\psi \theta'(t)C_\theta + C_\phi C_\psi m_{mau} S_\theta S_\phi S_\psi \theta'(t)C_\theta + C_\phi C_\psi m_v S_\theta S_\phi S_\psi \theta'(t)C_\theta \\
& - C_\phi C_\psi m_{a1} S_\theta S_\phi \phi'(t)C_\theta - C_\phi C_\psi (m_1 + m_2) S_\theta S_\phi \phi'(t)C_\theta - C_\phi C_\psi m_{mau} S_\theta S_\phi \phi'(t)C_\theta \\
& - C_\phi C_\psi m_{vau} S_\theta S_\phi \phi'(t)C_\theta + C_\phi^2 m_{a1} S_\psi \phi'(t)C_\theta + C_\phi^2 (m_1 + m_2) S_\psi \phi'(t)C_\theta + C_\phi^2 m_{mau} S_\psi \phi'(t)C_\theta \\
& + C_\phi^2 m_v S_\psi \phi'(t)C_\theta + C_\phi^2 m_{vau} S_\psi \phi'(t)C_\theta - m_{a1} S_\phi S_\psi^2 \phi'(t)C_\theta - (m_1 + m_2) S_\phi S_\psi^2 \phi'(t)C_\theta \\
& - m_{mau} S_\phi S_\psi^2 \phi'(t)C_\theta - m_v S_\phi S_\psi^2 \phi'(t)C_\theta - m_{vau} S_\phi S_\psi^2 \phi'(t)C_\theta + C_\phi C_\psi m_{a1} S_\phi \psi'(t)C_\theta \\
& + C_\phi C_\psi (m_1 + m_2) S_\phi \psi'(t)C_\theta + C_\phi C_\psi m_{mau} S_\phi \psi'(t)C_\theta + C_\phi C_\psi m_{vau} S_\phi \psi'(t)C_\theta + C_\phi C_\psi m_v S_\phi \psi'(t)C_\theta \\
& - C_\phi^2 m_{a1} S_\theta S_\psi \psi'(t)C_\theta - C_\phi C_\psi m_{a1} S_\theta S_\psi \psi'(t)C_\theta - C_\phi^2 (m_1 + m_2) S_\theta S_\psi \psi'(t)C_\theta \\
& - C_\phi^2 m_{mau} S_\theta S_\psi \psi'(t)C_\theta - C_\phi C_\psi m_{mau} S_\theta S_\psi \psi'(t)C_\theta - C_\phi^2 m_v S_\theta S_\psi \psi'(t)C_\theta - C_\phi C_\psi m_v S_\theta S_\psi \psi'(t)C_\theta \\
& - C_\phi^2 m_{vau} S_\theta S_\psi \psi'(t)C_\theta - C_\phi C_\psi m_{vau} S_\theta S_\psi \psi'(t)C_\theta - p(t)C_\psi J_{xy} S_\phi^2 - r(t)(-J_{yz}(C_\theta C_\phi^2 \\
& + S_\theta S_\phi S_\psi C_\phi - C_\psi S_\phi^2) + C_\theta C_\phi J_{az}(C_\theta S_\phi + C_\phi C_\psi m_{vau} S_\theta S_\phi S_\psi \theta'(t)C_\theta - C_\phi C_\psi m_v S_\theta S_\phi \phi'(t)C_\theta \\
& + C_\psi S_\phi - C_\phi S_\theta S_\psi) + C_\theta C_\phi J_{zz}(C_\theta S_\phi + C_\psi S_\phi - C_\phi S_\theta S_\psi)) + q(t)(-J_{yy}(C_\theta C_\phi^2 + S_\theta S_\phi S_\psi C_\phi \\
& - C_\phi C_\psi (m_1 + m_2) S_\theta S_\psi \psi'(t)C_\theta - C_\psi S_\phi^2) - J_{ay}(C_\theta C_\phi^2 + S_\phi(C_\phi S_\theta S_\psi - C_\psi S_\phi)) \\
& - C_\phi^2 C_\psi m_{vau} S_\theta S_\psi \psi'(t) + C_\theta C_\phi J_{yz}(C_\theta S_\phi + C_\psi S_\phi - C_\phi S_\theta S_\psi)) + p(t)C_\phi J_{xy} S_\theta S_\phi S_\psi \\
& + C_\phi C_\psi m_v S_\theta S_\phi z_g v(t) + m_v S_\phi^2 S_\psi z_g v(t) - C_\psi m_v S_\phi^2 x_g w(t) + C_\phi m_v S_\theta S_\phi S_\psi x_g w(t) \\
& + C_\psi m_v S_\theta S_\phi y_g w(t) - C_\phi m_v S_\theta^2 S_\psi y_g w(t) + C_\phi^2 C_\psi m_v S_\theta z_g w(t) + C_\phi m_v S_\phi S_\psi z_g w(t) \\
& - C_\phi C_\psi^2 m_{a1} S_\theta^2 \theta'(t) - C_\phi C_\psi^2 (m_1 + m_2) S_\theta^2 \theta'(t) - C_\phi C_\psi^2 m_{mau} S_\theta^2 \theta'(t) - C_\phi C_\psi^2 m_v S_\theta^2 \theta'(t) \\
& - C_\phi C_\psi^2 m_{vau} S_\theta^2 \theta'(t) - C_\psi m_{a1} S_\theta S_\phi S_\psi \theta'(t) - C_\psi (m_1 + m_2) S_\theta S_\phi S_\psi \theta'(t) - C_\psi m_{mau} S_\theta S_\phi S_\psi \theta'(t) \\
& - C_\psi m_v S_\theta S_\phi \theta'(t) - C_\psi m_{vau} S_\theta S_\phi S_\psi \theta'(t) + C_\phi m_{a1} S_\theta S_\phi S_\psi^2 \phi'(t) + C_\phi (m_1 + m_2) S_\theta S_\phi S_\psi^2 \phi'(t) \\
& + C_\phi m_{mau} S_\theta S_\phi S_\psi^2 \phi'(t) + C_\phi m_v S_\theta S_\phi S_\psi^2 \phi'(t) + C_\phi m_{vau} S_\theta S_\phi S_\psi^2 \phi'(t) - C_\phi C_\psi^2 m_{a1} S_\theta S_\phi \phi'(t) \\
& - C_\phi C_\psi^2 (m_1 + m_2) S_\theta S_\phi \phi'(t) - C_\phi C_\psi^2 m_{mau} S_\theta S_\phi \phi'(t) - C_\phi C_\psi^2 m_v S_\theta S_\phi \phi'(t) - C_\phi C_\psi^2 m_{vau} S_\theta S_\phi \phi'(t) \\
& + C_\phi^2 C_\psi m_{a1} S_\theta^2 S_\psi \phi'(t) + C_\phi^2 C_\psi (m_1 + m_2) S_\theta^2 S_\psi \phi'(t) + C_\phi^2 C_\psi m_{mau} S_\theta^2 S_\psi \phi'(t) + C_\phi^2 C_\psi m_v S_\theta^2 S_\psi \phi'(t) \\
& + C_\phi^2 C_\psi m_{vau} S_\theta^2 S_\psi \phi'(t) - C_\psi m_{a1} S_\phi^2 S_\psi \phi'(t) - C_\psi (m_1 + m_2) S_\phi^2 S_\psi \phi'(t) - C_\psi m_{mau} S_\phi^2 S_\psi \phi'(t) \\
& - C_\psi m_v S_\phi^2 S_\psi \phi'(t) - C_\psi m_{vau} S_\phi^2 S_\psi \phi'(t) - C_\phi m_{a1} S_\theta^2 S_\psi^2 \psi'(t) - C_\phi (m_1 + m_2) S_\theta^2 S_\phi S_\psi^2 \psi'(t) \\
& - C_\phi m_{mau} S_\theta^2 S_\phi S_\psi^2 \psi'(t) - C_\phi m_v S_\theta^2 S_\phi S_\psi^2 \psi'(t) - C_\phi m_{vau} S_\theta^2 S_\phi S_\psi^2 \psi'(t) + C_\phi C_\psi^2 m_{a1} S_\phi \psi'(t) \\
& + C_\phi C_\psi^2 (m_1 + m_2) S_\phi \psi'(t) + C_\phi C_\psi^2 m_{mau} S_\phi \psi'(t) + C_\phi C_\psi^2 m_v S_\phi \psi'(t) + C_\phi C_\psi^2 m_{vau} S_\phi \psi'(t) \\
& + C_\psi m_{a1} S_\theta^2 S_\psi \psi'(t) + C_\psi (m_1 + m_2) S_\theta^2 S_\psi \psi'(t) + C_\psi m_{mau} S_\theta^2 S_\psi \psi'(t) + C_\psi m_v S_\theta^2 S_\psi \psi'(t) \\
& + C_\psi m_{vau} S_\theta^2 S_\psi \psi'(t) - C_\phi^2 C_\psi m_{a1} S_\theta S_\psi \psi'(t) - C_\phi^2 C_\psi (m_1 + m_2) S_\theta S_\psi \psi'(t) - C_\phi^2 C_\psi m_{mau} S_\theta S_\psi \psi'(t) \\
& - C_\phi^2 C_\psi m_v S_\theta S_\psi \psi'(t),
\end{aligned}$$

$$c_{4,1} = m_v(C_\phi z_g r(t) + C_\theta y_g \theta'(t) + z_g q(t) S_\phi),$$

$$\begin{aligned} c_{5,1} = & m_v(z_g q(t) S_\phi^2 \tan(\theta(t)) (C_\phi (\sec(\theta(t)) + 1) + 1) + C_\phi z_g r(t) S_\phi \tan(\theta(t)) (C_\phi (\sec(\theta(t)) \\ & + 1) + 1) + C_\phi y_g S_\theta S_\phi \theta'(t) + C_\phi y_g S_\phi \theta'(t) \tan(\theta(t)) - C_\phi z_g S_\phi \sec(\theta(t)) \phi'(t) \\ & - C_\phi z_g S_\phi \phi'(t) - C_\theta C_\phi^2 y_g \phi'(t) + y_g S_\theta S_\phi \theta'(t) + y_g S_\phi^2 \phi'(t)), \end{aligned}$$

$$\begin{aligned} c_{6,1} = & m_v(z_g q(t) S_\phi \tan(\theta(t)) (C_\phi^2 \sec(\theta(t)) + C_\phi - S_\phi^2) + C_\phi z_g r(t) \tan(\theta(t)) (C_\phi^2 \sec(\theta(t)) \\ & + C_\phi - S_\phi^2) + C_\phi y_g S_\theta \theta'(t) + C_\theta C_\phi y_g S_\phi \phi'(t) + C_\phi y_g S_\phi \phi'(t) \\ & + C_\phi^2 y_g \theta'(t) \tan(\theta(t)) - C_\phi^2 z_g \sec(\theta(t)) \phi'(t) - y_g S_\theta S_\phi^2 \theta'(t) + z_g S_\phi^2 \phi'(t)), \end{aligned}$$

$$\begin{aligned} c_{1,2} = & -r(t) C_\phi C_\psi J_{az} C_\theta^2 - r(t) C_\phi C_\psi J_{zz} C_\theta^2 + C_\phi m_v S_\psi x_g u(t) C_\theta^2 \\ & + C_\phi C_\psi m_v y_g u(t) C_\theta^2 + C_\psi m_v S_\phi x_g w(t) C_\theta^2 \\ & + (m_1 + m_2) S_\phi S_\psi^2 \theta'(t) C_\theta^2 + m_{\text{mav}} S_\phi S_\psi^2 \theta'(t) C_\theta^2 + m_v S_\phi S_\psi^2 \theta'(t) C_\theta^2 + m_{\text{vav}} S_\phi S_\psi^2 \theta'(t) C_\theta^2 \\ & + C_\psi^2 (m_1 + m_2) \psi'(t) C_\theta^2 + C_\psi^2 m_{\text{mav}} \psi'(t) C_\theta^2 + C_\psi^2 m_v \psi'(t) C_\theta^2 + C_\psi^2 m_{\text{vav}} \psi'(t) C_\theta^2 \\ & - r(t) C_\psi J_{yz} S_\phi C_\theta + m_v S_\theta S_\phi x_g u(t) C_\theta + C_\psi m_v S_\phi z_g u(t) C_\theta - C_\psi m_v S_\phi x_g w(t) C_\theta \\ & + m_v S_\theta S_\psi x_g w(t) C_\theta + m_v S_\psi z_g w(t) C_\theta - C_\phi (m_1 + m_2) S_\theta S_\psi \theta'(t) C_\theta - C_\psi (m_1 + m_2) S_\theta S_\psi \theta'(t) C_\theta \\ & - C_\phi m_{\text{mav}} S_\theta S_\psi \theta'(t) C_\theta - C_\psi m_{\text{mav}} S_\theta S_\psi \theta'(t) C_\theta - C_\phi m_v S_\theta S_\psi \theta'(t) C_\theta - C_\psi m_v S_\theta S_\psi \theta'(t) C_\theta \\ & - C_\phi m_{\text{vav}} S_\theta S_\psi \theta'(t) C_\theta - C_\psi m_{\text{vav}} S_\theta S_\psi \theta'(t) C_\theta + C_\phi (m_1 + m_2) S_\theta S_\psi^2 \phi'(t) C_\theta + C_\phi m_{\text{mav}} S_\theta S_\psi^2 \phi'(t) C_\theta \\ & + C_\phi m_v S_\theta S_\psi^2 \phi'(t) C_\theta + C_\phi m_{\text{vav}} S_\theta S_\psi^2 \phi'(t) C_\theta - C_\psi (m_1 + m_2) S_\phi S_\psi \phi'(t) C_\theta - C_\psi m_{\text{mav}} S_\phi S_\psi \phi'(t) C_\theta \\ & - C_\psi m_v S_\phi S_\psi \phi'(t) C_\theta - C_\psi m_{\text{vav}} S_\phi S_\psi \phi'(t) C_\theta - C_\phi (m_1 + m_2) S_\psi^2 \psi'(t) C_\theta - C_\phi m_{\text{mav}} S_\psi^2 \psi'(t) C_\theta \\ & - C_\phi m_v S_\psi^2 \psi'(t) C_\theta - C_\phi m_{\text{vav}} S_\psi^2 \psi'(t) C_\theta + C_\psi (m_1 + m_2) S_\theta S_\phi S_\psi \psi'(t) C_\theta + C_\psi m_{\text{mav}} S_\theta S_\phi S_\psi \psi'(t) C_\theta \\ & + C_\psi m_v S_\theta S_\phi S_\psi \psi'(t) C_\theta + C_\psi m_{\text{vav}} S_\theta S_\phi S_\psi \psi'(t) C_\theta + r(t) J_{az} S_\theta^2 + r(t) J_{zz} S_\theta^2 \\ & + r(t) J_{xz} S_\theta - p(t) (-C_\phi C_\psi J_{xz} C_\theta^2 + C_\psi J_{xy} S_\phi C_\theta + J_{xz} S_\theta^2 \\ & + J_{ax} S_\theta + J_{xx} S_\theta) + q(t) (C_\phi C_\psi J_{yz} C_\theta^2 + C_\psi J_{ay} S_\phi C_\theta \\ & + C_\psi J_{yy} S_\phi C_\theta - J_{yz} S_\theta^2 + J_{xy} S_\theta) - m_v S_\theta^2 y_g u(t) - m_v S_\theta y_g w(t) \\ & + C_\phi C_\psi (m_1 + m_2) S_\theta \phi'(t) + C_\phi C_\psi m_{\text{mav}} S_\theta \phi'(t) + C_\phi C_\psi m_v S_\theta \phi'(t) + C_\phi C_\psi m_{\text{vav}} S_\theta \phi'(t) \\ & + (m_1 + m_2) S_\theta^2 S_\phi S_\psi \phi'(t) + m_{\text{mav}} S_\theta^2 S_\phi S_\psi \phi'(t) + m_v S_\theta^2 S_\phi S_\psi \phi'(t) + m_{\text{vav}} S_\theta^2 S_\phi S_\psi \phi'(t) \\ & - C_\phi C_\psi (m_1 + m_2) S_\theta^2 \psi'(t) - C_\phi C_\psi m_{\text{mav}} S_\theta^2 \psi'(t) - C_\phi C_\psi m_v S_\theta^2 \psi'(t) - C_\phi C_\psi m_{\text{vav}} S_\theta^2 \psi'(t) \\ & - (m_1 + m_2) S_\theta S_\phi S_\psi \psi'(t) - m_{\text{mav}} S_\theta S_\phi S_\psi \psi'(t) - m_v S_\theta S_\phi S_\psi \psi'(t) - m_{\text{vav}} S_\theta S_\phi S_\psi \psi'(t), \end{aligned}$$

$$\begin{aligned}
c_{2,2} = & -m_v S_\phi^2 x_g u(t) C_\theta^2 + C_\phi(m_1 + m_2) S_\phi S_\psi \theta'(t) C_\theta^2 + C_\phi m_{\text{mav}} S_\phi S_\psi \theta'(t) C_\theta^2 + C_\phi m_v S_\phi S_\psi \theta'(t) C_\theta^2 \\
& + C_\phi m_{\text{vav}} S_\phi S_\psi \theta'(t) C_\theta^2 - r(t) J_{xz} S_\phi C_\theta - r(t) J_{az} S_\theta S_\phi C_\theta - r(t) C_\phi C_\psi J_{az} S_\theta S_\phi C_\theta \\
& - r(t) J_{zz} S_\theta S_\phi C_\theta - r(t) C_\phi C_\psi J_{zz} S_\theta S_\phi C_\theta + r(t) C_\phi^2 J_{az} S_\psi C_\theta + r(t) C_\phi^2 J_{zz} S_\psi C_\theta \\
& + C_\phi^2 C_\psi m_v x_g u(t) C_\theta + C_\phi m_v S_\theta S_\phi S_\psi x_g u(t) C_\theta + C_\phi C_\psi m_v S_\theta S_\phi y_g u(t) C_\theta + m_v S_\theta S_\phi y_g u(t) C_\theta \\
& - C_\phi^2 m_v S_\psi y_g u(t) C_\theta + C_\psi m_v S_\theta S_\phi S_\psi x_g w(t) C_\theta - C_\phi m_v S_\phi S_\psi x_g w(t) C_\theta + m_v S_\phi y_g w(t) C_\theta \\
& + (m_1 + m_2) S_\theta S_\phi^2 S_\psi^2 \theta'(t) C_\theta + m_{\text{mav}} S_\theta S_\phi^2 S_\psi^2 \theta'(t) C_\theta + m_v S_\theta S_\phi^2 S_\psi^2 \theta'(t) C_\theta + m_{\text{vav}} S_\theta S_\phi^2 S_\psi^2 \theta'(t) C_\theta \\
& + C_\phi C_\psi (m_1 + m_2) S_\phi S_\psi \theta'(t) C_\theta + C_\phi C_\psi m_{\text{mav}} S_\phi S_\psi \theta'(t) C_\theta + C_\phi C_\psi m_v S_\phi S_\psi \theta'(t) C_\theta \\
& - C_\phi C_\psi (m_1 + m_2) S_\phi \phi'(t) C_\theta - C_\phi C_\psi m_{\text{mav}} S_\phi \phi'(t) C_\theta - C_\phi C_\psi m_v S_\phi \phi'(t) C_\theta - C_\phi C_\psi m_{\text{vav}} S_\phi \phi'(t) C_\theta \\
& - (m_1 + m_2) S_\theta S_\phi^2 S_\psi \phi'(t) C_\theta - m_{\text{mav}} S_\theta S_\phi^2 S_\psi \phi'(t) C_\theta - m_v S_\theta S_\phi^2 S_\psi \phi'(t) C_\theta - m_{\text{vav}} S_\theta S_\phi^2 S_\psi \phi'(t) C_\theta \\
& + C_\psi^2 (m_1 + m_2) S_\theta S_\phi \psi'(t) C_\theta + C_\phi C_\psi (m_1 + m_2) S_\theta S_\phi \psi'(t) C_\theta + C_\psi^2 m_{\text{mav}} S_\theta S_\phi \psi'(t) C_\theta + C_\phi C_\psi m_{\text{vav}} S_\theta S_\phi \psi'(t) C_\theta \\
& + (m_1 + m_2) S_\phi^2 S_\psi \psi'(t) C_\theta + m_{\text{mav}} S_\phi^2 S_\psi \psi'(t) C_\theta + m_v S_\phi^2 S_\psi \psi'(t) C_\theta + m_{\text{vav}} S_\phi^2 S_\psi \psi'(t) C_\theta \\
& - C_\phi C_\psi (m_1 + m_2) S_\psi \psi'(t) C_\theta - C_\phi C_\psi m_{\text{mav}} S_\psi \psi'(t) C_\theta - C_\phi C_\psi m_v S_\psi \psi'(t) C_\theta - C_\phi C_\psi m_{\text{vav}} S_\psi \psi'(t) C_\theta \\
& - r(t) C_\psi J_{yz} S_\theta S_\phi^2 + p(t) (-C_\theta J_{xz} S_\psi C_\phi^2 + C_\theta C_\psi J_{xz} S_\theta S_\phi C_\phi + J_{xy} S_\phi S_\psi C_\phi \\
& - C_\psi J_{xy} S_\theta S_\phi^2 + C_\theta J_{ax} S_\phi + C_\theta J_{xx} S_\phi + C_\theta J_{xz} S_\theta S_\phi) + r(t) C_\phi J_{yz} S_\phi S_\psi \\
& + q(t) (-C_\theta J_{yz} S_\psi C_\phi^2 + C_\theta C_\psi J_{yz} S_\theta S_\phi C_\phi - J_{yy} S_\phi S_\psi C_\phi + C_\psi J_{yy} S_\theta S_\phi^2 \\
& - C_\theta J_{xy} S_\phi + C_\theta J_{yz} S_\theta S_\phi + J_{ay} S_\phi (C_\psi S_\theta S_\phi - C_\phi S_\psi)) + C_\psi m_v S_\theta S_\phi^2 z_g u(t) \\
& - C_\phi m_v S_\phi S_\psi z_g u(t) - C_\psi m_v S_\theta S_\phi^2 x_g w(t) + C_\phi C_\psi m_v S_\theta x_g w(t) + m_v S_\theta^2 S_\phi S_\psi x_g w(t) \\
& + C_\phi m_v S_\phi S_\psi x_g w(t) + C_\phi C_\psi m_v z_g w(t) + m_v S_\theta S_\phi S_\psi z_g w(t) + C_\phi (m_1 + m_2) S_\theta S_\psi^2 \theta'(t) \\
& + C_\phi m_v S_\theta S_\psi \theta'(t) + C_\phi m_{\text{vav}} S_\theta S_\psi \theta'(t) - C_\psi (m_1 + m_2) S_\theta^2 S_\phi S_\psi \theta'(t) - C_\psi m_{\text{mav}} S_\theta^2 S_\phi S_\psi \theta'(t) \\
& - C_\psi m_v S_\theta^2 S_\phi \theta'(t) - C_\psi m_{\text{vav}} S_\theta^2 S_\phi S_\psi \theta'(t) + C_\phi (m_1 + m_2) S_\theta^2 S_\phi S_\psi \phi'(t) + C_\phi m_{\text{mav}} S_\theta^2 S_\phi S_\psi \phi'(t) \\
& + C_\phi m_v S_\theta^2 S_\phi \phi'(t) + C_\phi m_{\text{vav}} S_\theta^2 S_\phi S_\psi \phi'(t) - C_\phi C_\psi^2 (m_1 + m_2) S_\phi \phi'(t) - C_\phi C_\psi^2 m_{\text{mav}} S_\phi \phi'(t) \\
& - C_\phi C_\psi^2 m_v S_\phi \phi'(t) - C_\phi C_\psi^2 m_{\text{vav}} S_\phi \phi'(t) - C_\psi (m_1 + m_2) S_\theta S_\phi^2 S_\psi \phi'(t) - C_\psi m_{\text{mav}} S_\theta S_\phi^2 S_\psi \phi'(t) \\
& - C_\psi m_v S_\theta S_\phi^2 S_\psi \phi'(t) - C_\psi m_{\text{vav}} S_\theta S_\phi S_\psi^2 \phi'(t) + C_\phi C_\psi^2 (m_1 + m_2) S_\theta S_\phi \psi'(t) + C_\phi^2 C_\psi m_{\text{mav}} S_\theta S_\psi \phi'(t) \\
& + C_\phi^2 C_\psi m_v S_\theta S_\psi \phi'(t) + C_\phi^2 C_\psi m_{\text{vav}} S_\theta S_\psi \phi'(t) - C_\phi (m_1 + m_2) S_\theta S_\phi S_\psi^2 \psi'(t) - C_\phi m_{\text{mav}} S_\theta S_\phi S_\psi^2 \psi'(t) \\
& - C_\phi m_v S_\theta S_\phi S_\psi^2 \psi'(t) - C_\phi m_{\text{vav}} S_\theta S_\phi S_\psi^2 \psi'(t) + C_\phi C_\psi^2 (m_1 + m_2) S_\theta S_\phi \psi'(t) + C_\phi C_\psi^2 m_{\text{mav}} S_\theta S_\phi \psi'(t) \\
& + C_\phi C_\psi^2 m_v S_\theta S_\phi \psi'(t) + C_\phi C_\psi^2 m_{\text{vav}} S_\theta S_\phi \psi'(t) + C_\psi (m_1 + m_2) S_\theta^2 S_\phi^2 S_\psi \psi'(t) + C_\psi m_{\text{mav}} S_\theta^2 S_\phi^2 S_\psi \psi'(t) \\
& + C_\psi m_v S_\theta^2 S_\phi S_\psi \psi'(t) + C_\psi m_{\text{vav}} S_\theta^2 S_\phi^2 S_\psi \psi'(t) - C_\phi^2 C_\psi (m_1 + m_2) S_\psi \psi'(t) - C_\phi^2 C_\psi m_{\text{mav}} S_\psi \psi'(t) \\
& - C_\phi^2 C_\psi m_v S_\psi \psi'(t) - C_\phi^2 C_\psi m_{\text{vav}} S_\psi \psi'(t) + C_\phi C_\psi m_{\text{vav}} S_\phi S_\psi \theta'(t) C_\theta + C_\phi C_\psi m_{\text{mav}} S_\theta S_\phi \psi'(t) C_\theta \\
& + C_\phi m_{\text{mav}} S_\theta S_\psi^2 \theta'(t),
\end{aligned}$$

$$\begin{aligned}
c_{3,2} = & -C_\phi m_v S_\phi x_g u(t) C_\theta^2 + C_\phi^2 (m_1 + m_2) S_\psi \theta'(t) C_\theta^2 + C_\phi^2 m_{\text{mav}} S_\psi \theta'(t) C_\theta^2 + C_\phi^2 m_v S_\psi \theta'(t) C_\theta^2 \\
& + C_\phi m_{\text{vav}} S_\theta S_\phi S_\psi^2 \theta'(t) C_\theta + C_\phi^2 m_{\text{vav}} S_\psi \theta'(t) C_\theta^2 - r(t) C_\phi J_{xz} S_\theta C_\theta - r(t) C_\phi J_{az} S_\theta C_\theta - r(t) C_\phi^2 C_\psi J_{az} S_\theta C_\theta \\
& - r(t) C_\phi J_{zz} S_\theta C_\theta - r(t) C_\phi^2 C_\psi J_{zz} S_\theta C_\theta - r(t) C_\phi J_{az} S_\phi S_\psi C_\theta - r(t) C_\phi J_{zz} S_\phi S_\psi C_\theta \\
& - C_\phi C_\psi m_v S_\phi x_g u(t) C_\theta + C_\phi^2 m_v S_\theta S_\psi x_g u(t) C_\theta + C_\phi m_v S_\theta y_g u(t) C_\theta + C_\phi^2 C_\psi m_v S_\theta y_g u(t) C_\theta \\
& + C_\phi m_v S_\phi y_g u(t) C_\theta + C_\phi C_\psi m_v S_\theta S_\phi w(t) C_\theta + m_v S_\phi^2 S_\psi x_g w(t) C_\theta + C_\phi m_v y_g w(t) C_\theta \\
& + C_\phi (m_1 + m_2) S_\theta S_\phi S_\psi^2 \theta'(t) C_\theta + C_\phi m_{\text{mav}} S_\theta S_\phi S_\psi^2 \theta'(t) C_\theta + C_\phi m_v S_\theta S_\phi S_\psi^2 \theta'(t) C_\theta \\
& - C_\psi (m_1 + m_2) S_\phi^2 S_\psi \theta'(t) C_\theta - C_\psi m_{\text{mav}} S_\phi^2 S_\psi \theta'(t) C_\theta - C_\psi m_v S_\phi^2 S_\psi \theta'(t) C_\theta - C_\psi m_{\text{vav}} S_\phi^2 S_\psi \theta'(t) C_\theta \\
& - C_\phi^2 C_\psi (m_1 + m_2) \phi'(t) C_\theta - C_\phi^2 C_\psi m_{\text{mav}} \phi'(t) C_\theta - C_\phi^2 C_\psi m_v \phi'(t) C_\theta - C_\phi^2 C_\psi m_{\text{vav}} \phi'(t) C_\theta \\
& - C_\phi (m_1 + m_2) S_\theta S_\phi S_\psi \phi'(t) C_\theta - C_\phi m_{\text{mav}} S_\theta S_\phi S_\psi \phi'(t) C_\theta - C_\phi m_v S_\theta S_\phi S_\psi \phi'(t) C_\theta \\
& + C_\phi C_\psi^2 (m_1 + m_2) S_\theta \psi'(t) C_\theta + C_\phi^2 C_\psi (m_1 + m_2) S_\theta \psi'(t) C_\theta + C_\phi C_\psi^2 m_{\text{mav}} S_\theta \psi'(t) C_\theta \\
& + C_\phi C_\psi^2 m_v S_\theta \psi'(t) C_\theta + C_\phi^2 C_\psi m_v S_\theta \psi'(t) C_\theta + C_\phi C_\psi^2 m_{\text{vav}} S_\theta \psi'(t) C_\theta + C_\phi C_\psi^2 m_{\text{vav}} S_\theta \psi'(t) C_\theta \\
& + C_\phi (m_1 + m_2) S_\phi S_\psi \psi'(t) C_\theta + C_\psi (m_1 + m_2) S_\phi S_\psi \psi'(t) C_\theta + C_\phi m_{\text{mav}} S_\phi S_\psi \psi'(t) C_\theta \\
& + C_\phi m_v S_\phi S_\psi \psi'(t) C_\theta + C_\psi m_v S_\phi S_\psi \psi'(t) C_\theta + C_\phi m_{\text{vav}} S_\phi S_\psi \psi'(t) C_\theta + C_\psi m_{\text{vav}} S_\phi S_\psi \psi'(t) C_\theta \\
& - r(t) C_\phi C_\psi J_{yz} S_\theta S_\phi - r(t) J_{yz} S_\phi^2 S_\psi + p(t) (C_\theta C_\psi J_{xz} S_\theta C_\phi^2 + C_\theta J_{ax} C_\phi - C_\phi m_{\text{vav}} S_\theta S_\phi S_\psi \phi'(t) C_\theta \\
& + C_\phi^2 C_\psi m_{\text{mav}} S_\theta \psi'(t) C_\theta + C_\theta J_{xx} C_\phi + C_\theta J_{xz} S_\theta C_\phi - C_\psi J_{xy} S_\theta S_\phi C_\phi + C_\theta J_{xz} S_\phi S_\psi C_\phi \\
& + C_\psi m_{\text{mav}} S_\phi S_\psi \psi'(t) C_\theta - J_{xy} S_\phi^2 S_\psi) + q(t) (C_\theta C_\psi J_{yz} S_\theta C_\phi^2 - C_\theta J_{xy} C_\phi + C_\theta J_{yz} S_\theta C_\phi \\
& + C_\psi J_{yy} S_\theta S_\phi C_\phi + C_\theta J_{yz} S_\phi S_\psi C_\phi + J_{yy} S_\phi^2 S_\psi + J_{ay} S_\phi (C_\phi C_\psi S_\theta + C_\phi C_\psi (m_1 + m_2) S_\phi S_\psi \psi'(t) \\
& + S_\phi S_\psi)) + C_\phi C_\psi m_v S_\theta S_\phi z_g u(t) + m_v S_\phi^2 S_\psi z_g u(t) - C_\phi C_\psi m_v S_\theta x_g w(t) - 2C_\phi C_\psi m_v S_\theta S_\phi S_\psi \phi'(t) \\
& - C_\psi m_v S_\theta S_\phi x_g w(t) + C_\phi m_v S_\theta^2 S_\psi x_g w(t) - m_v S_\phi^2 S_\psi x_g w(t) - C_\psi m_v S_\phi z_g w(t) \\
& + C_\phi m_v S_\theta S_\psi z_g w(t) - (m_1 + m_2) S_\theta S_\phi S_\psi^2 \theta'(t) - m_{\text{mav}} S_\theta S_\phi S_\psi^2 \theta'(t) - m_v S_\theta S_\phi S_\psi^2 \theta'(t) \\
& - m_{\text{vav}} S_\theta S_\phi S_\psi^2 \theta'(t) - C_\phi C_\psi (m_1 + m_2) S_\theta^2 S_\psi \theta'(t) - C_\phi C_\psi m_{\text{mav}} S_\theta^2 S_\psi \theta'(t) - C_\phi C_\psi m_v S_\theta^2 S_\psi \theta'(t) \\
& - C_\phi C_\psi m_{\text{vav}} S_\theta^2 S_\psi \theta'(t) + C_\psi^2 (m_1 + m_2) S_\phi^2 \phi'(t) + C_\psi^2 m_{\text{mav}} S_\phi^2 \phi'(t) + C_\psi^2 m_v S_\phi^2 \phi'(t) \\
& + C_\psi^2 m_{\text{vav}} S_\phi^2 \phi'(t) + C_\phi^2 (m_1 + m_2) S_\theta^2 S_\psi^2 \phi'(t) + C_\phi^2 m_{\text{mav}} S_\theta^2 S_\psi^2 \phi'(t) + C_\phi^2 m_v S_\theta^2 S_\psi^2 \phi'(t) \\
& + C_\phi^2 m_{\text{vav}} S_\theta^2 S_\psi^2 \phi'(t) - 2C_\phi C_\psi (m_1 + m_2) S_\theta S_\phi S_\psi \phi'(t) - 2C_\phi C_\psi m_{\text{mav}} S_\theta S_\phi S_\psi \phi'(t) \\
& - 2C_\phi C_\psi m_{\text{vav}} S_\theta S_\phi S_\psi \phi'(t) - C_\psi^2 (m_1 + m_2) S_\theta S_\phi^2 \psi'(t) - C_\psi^2 m_{\text{mav}} S_\theta S_\phi^2 \psi'(t) - C_\psi^2 m_v S_\theta S_\phi^2 \psi'(t) \\
& - C_\phi^2 (m_1 + m_2) S_\theta S_\psi^2 \psi'(t) - C_\phi^2 m_{\text{mav}} S_\theta S_\psi^2 \psi'(t) - C_\phi^2 m_v S_\theta S_\psi^2 \psi'(t) - C_\phi^2 m_{\text{vav}} S_\theta S_\psi^2 \psi'(t) \\
& + C_\phi C_\psi m_{\text{mav}} S_\theta^2 S_\phi S_\psi \psi'(t) + C_\phi C_\psi m_v S_\theta^2 S_\phi S_\psi \psi'(t) + C_\phi C_\psi m_{\text{vav}} S_\theta^2 S_\phi S_\psi \psi'(t) \\
& + C_\phi C_\psi m_{\text{mav}} S_\phi S_\psi \psi'(t) + C_\phi C_\psi m_v S_\phi S_\psi \psi'(t) + C_\phi C_\psi m_{\text{vav}} S_\phi S_\psi \psi'(t) - C_\psi^2 m_{\text{vav}} S_\theta S_\phi^2 \psi'(t) \\
& + C_\phi C_\psi (m_1 + m_2) S_\theta^2 S_\phi S_\psi \psi'(t),
\end{aligned}$$

$$c_{4,2} = -C_\theta x_g m_v \theta'(t),$$

$$c_{5,2} = m_v (C_\phi z_g r(t) + x_g (-S_\phi \theta'(t)((C_\phi + 1)S_\theta + C_\phi \tan(\theta(t))) + \phi'(t)(S_\phi^2 - C_\theta C_\phi^2))) - z_g q(t) S_\phi \sec(\theta(t)),$$

$$c_{6,2} = m_v(C_\phi z_g(-q(t)) \sec(\theta(t)) - x_g(\theta'(t)(C_\phi S_\theta + C_\phi^2 \tan(\theta(t)) - S_\theta S_\phi^2) + (C_\theta + 1)C_\phi S_\phi \phi'(t)) - z_g r(t) S_\phi),$$

$$\begin{aligned} c_{1,3} = & -C_\theta p(t)(J_{ax}S_\psi + C_\theta C_\psi J_{xz}S_\phi + C_\psi C_\phi J_{xy} + J_{xx}S_\psi + J_{xz}S_\theta S_\psi) + C_\theta q(t)(J_{ay}C_\psi C_\phi \\ & - C_\theta C_\psi J_{yz}S_\phi + C_\psi C_\phi J_{yy} + J_{xy}S_\psi - J_{yz}S_\theta S_\psi) + J_{az}C_\theta r(t)S_\theta S_\psi + J_{az}C_\theta^2 C_\psi r(t)S_\phi + C_\theta x_g m_v S_\psi S_\phi u(t) \\ & - C_\phi x_g m_v S_\theta u(t) - C_\theta y_g m_v S_\theta S_\psi u(t) - C_\theta^2 C_\psi y_g m_v S_\phi u(t) + C_\theta x_g m_v S_\theta S_\psi v(t) \\ & + C_\theta^2 C_\psi x_g m_v S_\phi v(t) - C_\theta C_\psi x_g m_v S_\phi v(t) + C_\theta z_g m_v S_\psi v(t) + C_\theta C_\psi C_\phi z_g m_v u(t) \\ & + C_\theta J_{xz}r(t)S_\psi + C_\theta J_{zz}r(t)S_\theta S_\psi + C_\theta^2 C_\psi J_{zz}r(t)S_\phi - C_\theta C_\psi C_\phi J_{yz}r(t) \\ & - C_\theta m_{maw} S_\theta S_\psi S_\phi \theta'(t) + C_\phi m_{maw} S_\theta^2 \theta'(t) + C_\theta^2 C_\phi m_{maw} S_\psi \phi'(t) + C_\theta m_{maw} S_\theta S_\phi \phi'(t) \\ & - C_\theta^2 C_\psi m_{maw} \theta'(t) - (m_1 + m_2)C_\theta S_\theta S_\psi S_\phi \theta'(t) + (m_1 + m_2)C_\phi S_\theta^2 \theta'(t) + (m_1 + m_2)C_\theta^2 C_\phi S_\psi \phi'(t) \\ & - C_\theta m_v S_\theta S_\psi S_\phi \theta'(t) + C_\phi m_v S_\theta^2 \theta'(t) + C_\theta^2 C_\phi m_v S_\psi \phi'(t) + C_\theta m_v S_\theta S_\phi \phi'(t) \\ & - C_\theta m_{vaw} S_\theta S_\psi S_\phi \theta'(t) + C_\phi m_{vaw} S_\theta^2 \theta'(t) + C_\theta^2 C_\phi m_{vaw} S_\psi \phi'(t) + C_\theta m_{vaw} S_\theta S_\phi \phi'(t) \\ & - (m_1 + m_2)C_\theta^2 C_\psi \theta'(t) - C_\theta^2 C_\psi m_v \theta'(t) - C_\theta^2 C_\psi m_{vaw} \theta'(t) - y_g m_v S_\theta v(t) \\ & + (m_1 + m_2)C_\theta S_\theta S_\phi \phi'(t), \end{aligned}$$

$$\begin{aligned} c_{2,3} = & r(t)J_{yz}S_\psi C_\phi^2 - m_v S_\psi z_g u(t)C_\phi^2 + C_\theta C_\psi(m_1 + m_2)\phi'(t)C_\phi^2 + C_\theta C_\psi m_{maw} \phi'(t)C_\phi^2 \\ & + C_\theta C_\psi m_v \phi'(t)C_\phi^2 + C_\theta C_\psi m_{vaw} \phi'(t)C_\phi^2 + r(t)C_\psi J_{xz}C_\phi + r(t)C_\psi J_{az}S_\theta C_\phi \\ & + r(t)C_\psi J_{zz}S_\theta C_\phi - r(t)C_\psi J_{yz}S_\theta S_\phi C_\phi - r(t)C_\theta J_{az}S_\phi S_\psi C_\phi - r(t)C_\theta J_{zz}S_\phi S_\psi C_\phi \\ & + C_\theta m_v S_\phi x_g u(t)C_\phi + C_\psi m_v S_\phi x_g u(t)C_\phi - C_\psi m_v S_\theta y_g u(t)C_\phi + C_\theta m_v S_\phi S_\psi y_g u(t)C_\phi \\ & + C_\psi m_v S_\theta S_\phi z_g u(t)C_\phi + C_\psi m_v S_\theta x_g v(t)C_\phi - C_\theta m_v S_\phi S_\psi x_g v(t)C_\phi + m_v S_\phi S_\psi x_g v(t)C_\phi \\ & + C_\psi m_v z_g v(t)C_\phi - C_\theta(m_1 + m_2)S_\theta S_\phi \theta'(t)C_\phi - C_\psi(m_1 + m_2)S_\theta S_\phi \theta'(t)C_\phi - C_\theta m_{maw} S_\theta S_\phi \theta'(t)C_\phi \\ & - C_\psi m_{maw} S_\theta S_\phi \theta'(t)C_\phi - C_\theta m_v S_\theta S_\phi \theta'(t)C_\phi - C_\psi m_v S_\theta S_\phi \theta'(t)C_\phi - C_\theta m_{vaw} S_\theta S_\phi \theta'(t)C_\phi \\ & - C_\psi m_{vaw} S_\theta S_\phi \theta'(t)C_\phi + C_\theta(m_1 + m_2)S_\psi \theta'(t)C_\phi + C_\theta m_{maw} S_\psi \theta'(t)C_\phi + C_\theta m_v S_\psi \theta'(t)C_\phi \\ & + C_\theta m_{vaw} S_\psi \theta'(t)C_\phi + C_\theta(m_1 + m_2)S_\theta S_\phi S_\psi \phi'(t)C_\phi + C_\theta m_{maw} S_\theta S_\phi S_\psi \phi'(t)C_\phi + C_\theta m_v S_\theta S_\phi S_\psi \phi'(t)C_\phi \\ & + C_\theta m_{vaw} S_\theta S_\phi S_\psi \phi'(t)C_\phi + r(t)C_\theta C_\psi J_{az}S_\theta S_\phi^2 + r(t)C_\theta C_\psi J_{zz}S_\theta S_\phi^2 + r(t)J_{az}S_\theta^2 S_\phi S_\psi \\ & + r(t)J_{zz}S_\theta^2 S_\phi S_\psi + r(t)J_{xz}S_\theta S_\phi S_\psi + q(t)(-J_{yy}S_\psi C_\phi^2 - C_\psi J_{yz}S_\theta C_\phi \\ & + C_\psi J_{yy}S_\theta S_\phi C_\phi + C_\theta J_{yz}S_\phi S_\psi C_\phi + J_{ay}(C_\psi S_\theta S_\phi - C_\phi S_\psi)C_\phi \\ & - C_\theta C_\psi J_{yz}S_\theta S_\phi^2 - J_{yz}S_\theta^2 S_\phi S_\psi + J_{xy}(C_\phi C_\psi + S_\theta S_\phi S_\psi)) + p(t)(J_{xy}S_\psi C_\phi^2 \\ & - C_\psi J_{xz}S_\theta C_\phi - C_\psi J_{xy}S_\theta S_\phi C_\phi + C_\theta J_{xz}S_\phi S_\psi C_\phi - C_\theta C_\psi J_{xz}S_\theta S_\phi^2 \\ & - J_{xz}S_\theta^2 S_\phi S_\psi - J_{ax}(C_\phi C_\psi + S_\theta S_\phi S_\psi) - J_{xx}(C_\phi C_\psi + S_\theta S_\phi S_\psi)) \\ & + m_v S_\theta S_\phi^2 S_\psi x_g u(t) - C_\theta C_\psi m_v S_\theta S_\phi^2 y_g u(t) - m_v S_\theta^2 S_\phi S_\psi y_g u(t) + C_\theta C_\psi m_v S_\theta S_\phi^2 x_g v(t) \\ & - C_\psi m_v S_\theta S_\phi^2 x_g v(t) + m_v S_\theta^2 S_\phi S_\psi x_g v(t) + C_\theta m_v S_\phi y_g v(t) + m_v S_\theta S_\phi S_\psi z_g v(t) \\ & - C_\theta C_\psi(m_1 + m_2)S_\theta S_\phi \theta'(t) - C_\theta C_\psi m_{maw} S_\theta S_\phi \theta'(t) - C_\theta C_\psi m_v S_\theta S_\phi \theta'(t) - C_\theta C_\psi m_{vaw} S_\theta S_\phi \theta'(t) \\ & - (m_1 + m_2)S_\theta^2 S_\phi^2 S_\psi \theta'(t) - m_{maw} S_\theta^2 S_\phi^2 S_\psi \theta'(t) - m_v S_\theta^2 S_\phi^2 S_\psi \theta'(t) - m_{vaw} S_\theta^2 S_\phi^2 S_\psi \theta'(t) \\ & - C_\theta^2(m_1 + m_2)S_\phi^2 \phi'(t) - C_\theta^2 m_{maw} S_\phi^2 \phi'(t) - C_\theta^2 m_v S_\phi^2 \phi'(t) - C_\theta^2 m_{vaw} S_\phi^2 \phi'(t), \end{aligned}$$

$$\begin{aligned}
c_{3,3} = & -C_\phi(m_1 + m_2)S_\phi\phi'(t)C_\theta^2 - C_\phi m_{\text{maw}}S_\phi\phi'(t)C_\theta^2 - C_\phi m_v S_\phi\phi'(t)C_\theta^2 - C_\phi m_{\text{vaw}}S_\phi\phi'(t)C_\theta^2 \\
& + r(t)C_\phi C_\psi J_{zz}S_\theta S_\phi C_\theta + r(t)J_{az}S_\phi^2 S_\psi C_\theta + r(t)J_{zz}S_\phi^2 S_\psi C_\theta + C_\phi^2 m_v x_g u(t)C_\theta - C_\phi C_\psi m_v S_\theta S_\phi y_g u(t)C_\theta \\
& - m_v S_\phi^2 S_\psi y_g u(t)C_\theta + C_\phi C_\psi m_v S_\theta S_\phi x_g v(t)C_\theta + m_v S_\phi^2 S_\psi x_g v(t)C_\theta + C_\phi m_v y_g v(t)C_\theta \\
& - C_\phi^2(m_1 + m_2)S_\theta\theta'(t)C_\theta - C_\phi C_\psi(m_1 + m_2)S_\theta\theta'(t)C_\theta - C_\phi^2 m_{\text{maw}}S_\theta\theta'(t)C_\theta - C_\phi C_\psi m_{\text{maw}}S_\theta\theta'(t)C_\theta \\
& - C_\phi^2 m_v S_\theta\theta'(t)C_\theta - C_\phi C_\psi m_v S_\theta\theta'(t)C_\theta - C_\phi^2 m_{\text{vaw}}S_\theta\theta'(t)C_\theta - C_\phi C_\psi m_{\text{vaw}}S_\theta\theta'(t)C_\theta \\
& - (m_1 + m_2)S_\phi S_\psi \theta'(t)C_\theta - m_{\text{maw}}S_\phi S_\psi \theta'(t)C_\theta - m_v S_\phi S_\psi \theta'(t)C_\theta - m_{\text{vaw}}S_\phi S_\psi \theta'(t)C_\theta \\
& - C_\phi C_\psi(m_1 + m_2)S_\phi\phi'(t)C_\theta - C_\phi C_\psi m_{\text{maw}}S_\phi\phi'(t)C_\theta - C_\phi C_\psi m_v S_\phi\phi'(t)C_\theta - C_\phi C_\psi m_{\text{vaw}}S_\phi\phi'(t)C_\theta \\
& + C_\phi^2(m_1 + m_2)S_\theta S_\psi \phi'(t)C_\theta + C_\phi^2 m_{\text{maw}}S_\theta S_\psi \phi'(t)C_\theta + C_\phi^2 m_v S_\theta S_\psi \phi'(t)C_\theta + C_\phi^2 m_{\text{vaw}}S_\theta S_\psi \phi'(t)C_\theta \\
& - r(t)C_\phi^2 C_\psi J_{yz}S_\theta - r(t)C_\psi J_{xz}S_\phi - r(t)C_\psi J_{az}S_\theta S_\phi - r(t)C_\psi J_{zz}S_\theta S_\phi - p(t)(C_\psi J_{xy}S_\theta C_\phi^2 \\
& + r(t)C_\phi C_\psi J_{az}S_\theta S_\phi C_\theta + C_\theta C_\psi J_{xz}S_\theta S_\phi C_\phi + J_{xz}S_\theta^2 S_\psi C_\phi + J_{xy}S_\phi S_\psi C_\phi - C_\psi J_{xz}S_\theta S_\phi \\
& + J_{ax}(C_\phi S_\theta S_\psi - C_\psi S_\phi) + J_{xx}(C_\phi S_\theta S_\psi - C_\psi S_\phi) + C_\theta J_{xz}S_\phi^2 S_\psi \\
& + q(t)(C_\psi J_{yy}S_\theta C_\phi^2 - C_\theta C_\psi J_{yz}S_\theta S_\phi C_\phi - J_{yz}S_\theta^2 S_\psi C_\phi + J_{yy}S_\phi S_\psi C_\phi \\
& + J_{ay}(C_\phi C_\psi S_\theta + S_\phi S_\psi)C_\phi + C_\psi J_{yz}S_\theta S_\phi + J_{xy}(C_\phi S_\theta S_\psi - C_\psi S_\phi) \\
& - C_\theta J_{yz}S_\phi^2 S_\psi) + r(t)C_\phi J_{az}S_\theta^2 S_\psi + r(t)C_\phi J_{zz}S_\theta^2 S_\psi + r(t)C_\phi J_{xz}S_\theta S_\psi \\
& - r(t)C_\phi J_{yz}S_\phi S_\psi - C_\psi m_v S_\phi^2 x_g u(t) + C_\phi m_v S_\theta S_\phi S_\psi x_g u(t) + C_\psi m_v S_\theta S_\phi y_g u(t) - C_\phi m_v S_\theta^2 S_\psi y_g u(t) \\
& + C_\phi^2 C_\psi m_v S_\theta z_g u(t) + C_\phi m_v S_\phi S_\psi z_g u(t) - C_\phi C_\psi m_v S_\theta S_\phi x_g v(t) - C_\psi m_v S_\theta S_\phi x_g v(t) \\
& + C_\phi m_v S_\theta^2 S_\psi x_g v(t) - m_v S_\phi^2 S_\psi x_g v(t) - C_\psi m_v S_\phi z_g v(t) + C_\phi m_v S_\theta S_\psi z_g v(t) + C_\psi(m_1 + m_2)S_\theta^2 S_\phi \theta'(t) \\
& + C_\psi m_{\text{maw}}S_\theta S_\phi \theta'(t) + C_\psi m_v S_\theta S_\phi \theta'(t) + C_\psi m_{\text{vaw}}S_\theta S_\phi \theta'(t) - C_\phi(m_1 + m_2)S_\theta^2 S_\phi S_\psi \theta'(t) \\
& - C_\phi m_{\text{maw}}S_\theta^2 S_\phi S_\psi \theta'(t) - C_\phi m_v S_\theta^2 S_\phi S_\psi \theta'(t) - C_\phi m_{\text{vaw}}S_\theta^2 S_\phi S_\psi \theta'(t),
\end{aligned}$$

$$c_{4,3} = x_g(-m_v)(C_\phi r(t) + q(t)S_\phi),$$

$$\begin{aligned}
c_{5,3} = & -m_v(S_\phi(q(t)(x_g S_\phi \tan(\theta(t))(C_\phi(\sec(\theta(t)) + 1) + 1) - y_g \sec(\theta(t))) - C_\phi x_g(\sec(\theta(t)) + 1)\phi'(t)) \\
& + C_\phi r(t)(x_g S_\phi \tan(\theta(t))(C_\phi(\sec(\theta(t)) + 1) + 1) + y_g)),
\end{aligned}$$

$$\begin{aligned}
c_{6,3} = & m_v(q(t)(x_g S_\phi \tan(\theta(t))(C_\phi^2(-\sec(\theta(t))) - C_\phi + S_\phi^2) + C_\phi y_g \sec(\theta(t))) + r(t)(y_g S_\phi \\
& - C_\phi x_g \tan(\theta(t))(C_\phi^2 \sec(\theta(t)) + C_\phi - S_\phi^2)) + x_g \phi'(t)(C_\phi^2 \sec(\theta(t)) - S_\phi^2)),
\end{aligned}$$

$$\begin{aligned}
c_{1,4} = & -J_{\text{ax}} C_\theta S_\psi w(t) - J_{\text{ax}} S_\theta v(t) - C_\theta y_g m_v S_\theta S_\psi S_\phi \theta'(t) + C_\phi y_g m_v S_\theta^2 \theta'(t) - C_\theta^2 C_\psi^2 z_g m_v \psi'(t) \\
& + C_\theta^2 C_\phi y_g m_v S_\psi \phi'(t) + C_\theta y_g m_v S_\theta S_\phi \phi'(t) + C_\theta C_\psi z_g m_v S_\theta S_\psi \theta'(t) + C_\theta^2 (-z_g) m_v S_\psi^2 S_\phi \theta'(t) \\
& + C_\theta C_\phi z_g m_v S_\theta S_\psi \theta'(t) - C_\theta C_\phi z_g m_v S_\theta S_\psi^2 \phi'(t) + C_\theta C_\psi z_g m_v S_\psi S_\phi \phi'(t) - C_\psi C_\phi z_g m_v S_\theta \phi'(t) \\
& - C_\theta C_\psi z_g m_v S_\theta S_\psi S_\phi \psi'(t) + C_\theta C_\phi z_g m_v S_\psi^2 \psi'(t) + C_\psi C_\phi z_g m_v S_\theta^2 \psi'(t) - C_\theta^2 C_\psi y_g m_v \theta'(t) \\
& - J_{\text{xy}} (u(t)(C_\phi S_\theta - C_\theta S_\psi S_\phi) + C_\theta C_\psi (C_\phi w(t) + S_\phi v(t))) - J_{\text{xz}} (C_\theta u(t)(C_\theta C_\phi S_\psi \\
& + S_\theta S_\phi) + v(t)(S_\theta^2 - C_\theta^2 C_\psi C_\phi) + C_\theta w(t)(C_\theta C_\psi S_\phi + S_\theta S_\psi)) \\
& - C_\theta J_{\text{xx}} S_\psi w(t) - z_g m_v S_\theta^2 S_\psi S_\phi \phi'(t) + z_g m_v S_\theta S_\psi S_\phi \psi'(t) - J_{\text{xx}} S_\theta v(t),
\end{aligned}$$

$$\begin{aligned}
c_{2,4} = & -C_\phi m_v S_\phi S_\psi z_g \theta'(t) C_\theta^2 - m_v S_\phi^2 y_g \phi'(t) C_\theta^2 + J_{\text{ax}} S_\phi v(t) C_\theta + J_{\text{xx}} S_\phi v(t) C_\theta \\
& - C_\phi m_v S_\theta S_\phi y_g \theta'(t) C_\theta - C_\psi m_v S_\theta S_\phi y_g \theta'(t) C_\theta + C_\phi m_v S_\psi y_g \theta'(t) C_\theta - m_v S_\theta S_\phi^2 S_\psi^2 z_g \theta'(t) C_\theta \\
& - C_\phi C_\psi m_v S_\phi S_\psi z_g \theta'(t) C_\theta + C_\phi^2 C_\psi m_v y_g \phi'(t) C_\theta + C_\phi m_v S_\theta S_\phi S_\psi y_g \phi'(t) C_\theta + C_\phi C_\psi m_v S_\phi z_g \phi'(t) C_\theta \\
& + m_v S_\theta S_\phi^2 S_\psi z_g \phi'(t) C_\theta - C_\psi^2 m_v S_\theta S_\phi z_g \psi'(t) C_\theta - C_\phi C_\psi m_v S_\theta S_\phi z_g \psi'(t) C_\theta - m_v S_\phi^2 S_\psi z_g \psi'(t) C_\theta \\
& + C_\phi C_\psi m_v S_\psi z_g \psi'(t) C_\theta - C_\phi C_\psi J_{\text{ax}} w(t) - C_\phi C_\psi J_{\text{xx}} w(t) - J_{\text{ax}} S_\theta S_\phi S_\psi w(t) \\
& - J_{\text{xx}} S_\theta S_\phi S_\psi w(t) + J_{\text{xz}} (C_\theta (-C_\psi C_\phi^2 - S_\theta S_\phi S_\psi C_\phi + C_\theta S_\phi^2) u(t) \\
& + C_\theta ((C_\phi C_\psi + 1) S_\theta S_\phi - C_\phi^2 S_\psi) v(t) - (S_\theta S_\phi (C_\theta C_\psi S_\phi + S_\theta S_\psi) \\
& + C_\phi (C_\psi S_\theta - C_\theta S_\phi S_\psi)) w(t)) + J_{\text{xy}} (S_\phi (C_\theta C_\phi + C_\psi C_\phi + S_\theta S_\phi S_\psi) u(t) \\
& - (C_\psi S_\theta S_\phi - C_\phi S_\psi) (S_\phi v(t) + C_\phi w(t))) - C_\phi C_\psi m_v S_\theta S_\phi y_g \theta'(t) - m_v S_\theta^2 S_\phi^2 S_\psi y_g \theta'(t) \\
& - C_\phi m_v S_\theta S_\phi^2 z_g \theta'(t) + C_\psi m_v S_\theta^2 S_\phi S_\psi z_g \theta'(t) - C_\phi m_v S_\theta^2 S_\phi S_\psi^2 z_g \phi'(t) + C_\phi C_\psi^2 m_v S_\phi z_g \phi'(t) \\
& + C_\psi m_v S_\theta S_\phi^2 S_\psi z_g \phi'(t) - C_\phi^2 C_\psi m_v S_\theta S_\psi z_g \phi'(t) + C_\phi m_v S_\theta S_\phi S_\psi^2 z_g \psi'(t) - C_\phi C_\psi^2 m_v S_\theta S_\phi z_g \psi'(t) \\
& - C_\psi m_v S_\theta^2 S_\phi^2 S_\psi z_g \psi'(t) + C_\phi^2 C_\psi m_v S_\psi z_g \psi'(t),
\end{aligned}$$

$$\begin{aligned}
c_{3,4} = & -C_\phi^2 m_v S_\psi z_g \theta'(t) C_\theta^2 - C_\phi m_v S_\phi y_g \phi'(t) C_\theta^2 + C_\phi J_{\text{ax}} v(t) C_\theta + C_\phi J_{\text{xx}} v(t) C_\theta \\
& - C_\phi^2 m_v S_\theta y_g \theta'(t) C_\theta - C_\phi C_\psi m_v S_\theta y_g \theta'(t) C_\theta - m_v S_\phi S_\psi y_g \theta'(t) C_\theta - C_\phi m_v S_\theta S_\phi S_\psi^2 z_g \theta'(t) C_\theta \\
& + C_\psi m_v S_\phi^2 S_\psi z_g \theta'(t) C_\theta - C_\phi C_\psi m_v S_\phi y_g \phi'(t) C_\theta + C_\phi^2 m_v S_\theta S_\psi y_g \phi'(t) C_\theta + C_\phi^2 C_\psi m_v z_g \phi'(t) C_\theta \\
& + C_\phi m_v S_\theta S_\phi S_\psi z_g \phi'(t) C_\theta - C_\phi C_\psi^2 m_v S_\theta z_g \psi'(t) C_\theta - C_\phi^2 C_\psi m_v S_\theta z_g \psi'(t) C_\theta - C_\phi m_v S_\phi S_\psi z_g \psi'(t) C_\theta \\
& - C_\psi m_v S_\phi S_\psi z_g \psi'(t) C_\theta + C_\psi J_{\text{ax}} S_\phi w(t) + C_\psi J_{\text{xx}} S_\phi w(t) - C_\phi J_{\text{ax}} S_\theta S_\psi w(t) \\
& - C_\phi J_{\text{xx}} S_\theta S_\psi w(t) + J_{\text{xz}} (C_\theta C_\phi (C_\theta S_\phi + C_\psi S_\phi - C_\phi S_\theta S_\psi) u(t) + C_\theta C_\phi ((C_\phi C_\psi \\
& + 1) S_\theta + S_\phi S_\psi) v(t) - ((C_\theta C_\phi - 1) C_\psi S_\theta S_\phi + (C_\phi S_\theta^2 \\
& + C_\theta S_\phi^2) S_\psi) w(t)) + J_{\text{xy}} ((C_\theta C_\phi^2 + S_\phi (C_\phi S_\theta S_\psi - C_\psi S_\phi)) u(t) - (C_\phi C_\psi S_\theta \\
& + S_\phi S_\psi) (S_\phi v(t) + C_\phi w(t))) + C_\psi m_v S_\theta S_\phi^2 y_g \theta'(t) - C_\phi m_v S_\theta^2 S_\phi S_\psi y_g \theta'(t) + m_v S_\theta S_\phi S_\psi^2 z_g \theta'(t) \\
& + C_\phi C_\psi m_v S_\theta^2 S_\psi z_g \theta'(t) - C_\psi^2 m_v S_\theta^2 z_g \phi'(t) - C_\phi^2 m_v S_\theta^2 S_\psi^2 z_g \phi'(t) + 2C_\phi C_\psi m_v S_\theta S_\phi S_\psi z_g \phi'(t) \\
& + C_\psi^2 m_v S_\theta S_\phi^2 z_g \psi'(t) + C_\phi^2 m_v S_\theta S_\psi^2 z_g \psi'(t) - C_\phi C_\psi m_v S_\theta S_\phi S_\psi z_g \psi'(t) - C_\phi C_\psi m_v S_\phi S_\psi z_g \psi'(t),
\end{aligned}$$

$$c_{4,4} = C_\phi J_{\text{xy}} (-r(t)) + C_\theta J_{\text{xz}} \theta'(t) - J_{\text{xy}} q(t) S_\phi,$$

$$\begin{aligned}
c_{5,4} = & -q(t)S_\phi(-J_{ax}\sec(\theta(t)) + J_{xy}S_\phi\tan(\theta(t))(C_\phi(\sec(\theta(t)) + 1) + 1) - J_{xx}\sec(\theta(t))) \\
& - C_\phi r(t)(J_{ax} + J_{xy}S_\phi\tan(\theta(t))(C_\phi(\sec(\theta(t)) + 1) + 1) + J_{xx}) + C_\phi J_{xy}S_\phi\sec(\theta(t))\phi'(t) \\
& + C_\phi J_{xy}S_\phi\phi''(t) + C_\phi J_{xz}S_\theta S_\phi\theta'(t) + C_\phi J_{xz}S_\phi\theta'(t)\tan(\theta(t)) - C_\phi C_\phi^2 J_{xz}\phi'(t) \\
& + J_{xz}S_\theta S_\phi\theta'(t) + J_{xz}S_\phi^2\phi'(t),
\end{aligned}$$

$$\begin{aligned}
c_{6,4} = & q(t)(J_{ax}C_\phi\sec(\theta(t)) + J_{xy}S_\phi\tan(\theta(t))(C_\phi^2(-\sec(\theta(t))) - C_\phi + S_\phi^2) \\
& + C_\phi J_{xx}\sec(\theta(t))) + r(t)(J_{ax}S_\phi - C_\phi J_{xy}\tan(\theta(t))(C_\phi^2\sec(\theta(t)) + C_\phi - S_\phi^2) \\
& + J_{xx}S_\phi) + C_\phi J_{xz}S_\theta\theta'(t) + C_\theta C_\phi J_{xz}S_\phi\phi'(t) + C_\phi J_{xz}S_\phi\phi'(t) \\
& + C_\phi^2 J_{xy}\sec(\theta(t))\phi'(t) + C_\phi^2 J_{xz}\theta'(t)\tan(\theta(t)) - J_{xy}S_\phi^2\phi'(t) - J_{xz}S_\theta S_\phi^2\theta'(t),
\end{aligned}$$

$$\begin{aligned}
c_{1,5} = & J_{ay}(u(t)(C_\phi S_\theta - C_\theta S_\psi S_\phi) + C_\theta C_\psi(C_\phi w(t) + S_\phi v(t))) + C_\theta x_g m_v S_\theta S_\psi S_\phi \theta'(t) \\
& - C_\phi x_g m_v S_\theta^2 \theta'(t) - C_\theta^2 C_\phi x_g m_v S_\psi \phi'(t) - C_\theta x_g m_v S_\theta S_\phi \phi'(t) - C_\theta C_\psi^2 z_g m_v S_\theta \theta'(t) \\
& + C_\theta^2 C_\psi z_g m_v S_\psi S_\phi \theta'(t) - C_\theta C_\psi C_\phi z_g m_v S_\theta \theta'(t) - C_\theta^2 C_\psi z_g m_v S_\psi \psi'(t) + C_\theta z_g m_v S_\psi^2 S_\phi \phi'(t) \\
& + C_\theta C_\psi C_\phi z_g m_v S_\theta S_\psi \phi'(t) - C_\phi z_g m_v S_\theta S_\psi \phi'(t) + C_\psi z_g m_v S_\theta^2 S_\phi \phi'(t) - C_\theta z_g m_v S_\theta S_\psi^2 S_\phi \psi'(t) \\
& - C_\psi z_g m_v S_\theta S_\phi \psi'(t) + C_\phi z_g m_v S_\theta^2 S_\psi \psi'(t) + C_\theta^2 C_\psi x_g m_v \theta'(t) + J_{yy}(u(t)(C_\phi S_\theta - C_\theta S_\psi S_\phi) \\
& + C_\theta C_\psi(C_\phi w(t) + S_\phi v(t))) + C_\theta^2(-C_\phi) J_{yz} S_\psi u(t) - C_\theta J_{yz} S_\theta S_\phi u(t) \\
& + C_\theta J_{xy} S_\psi w(t) - C_\theta J_{yz} S_\theta S_\psi w(t) - C_\theta^2 C_\psi J_{yz} S_\phi w(t) + C_\theta^2 C_\psi C_\phi J_{yz} v(t) \\
& - C_\theta C_\psi C_\phi z_g m_v S_\psi \psi'(t) + J_{xy} S_\theta v(t) - J_{yz} S_\theta^2 v(t),
\end{aligned}$$

$$\begin{aligned}
c_{2,5} = & J_{yz} S_\phi^2 u(t) C_\theta^2 + C_\phi C_\psi m_v S_\phi z_g \theta'(t) C_\theta^2 + m_v S_\phi^2 x_g \phi'(t) C_\theta^2 - C_\phi^2 C_\psi J_{yz} u(t) C_\theta \\
& - C_\phi J_{yz} S_\theta S_\phi S_\psi u(t) C_\theta - J_{xy} S_\phi v(t) C_\theta + C_\phi C_\psi J_{yz} S_\theta S_\phi v(t) C_\theta + J_{yz} S_\theta S_\phi v(t) C_\theta \\
& - C_\phi^2 J_{yz} S_\psi v(t) C_\theta - C_\psi J_{yz} S_\theta S_\phi^2 w(t) C_\theta + C_\phi J_{yz} S_\phi S_\psi w(t) C_\theta + C_\phi m_v S_\theta S_\phi x_g \theta'(t) C_\theta \\
& + C_\psi m_v S_\theta S_\phi x_g \theta'(t) C_\theta - C_\phi m_v S_\psi x_g \theta'(t) C_\theta + C_\phi C_\psi^2 m_v S_\phi z_g \theta'(t) C_\theta + C_\psi m_v S_\theta S_\phi^2 S_\psi z_g \theta'(t) C_\theta \\
& - C_\phi^2 C_\psi m_v x_g \phi'(t) C_\theta - C_\phi m_v S_\theta S_\phi x_g \phi'(t) C_\theta - C_\psi m_v S_\theta S_\phi^2 z_g \phi'(t) C_\theta + C_\phi m_v S_\phi S_\psi z_g \phi'(t) C_\theta \\
& + C_\psi m_v S_\phi^2 z_g \psi'(t) C_\theta + C_\phi m_v S_\psi^2 z_g \psi'(t) C_\theta - C_\phi m_v S_\theta S_\phi S_\psi z_g \psi'(t) C_\theta - C_\psi m_v S_\theta S_\phi S_\psi z_g \psi'(t) C_\theta \\
& - J_{ay} S_\phi (C_\theta C_\phi + C_\psi C_\phi \\
& + S_\theta S_\phi S_\psi) u(t) - J_{yy} S_\phi (C_\theta C_\phi + C_\psi C_\phi + S_\theta S_\phi S_\psi) u(t) \\
& + C_\phi C_\psi J_{xy} w(t) - C_\phi C_\psi J_{yz} S_\theta w(t) - J_{yz} S_\theta^2 S_\phi S_\psi w(t) + J_{xy} S_\theta S_\phi S_\psi w(t) \\
& + J_{ay} (C_\psi S_\theta S_\phi - C_\phi S_\psi) (S_\phi v(t) + C_\phi w(t)) + J_{yy} (C_\psi S_\theta S_\phi - C_\phi S_\psi) (S_\phi v(t) \\
& + C_\phi w(t)) + C_\phi C_\psi m_v S_\theta S_\phi x_g \theta'(t) + m_v S_\theta^2 S_\phi^2 S_\psi x_g \theta'(t) - C_\psi^2 m_v S_\theta^2 S_\phi z_g \theta'(t) \\
& + C_\phi C_\psi m_v S_\theta S_\phi z_g \theta'(t) + m_v S_\theta S_\phi^2 S_\psi^2 z_g \phi'(t) + C_\phi^2 C_\psi^2 m_v S_\theta z_g \phi'(t) + C_\phi C_\psi m_v S_\theta^2 S_\phi S_\psi z_g \phi'(t) \\
& + C_\phi C_\psi m_v S_\phi S_\psi z_g \phi'(t) - m_v S_\theta^2 S_\phi^2 S_\psi^2 z_g \psi'(t) - C_\phi^2 C_\psi^2 m_v z_g \psi'(t) - 2C_\phi C_\psi m_v S_\theta S_\phi S_\psi z_g \psi'(t),
\end{aligned}$$

$$\begin{aligned}
c_{3,5} = & C_\phi J_{yz} S_\phi u(t) C_\theta^2 + C_\phi^2 C_\psi m_v z_g \theta'(t) C_\theta^2 + C_\phi m_v S_\phi x_g \phi'(t) C_\theta^2 + C_\phi C_\psi J_{yz} S_\phi u(t) C_\theta \\
& - C_\phi^2 J_{yz} S_\theta S_\psi u(t) C_\theta - C_\phi J_{xy} v(t) C_\theta + C_\phi J_{yz} S_\theta v(t) C_\theta + C_\phi^2 C_\psi J_{yz} S_\theta v(t) C_\theta \\
& + C_\phi J_{yz} S_\phi S_\psi v(t) C_\theta - C_\phi C_\psi J_{yz} S_\theta S_\phi w(t) C_\theta - J_{yz} S_\phi^2 S_\psi w(t) C_\theta + C_\phi^2 m_v S_\theta x_g \theta'(t) C_\theta \\
& + C_\phi C_\psi m_v S_\theta x_g \theta'(t) C_\theta + m_v S_\phi S_\psi x_g \theta'(t) C_\theta - C_\psi^2 m_v S_\phi^2 z_g \theta'(t) C_\theta + C_\phi C_\psi m_v S_\theta S_\phi S_\psi z_g \theta'(t) C_\theta \\
& + C_\phi C_\psi m_v S_\phi x_g \phi'(t) C_\theta - C_\phi^2 m_v S_\theta S_\psi x_g \phi'(t) C_\theta - C_\phi C_\psi m_v S_\theta S_\phi z_g \phi'(t) C_\theta + C_\phi^2 m_v S_\psi z_g \phi'(t) C_\theta \\
& - m_v S_\phi S_\psi^2 z_g \psi'(t) C_\theta + C_\phi C_\psi m_v S_\phi z_g \psi'(t) C_\theta - C_\phi^2 m_v S_\theta S_\psi z_g \psi'(t) C_\theta - C_\phi C_\psi m_v S_\theta S_\psi z_g \psi'(t) C_\theta \\
& + J_{ay}((C_\phi C_\psi S_\theta + S_\phi S_\psi)(S_\phi v(t) + C_\phi w(t)) - (C_\theta C_\phi^2 + S_\theta S_\phi S_\psi C_\phi \\
& - C_\psi S_\phi^2)u(t)) + J_{yy}((C_\phi C_\psi S_\theta + S_\phi S_\psi)(S_\phi v(t) + C_\phi w(t)) - (C_\theta C_\phi^2 \\
& + S_\theta S_\phi S_\psi C_\phi - C_\psi S_\phi^2)u(t)) - C_\psi J_{xy} S_\phi w(t) + C_\psi J_{yz} S_\theta S_\phi w(t) - C_\phi J_{yz} S_\theta^2 S_\psi w(t) \\
& + C_\phi J_{xy} S_\theta S_\psi w(t) - C_\psi m_v S_\theta S_\phi^2 x_g \theta'(t) + C_\phi m_v S_\theta^2 S_\phi S_\psi x_g \theta'(t) - C_\phi C_\psi^2 m_v S_\theta^2 z_g \theta'(t) \\
& - C_\psi m_v S_\theta S_\phi S_\psi z_g \theta'(t) + C_\phi m_v S_\theta S_\phi S_\psi^2 z_g \phi'(t) - C_\phi C_\psi^2 m_v S_\theta S_\phi z_g \phi'(t) + C_\phi^2 C_\psi m_v S_\theta^2 S_\psi z_g \phi'(t) \\
& - C_\psi m_v S_\phi^2 S_\psi z_g \phi'(t) - C_\phi m_v S_\theta^2 S_\phi S_\psi^2 z_g \psi'(t) + C_\phi C_\psi^2 m_v S_\phi z_g \psi'(t) + C_\psi m_v S_\theta S_\phi^2 S_\psi z_g \psi'(t) \\
& - C_\phi^2 C_\psi m_v S_\theta S_\psi z_g \psi'(t),
\end{aligned}$$

$$c_{4,5} = r(t)(J_{ay} C_\phi + C_\phi J_{yy} - J_{yz} S_\phi) + C_\theta J_{yz} \theta'(t) + z_g m_v S_\phi u(t) - x_g m_v S_\phi w(t) + J_{xy}(-p(t))S_\phi,$$

$$\begin{aligned}
c_{5,5} = & -p(t)S_\phi(-J_{ax} \sec(\theta(t)) + J_{xy} S_\phi \tan(\theta(t))(C_\phi(\sec(\theta(t)) + 1) + 1) - J_{xx} \sec(\theta(t))) \\
& + r(t)(S_\phi \sec(\theta(t))((C_\theta(C_\phi + 1) + C_\phi) \tan(\theta(t))(J_{ay} C_\phi + C_\phi J_{yy} - J_{yz} S_\phi) \\
& - J_{xz}) + C_\phi J_{xy}) - J_{ay} C_\phi S_\phi \sec(\theta(t))\phi'(t) - J_{ay} C_\phi S_\phi \phi'(t) \\
& + C_\phi z_g m_v S_\phi^2 u(t) \tan(\theta(t)) + C_\phi z_g m_v S_\phi^2 u(t) \tan(\theta(t)) \sec(\theta(t)) - C_\phi x_g m_v S_\phi^2 w(t) \tan(\theta(t)) \\
& - C_\phi J_{yy} S_\phi \sec(\theta(t))\phi'(t) - C_\phi J_{yy} S_\phi \phi'(t) + C_\phi J_{yz} S_\theta S_\phi \theta'(t) + C_\phi J_{yz} S_\phi \theta'(t) \tan(\theta(t)) \\
& - C_\theta C_\phi^2 J_{yz} \phi'(t) + z_g m_v S_\phi^2 u(t) \tan(\theta(t)) - x_g m_v S_\phi^2 w(t) \tan(\theta(t)) + y_g m_v S_\phi w(t) \sec(\theta(t)) \\
& - C_\phi x_g m_v S_\phi^2 w(t) \tan(\theta(t)) \sec(\theta(t)) - z_g m_v S_\phi v(t) \sec(\theta(t)) + J_{yz} S_\theta S_\phi \theta'(t) + J_{yz} S_\phi^2 \phi'(t),
\end{aligned}$$

$$\begin{aligned}
c_{6,5} = & p(t)(J_{ax} C_\phi \sec(\theta(t)) + J_{xy} S_\phi \tan(\theta(t))(C_\phi^2(-\sec(\theta(t))) - C_\phi + S_\phi^2) + C_\phi J_{xx} \sec(\theta(t))) \\
& + r(t)(\frac{1}{2} \tan(\theta(t))(C_\phi^2(2 \sec(\theta(t)) + 1) + 2C_\phi - S_\phi^2 - 1)(J_{ay} C_\phi + C_\phi J_{yy} - J_{yz} S_\phi) \\
& + C_\phi J_{xz}(-\sec(\theta(t))) - J_{xy} S_\phi) - J_{ay} C_\phi^2 \sec(\theta(t))\phi'(t) + J_{ay} S_\phi^2 \phi'(t) + C_\phi z_g m_v S_\phi u(t) \tan(\theta(t)) \\
& + C_\phi^2 z_g m_v S_\phi u(t) \tan(\theta(t)) \sec(\theta(t)) - C_\phi x_g m_v S_\phi w(t) \tan(\theta(t)) - C_\phi^2 x_g m_v S_\phi w(t) \tan(\theta(t)) \sec(\theta(t)) \\
& + C_\phi y_g m_v w(t) \sec(\theta(t)) - C_\phi z_g m_v v(t) \sec(\theta(t)) + C_\phi J_{yz} S_\theta \theta'(t) + C_\theta C_\phi J_{yz} S_\phi \phi'(t) + C_\phi J_{yz} S_\phi \phi'(t) \\
& - C_\phi^2 J_{yy} \sec(\theta(t))\phi'(t) + C_\phi^2 J_{yz} \theta'(t) \tan(\theta(t)) + z_g(-m_v) S_\phi^3 u(t) \tan(\theta(t)) + x_g m_v S_\phi^3 w(t) \tan(\theta(t)) \\
& + J_{yy} S_\phi^2 \phi'(t) - J_{yz} S_\theta S_\phi^2 \theta'(t),
\end{aligned}$$

$$\begin{aligned}
c_{1,6} = & C_\phi J_{zz} S_\psi u(t) C_\theta^2 - C_\psi C_\phi J_{zz} v(t) C_\theta^2 + C_\psi J_{zz} S_\phi w(t) C_\theta^2 - C_\psi y_g m_v S_\psi S_\phi \theta'(t) C_\theta^2 \\
& + m_v S_\phi S_\psi^2 x_g \theta'(t) C_\theta^2 + C_\psi^2 m_v x_g \psi'(t) C_\theta^2 + C_\psi m_v S_\psi y_g \psi'(t) C_\theta^2 + J_{zz} S_\theta S_\phi u(t) C_\theta \\
& + J_{xz} S_\psi w(t) C_\theta + J_{zz} S_\theta S_\psi w(t) C_\theta - C_\phi x_g m_v S_\theta S_\psi \theta'(t) C_\theta - C_\psi x_g m_v S_\theta S_\psi \theta'(t) C_\theta \\
& + C_\psi^2 m_v S_\theta y_g \theta'(t) C_\theta + C_\phi C_\psi m_v S_\theta y_g \theta'(t) C_\theta - y_g m_v S_\psi^2 S_\phi \phi'(t) C_\theta - C_\psi x_g m_v S_\psi S_\phi \phi'(t) C_\theta \\
& - C_\psi C_\phi y_g m_v S_\theta S_\psi \phi'(t) C_\theta + C_\phi m_v S_\theta S_\psi^2 x_g \phi'(t) C_\theta - C_\phi x_g m_v S_\psi^2 \psi'(t) C_\theta + C_\psi m_v S_\theta S_\phi S_\psi x_g \psi'(t) C_\theta \\
& + m_v S_\theta S_\phi S_\psi^2 y_g \psi'(t) C_\theta + C_\phi C_\psi m_v S_\psi y_g \psi'(t) C_\theta + J_{az}(C_\theta(S_\theta S_\phi + C_\theta C_\phi S_\psi) u(t) + (S_\theta^2 \\
& - C_\theta^2 C_\psi C_\phi) v(t) + C_\theta(C_\theta C_\psi S_\phi + S_\theta S_\psi) w(t)) + J_{zz} S_\theta^2 v(t) + J_{xz} S_\theta v(t) \\
& - J_{yz}((C_\phi S_\theta - C_\theta S_\psi S_\phi) u(t) + C_\theta C_\psi (S_\phi v(t) + C_\phi w(t))) - C_\psi y_g m_v S_\theta^2 S_\phi \phi'(t) \\
& + C_\phi C_\psi m_v S_\theta x_g \phi'(t) + m_v S_\theta^2 S_\phi S_\psi x_g \phi'(t) + C_\phi m_v S_\theta S_\psi y_g \phi'(t) - C_\psi C_\phi x_g m_v S_\theta^2 \psi'(t) \\
& - x_g m_v S_\theta S_\psi S_\phi \psi'(t) - C_\phi y_g m_v S_\theta^2 S_\psi \psi'(t) + C_\psi m_v S_\theta S_\phi y_g \psi'(t),
\end{aligned}$$

$$\begin{aligned}
c_{2,6} = & -J_{zz} S_\phi^2 u(t) C_\theta^2 + C_\phi m_v S_\phi S_\psi x_g \theta'(t) C_\theta^2 - C_\phi C_\psi m_v S_\phi y_g \theta'(t) C_\theta^2 \\
& + C_\phi^2 C_\psi J_{zz} u(t) C_\theta + C_\phi J_{zz} S_\theta S_\phi S_\psi u(t) C_\theta - J_{xz} S_\phi v(t) C_\theta - C_\phi C_\psi J_{zz} S_\theta S_\phi v(t) C_\theta \\
& - J_{zz} S_\theta S_\phi v(t) C_\theta + C_\phi^2 J_{zz} S_\psi v(t) C_\theta + C_\psi J_{zz} S_\theta S_\phi^2 w(t) C_\theta - C_\phi J_{zz} S_\phi S_\psi w(t) C_\theta \\
& + m_v S_\theta S_\phi^2 S_\psi^2 x_g \theta'(t) C_\theta + C_\phi C_\psi m_v S_\phi S_\psi x_g \theta'(t) C_\theta - C_\phi C_\psi^2 m_v S_\phi y_g \theta'(t) C_\theta - C_\psi m_v S_\theta S_\phi^2 S_\psi y_g \theta'(t) C_\theta \\
& - C_\phi C_\psi m_v S_\phi x_g \phi'(t) C_\theta - m_v S_\theta S_\phi^2 S_\psi x_g \phi'(t) C_\theta + C_\psi m_v S_\theta S_\phi^2 y_g \phi'(t) C_\theta - C_\phi m_v S_\phi S_\psi y_g \phi'(t) C_\theta \\
& + C_\psi^2 m_v S_\theta S_\phi x_g \psi'(t) C_\theta + C_\phi C_\psi m_v S_\theta S_\phi x_g \psi'(t) C_\theta + m_v S_\phi^2 S_\psi x_g \psi'(t) C_\theta - C_\phi C_\psi m_v S_\psi x_g \psi'(t) C_\theta \\
& - C_\psi m_v S_\phi^2 y_g \psi'(t) C_\theta - C_\phi m_v S_\psi^2 y_g \psi'(t) C_\theta + C_\phi m_v S_\theta S_\phi S_\psi y_g \psi'(t) C_\theta + C_\psi m_v S_\theta S_\phi S_\psi y_g \psi'(t) C_\theta \\
& + C_\phi C_\psi J_{xz} w(t) + C_\phi C_\psi J_{zz} S_\theta w(t) + J_{zz} S_\theta^2 S_\phi S_\psi w(t) + J_{xz} S_\theta S_\phi S_\psi w(t) \\
& + J_{az}(C_\theta(C_\psi C_\phi^2 + S_\theta S_\phi S_\psi C_\phi - C_\theta S_\phi^2) u(t) - C_\theta((C_\phi C_\psi + 1) S_\theta S_\phi \\
& - C_\phi^2 S_\psi) v(t) + (S_\theta S_\phi(C_\theta C_\psi S_\phi + S_\theta S_\psi) + C_\phi(C_\psi S_\theta - C_\theta S_\phi S_\psi)) w(t)) \\
& + J_{yz}(S_\phi(C_\theta C_\phi + C_\psi C_\phi + S_\theta S_\phi S_\psi) u(t) - (C_\psi S_\theta S_\phi - C_\phi S_\psi)(S_\phi v(t) \\
& + C_\phi w(t)) + C_\phi m_v S_\theta S_\psi^2 x_g \theta'(t) - C_\psi m_v S_\theta^2 S_\phi S_\psi x_g \theta'(t) + C_\psi^2 m_v S_\theta^2 S_\phi y_g \theta'(t) \\
& - C_\phi C_\psi m_v S_\theta S_\psi y_g \theta'(t) + C_\phi m_v S_\theta^2 S_\phi S_\psi^2 x_g \phi'(t) - C_\phi C_\psi^2 m_v S_\phi x_g \phi'(t) - C_\psi m_v S_\theta S_\phi^2 S_\psi x_g \phi'(t) \\
& + C_\phi^2 C_\psi m_v S_\theta S_\psi x_g \phi'(t) - m_v S_\theta S_\phi^2 S_\psi^2 y_g \phi'(t) - C_\phi^2 C_\psi^2 m_v S_\theta y_g \phi'(t) - C_\phi C_\psi m_v S_\theta^2 S_\phi S_\psi y_g \phi'(t) \\
& - C_\phi C_\psi m_v S_\phi S_\psi y_g \phi'(t) - C_\phi m_v S_\theta S_\phi S_\psi^2 x_g \psi'(t) + C_\phi C_\psi^2 m_v S_\theta S_\phi x_g \psi'(t) + C_\psi m_v S_\theta^2 S_\phi^2 S_\psi x_g \psi'(t) \\
& - C_\phi^2 C_\psi m_v S_\psi x_g \psi'(t) + m_v S_\theta^2 S_\phi^2 S_\psi^2 y_g \psi'(t) + C_\phi^2 C_\psi^2 m_v y_g \psi'(t) + 2C_\phi C_\psi m_v S_\theta S_\phi S_\psi y_g \psi'(t),
\end{aligned}$$

$$\begin{aligned}
c_{3,6} = & -C_\phi J_{zz} S_\phi u(t) C_\theta^2 + C_\phi^2 m_v S_\psi x_g \theta'(t) C_\theta^2 - C_\phi^2 C_\psi m_v y_g \theta'(t) C_\theta^2 - C_\phi C_\psi J_{zz} S_\phi u(t) C_\theta \\
& + C_\phi^2 J_{zz} S_\theta S_\psi u(t) C_\theta - C_\phi J_{xz} v(t) C_\theta - C_\phi J_{zz} S_\theta v(t) C_\theta - C_\phi^2 C_\psi J_{zz} S_\theta v(t) C_\theta \\
& - C_\phi J_{zz} S_\phi S_\psi v(t) C_\theta + C_\phi C_\psi J_{zz} S_\theta S_\phi w(t) C_\theta + J_{zz} S_\phi^2 S_\psi w(t) C_\theta + C_\phi m_v S_\theta S_\phi S_\psi^2 x_g \theta'(t) C_\theta \\
& - C_\psi m_v S_\phi^2 S_\psi x_g \theta'(t) C_\theta + C_\psi^2 m_v S_\phi^2 y_g \theta'(t) C_\theta - C_\phi^2 C_\psi m_v x_g \phi'(t) C_\theta - C_\phi m_v S_\theta S_\phi S_\psi x_g \phi'(t) C_\theta \\
& + C_\phi C_\psi m_v S_\theta S_\phi y_g \phi'(t) C_\theta - C_\phi^2 m_v S_\psi y_g \phi'(t) C_\theta + C_\phi C_\psi^2 m_v S_\theta x_g \psi'(t) C_\theta + C_\phi^2 C_\psi m_v S_\theta x_g \psi'(t) C_\theta \\
& + C_\phi m_v S_\phi S_\psi x_g \psi'(t) C_\theta + C_\psi m_v S_\phi S_\psi x_g \psi'(t) C_\theta + m_v S_\phi S_\psi^2 y_g \psi'(t) C_\theta - C_\phi C_\psi m_v S_\phi y_g \psi'(t) C_\theta \\
& + C_\phi^2 m_v S_\theta S_\psi y_g \psi'(t) C_\theta + C_\phi C_\psi m_v S_\theta S_\psi y_g \psi'(t) C_\theta - C_\psi J_{xz} S_\phi w(t) - C_\psi J_{zz} S_\theta S_\phi w(t) \\
& + C_\phi J_{zz} S_\theta^2 S_\psi w(t) + C_\phi J_{xz} S_\theta S_\psi w(t) - J_{az} (C_\theta C_\phi (C_\theta S_\phi + C_\psi S_\phi - C_\phi S_\theta S_\psi) u(t) \\
& + C_\theta C_\phi ((C_\phi C_\psi + 1) S_\theta + S_\phi S_\psi) v(t) - ((C_\theta C_\phi - 1) C_\psi S_\theta S_\phi \\
& + (C_\phi S_\theta^2 + C_\theta S_\phi^2) S_\psi) w(t)) + J_{yz} ((C_\theta C_\phi^2 + S_\phi (C_\phi S_\theta S_\psi - C_\psi S_\phi)) u(t) \\
& - (C_\phi C_\psi S_\theta + S_\phi S_\psi) (S_\phi v(t) + C_\phi w(t))) - m_v S_\theta S_\phi S_\psi^2 x_g \theta'(t) - C_\phi C_\psi m_v S_\theta^2 S_\psi x_g \theta'(t) \\
& + C_\phi C_\psi^2 m_v S_\theta^2 y_g \theta'(t) - \frac{1}{8} \sin(2\theta(t)) \sin(2\phi(t)) \sin(2\psi(t)) m_v y_g \theta'(t) + C_\psi m_v S_\theta S_\phi S_\psi y_g \theta'(t) \\
& + C_\psi^2 m_v S_\phi^2 x_g \phi'(t) + C_\phi^2 m_v S_\theta^2 S_\psi^2 x_g \phi'(t) - 2C_\phi C_\psi m_v S_\theta S_\phi S_\psi x_g \phi'(t) - C_\phi m_v S_\theta S_\phi S_\psi^2 y_g \phi'(t) \\
& + C_\phi C_\psi^2 m_v S_\theta S_\phi y_g \phi'(t) - C_\phi^2 C_\psi m_v S_\theta^2 S_\psi y_g \phi'(t) + C_\psi m_v S_\phi^2 S_\psi y_g \phi'(t) - C_\psi^2 m_v S_\theta S_\phi^2 x_g \psi'(t) \\
& - C_\phi^2 m_v S_\theta S_\psi^2 x_g \psi'(t) + C_\phi C_\psi m_v S_\theta^2 S_\phi S_\psi x_g \psi'(t) + C_\phi C_\psi m_v S_\phi S_\psi x_g \psi'(t) + C_\phi m_v S_\theta^2 S_\phi S_\psi^2 y_g \psi'(t) \\
& - C_\phi C_\psi^2 m_v S_\phi y_g \psi'(t) - C_\psi m_v S_\theta S_\phi^2 S_\psi y_g \psi'(t) + C_\phi^2 C_\psi m_v S_\theta S_\psi y_g \psi'(t),
\end{aligned}$$

$$\begin{aligned}
c_{4,6} = & q(t) (J_{ay} C_\phi + C_\phi J_{yy} - J_{yz} S_\phi) - J_{az} C_\theta \theta'(t) + C_\phi z_g m_v u(t) - C_\phi x_g m_v w(t) \\
& + C_\phi J_{xy} (-p(t)) - C_\theta J_{zz} \theta'(t),
\end{aligned}$$

$$\begin{aligned}
c_{5,6} = & -C_\phi p(t) (J_{ax} + J_{xy} S_\phi \tan(\theta(t)) (C_\phi (\sec(\theta(t)) + 1) + 1) + J_{xx}) + q(t) (S_\phi \sec(\theta(t)) ((C_\theta (C_\phi + 1) \\
& + C_\phi) \tan(\theta(t)) (J_{ay} C_\phi + C_\phi J_{yy} - J_{yz} S_\phi) - J_{xz}) + C_\phi J_{xy}) - J_{az} C_\phi S_\theta S_\phi \theta'(t) \\
& - J_{az} C_\phi S_\phi \theta'(t) \tan(\theta(t)) + J_{az} C_\theta C_\phi^2 \phi'(t) - J_{az} S_\theta S_\phi \theta'(t) - J_{az} S_\phi^2 \phi'(t) \\
& + C_\phi^2 z_g m_v S_\phi u(t) \tan(\theta(t)) + C_\phi z_g m_v S_\phi u(t) \tan(\theta(t)) + C_\phi^2 z_g m_v S_\phi u(t) \tan(\theta(t)) \sec(\theta(t)) \\
& - C_\phi^2 x_g m_v S_\phi w(t) \tan(\theta(t)) - C_\phi x_g m_v S_\phi w(t) \tan(\theta(t)) + C_\phi^2 (-x_g) m_v S_\phi w(t) \tan(\theta(t)) \sec(\theta(t)) \\
& - C_\phi y_g m_v w(t) + C_\phi z_g m_v v(t) + C_\phi J_{yz} S_\phi \sec(\theta(t)) \phi'(t) + C_\phi J_{yz} S_\phi \phi'(t) - C_\phi J_{zz} S_\theta S_\phi \theta'(t) \\
& - C_\phi J_{zz} S_\phi \theta'(t) \tan(\theta(t)) + C_\theta C_\phi^2 J_{zz} \phi'(t) - J_{zz} S_\theta S_\phi \theta'(t) - J_{zz} S_\phi^2 \phi'(t),
\end{aligned}$$

$$\begin{aligned}
c_{6,6} = & p(t)(J_{ax}S_\phi - C_\phi J_{xy} \tan(\theta(t))(C_\phi^2 \sec(\theta(t)) + C_\phi - S_\phi^2) + J_{xx}S_\phi) \\
& + q(t)\left(\frac{1}{2} \tan(\theta(t))(C_\phi^2(2 \sec(\theta(t)) + 1) + 2C_\phi - S_\phi^2 - 1)(J_{ay}C_\phi + C_\phi J_{yy} - J_{yz}S_\phi)\right. \\
& + C_\phi J_{xz}(-\sec(\theta(t))) - J_{xy}S_\phi) - J_{az}C_\phi S_\theta \theta'(t) - J_{az}C_\theta C_\phi S_\phi \phi'(t) \\
& - J_{az}C_\phi S_\phi \phi'(t) - J_{az}C_\phi^2 \theta'(t) \tan(\theta(t)) + J_{az}S_\theta S_\phi^2 \theta'(t) - C_\phi z_g m_v S_\phi^2 u(t) \tan(\theta(t)) \\
& + C_\phi x_g m_v S_\phi^2 w(t) \tan(\theta(t)) + C_\phi^2 z_g m_v u(t) \tan(\theta(t)) + C_\phi^3 z_g m_v u(t) \tan(\theta(t)) \sec(\theta(t)) \\
& - C_\phi^2 x_g m_v w(t) \tan(\theta(t)) + C_\phi^3 (-x_g) m_v w(t) \tan(\theta(t)) \sec(\theta(t)) - C_\phi J_{zz} S_\theta \theta'(t) - C_\theta C_\phi J_{zz} S_\phi \phi'(t) \\
& - C_\phi J_{zz} S_\phi \phi'(t) + C_\phi^2 J_{yz} \sec(\theta(t)) \phi'(t) - C_\phi^2 J_{zz} \theta'(t) \tan(\theta(t)) + y_g m_v S_\phi w(t) - z_g m_v S_\phi v(t) \\
& \left. - J_{yz} S_\phi^2 \phi'(t) + J_{zz} S_\theta S_\phi^2 \theta'(t), \right.
\end{aligned}$$

$$c_{7,7} = -2l_1 l_{c2} (m_{2,au} + m_2) \alpha'_2(t) \sin(\alpha_2(t)),$$

$$c_{7,8} = -2l_1 l_{c2} (m_{2,au} + m_2) \alpha'_1(t) \sin(\alpha_2(t)).$$

The D_v matrix is given as follows.

$$D_v = \begin{bmatrix} d_{1,1} & \cdots & d_{1,10} \\ \vdots & \ddots & \vdots \\ d_{10,1} & \cdots & d_{10,10} \end{bmatrix}, \quad (\text{A4})$$

where all the non-zero elements of D_v are:

$$\begin{aligned}
d_{1,1} &= \frac{1}{2} \rho A_{pu} C_d v_{uab} + C_u, \\
d_{2,2} &= \frac{1}{2} \rho A_p C_d v_{vab} + C_v, \\
d_{3,3} &= \frac{1}{2} \rho A_p C_d v_{wab} + C_w, \\
d_{4,4} &= \rho A_p C_d l_v^3 p(t) + C_p, \\
d_{5,5} &= \rho A_p C_d l_v^3 q(t) + C_q, \\
d_{6,6} &= \rho A_p C_d l_v^3 r(t) + C_r, \\
d_{7,7} &= \rho A_p C_d l_v^3 \alpha'_1(t) + C_{\alpha_1}, \\
d_{8,8} &= \rho A_p C_d l_v^3 \alpha'_2(t) + C_{\alpha_2}.
\end{aligned}$$

Now, the vector G_v is given as follows:

$$G_v = \begin{pmatrix} g \sin(\theta(t)) (m_v - \rho V_s) \\ g \cos(\theta(t)) \sin(\phi(t)) (\rho V_s - m_v) \\ g \cos(\theta(t)) \cos(\phi(t)) (\rho V_s - m_v) \\ g \cos(\theta(t)) ((z_g \sin(\phi(t)) - y_g \cos(\phi(t))) m_v + \rho (y_b \cos(\phi(t)) - z_b \sin(\phi(t))) V_s) \\ g m_v (\cos(\theta(t)) \cos(\phi(t)) x_g + \sin(\theta(t)) z_g) - g \rho V_s (\cos(\theta(t)) \cos(\phi(t)) x_b + \sin(\theta(t)) z_b) \\ g \rho V_s (\cos(\theta(t)) \sin(\phi(t)) x_b + \sin(\theta(t)) y_b) - g m_v (\cos(\theta(t)) \sin(\phi(t)) x_g + \sin(\theta(t)) y_g) \\ l_{c1} \sin(\alpha_1(t)) (m_1 - g \rho V_{m,1}) \\ (m_2 - g \rho V_{m,2}) (\sin(\alpha_1(t)) l_1 + \sin(\alpha_1(t) + \alpha_2(t)) l_{c2}) \\ 0 \\ 0 \end{pmatrix}. \quad (\text{A5})$$

The force vector F_v is given as follows:

$$F_v = \begin{pmatrix} \frac{1}{8} \pi \rho C_d d_v^2 U_u |u - U_u| + C_u U_u + f_u \\ \frac{1}{2} \rho C_d d_v l_v U_v |U_v - v| + C_v U_v + f_v + f_{vv} \\ \frac{1}{2} \rho C_d d_v l_v U_w |U_w - w| + C_w U_w + f_{vw} + f_w \\ \tau_p \\ \tau_q \\ \tau_r \\ \tau_1 \\ \tau_2 \\ -f_1 + \tau_{\beta 1} - \tau_1 \\ -f_2 + \tau_{\beta 2} - \tau_2 \end{pmatrix}. \quad (\text{A6})$$

Now, by substituting the coupling torques τ_1 and τ_2 given by (18)-(19), and the dynamic friction torques f_1 and f_2 given by (21)-(22) into (A6) reveals the following form of F_v :

$$F_v = \begin{pmatrix} \frac{1}{8} \pi \rho |u - U_u| C_d U_u d_v^2 + f_u + C_u U_u \\ f_v + f_{vv} + \frac{1}{2} (2C_v + \rho |v - U_v| C_d d_v l_v) U_v \\ f_{vw} + f_w + \frac{1}{2} (2C_w + \rho |w - U_w| C_d d_v l_v) U_w \\ \tau_p \\ \tau_q \\ \tau_r \\ \Gamma(\delta_1) + c_j \delta'_1(t) \\ \Gamma(\delta_2) + c_j \delta'_2(t) \\ -\Gamma(\delta_1) + \tau_{\beta 1} - \left(\frac{2}{e^{-\lambda \beta'_1} + 1} - 1 \right) (F_C + e^{-vs^{-\mu} = |\beta'_1|^{\mu}} F_S) - c_j \delta'_1(t) \\ -\Gamma(\delta_2) + \tau_{\beta 2} - \left(\frac{2}{e^{-\lambda \beta'_2} + 1} - 1 \right) (F_C + e^{-vs^{-\mu} = |\beta'_2|^{\mu}} F_S) - c_j \delta'_2(t) \end{pmatrix}, \quad (\text{A7})$$

where f_{vv} and f_{vw} are the lift forces (acting along the j - and k -axis, respectively) given by (28)-(29), $\delta_1 = N_r^{-1} \beta_1 - \alpha_1$, $\delta_2 = N_r^{-1} \beta_2 - \alpha_2$, and $\Gamma(\delta_1)$ and $\Gamma(\delta_2)$ are given by (19)-(20).