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An Erroneous Proposal To Allow For Travel Of An Observer Between Two Celestial Altitude Observations

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KEY WORDS

1. Celestial Navigation. 2. Astronomical Running Fix.

1. INTRODUCTION. K. H. Zevering has recently proposed a way to calculate the position of a moving observer, from the intersection of position circles, that differs from the methods hitherto accepted by mariners[1]. Zevering concludes, from discrepancies between his own results and that of other methods, that those others are in error. However, the discrepancies result from incorrect assumptions in his proposed method, which give rise to erroneous answers. My comments will be restricted to the simplest case of two observations only, and will assume that the Earth is, for our purposes, spherical.

2. THE STATIONARY OBSERVER. An observer determines the altitude H1 of a celestial body such as a star S1, which at the moment of measurement is at a position (Dec1, GHA1), with a corresponding Geographical Position (GP1) below it on the Earth's surface. He must then be somewhere on a circle, centred on GP1, with a radius of (90° – H1) degrees, where a degree corresponds to 60 nautical miles measured along the Earth's surface. That circle is the locus of all possible observers who measure that same altitude at that moment, but we know nothing, yet, about where on that circle he may be.

The observer then measures the altitude H2 of another body S2, situated then at (Dec2, GHA2), which puts him on another circle, radius (90° – H2), centred on GP2, the point below the second body at the moment of the second observation. If the observer had not moved between those two observations, perhaps because they were effectively simultaneous, or perhaps because his vessel was stationary, then he must be at one of the two intersections of those two circles. Commonsense will usually decide which one. There are several methods of computing the intersection points of those circles, which are equivalent and give the same answers. There is no disagreement about any of that, I suggest; it's common ground.

3. THE TRAVELLING OBSERVER, AND ZEVERING'S PROPOSAL. A rather more complex problem occurs when the observer travels across the Earth's surface, by a known distance d and course α, the "run", between two observations made at different times. Somehow, that travel has to be allowed for. I will address only Zevering's proposed solution to that problem, and how he adjusts GP1 to allow for the intervening travel.

Zevering, in his section 2, proposes to take the first position circle, centred on GP1, preserving its radius (90° – H1), and displaces its centre by the same distance d, and in the same direction α, as the movement of the observer between the two sights. His Table 1 shows how the position of the new centre is to be calculated. He now appears to assume that at the time of the second observation, the observer must be somewhere on that displaced position circle. We should be seeking the locus of all possible observers, who were somewhere on the first circle, and have since moved through that known course α and distance d. Then Zevering assumes that the intersections of that transferred position circle with the circle from a second observation can be calculated as before. This proposed method for adjusting the initial position circle for the intervening travel is what he terms the "GHA-Dec updating technique", or in his abbreviation "GD – UT".

I will show that the proposal does not work.

4. TESTING THE PROPOSED METHOD. We can test the method by applying it to a hypothetical case, in which the geometry is simple, and which has been designed to show up the defects in the procedure.

1. *An observer, at position P1, measures the altitude of a star S1, at* (*Dec1* = *0°, GHA1* = *0°*)*, to be 30°.*
2. *Then he travels due North by 60 nautical miles* (= *1°*)*, to P2.*
3. *From there, he observes another star S2* (*then at Dec2* = *N 1°, GHA2* = *W 45°*) *to be at an altitude of 45°. Where on Earth is he then?*

To start with, we will put the Zevering proposal to one side.

Because of the simple geometry, the result is almost self-evident. There are, in general, two possible solutions for P2. One is in the Northern hemisphere, at Lat = N 46°, Long = W 45°, in which case P1 was 1° further South, at Lat = N 45°, Long = W 45°. The other possible position for P2 is at Lat = S 44°, Long = W 45°, with P1 situated, 1° further South, at Lat = S 45°, Long = W 45°.

My aim here is simply to show up the errors in the Zevering method, rather than to choose between better alternatives, so I do not propose to discuss here how that result should be derived. However, a sceptical reader can easily check that the solutions listed above are indeed exact, using computed altitude tables or spherical trig. He will find that from either position for P2, the altitude of star S2 is indeed exactly 45°, so condition 3 has been met. From either position for P1, the altitude of star S1 is exactly 30°, so condition 1 has been met.

And of course P2 is 1° North of P1, so condition 2 has been met.

Those are the conditions that were specified. We have two exact solutions, then, and the observer has to choose the correct one from other evidence, which should present no difficulty.

5. THE ZEVERING ALTERNATIVE. This is set out in his section 4, which states:

> "*GD-UT as explained finds the locus of the transferred position circle by moving X (=GP) to X* for the magnitude and direction of the displacement (d and α) and by projecting the circle from X* with its given radius* ($Zd = 90° - H_0$) $= XZ = X^*Z^*$"

So, start with a circle, centred on Geographic Position GP1, directly below S1, at Lat = 0°, Long = 0°, which corresponds to his position X. That circle, with radius 60°, is the locus of all points at which the altitude of S1 is 30°. The observer's initial position, at P1 (Z, in his notation) must lie somewhere on that circle.

Next (and this is the false step), displace the centre of that circle Northward by 1°, to correspond to the observer's travel between observations. The new circle still has a diameter of 60°, corresponding to an altitude of 30°, but its centre is now at Lat = N 1°, W 0° (position X*). Zevering presumes that the final position P2 (his Z*) must be somewhere on that displaced circle.

Next, find the two intersections between that displaced circle and a new circle, centred on GP2, directly below S2, at Lat = N 1°, Long = W 45°, with a radius corresponding to 45° altitude. As Zevering states, those intersections can be found algebraically.

The Northern result of this calculation of P2 (or Z*) is Lat = N 45·998°, Long = W 45·429°.

The Southern alternative is at Lat = S 43·999°, Long = W 44·600°.

We can see that the latitudes are very close to the exact values found above, but the longitudes are seriously in error, with displacements from the true value of the order of 0·4°, which amounts to about 17 miles at that latitude. And this, after a run of only 60 miles.

6. WHAT HAS GONE WRONG? We can back-calculate to check, as before. The calculated altitude of star S2, from either position of P2, is exactly 45°, as it should be, so condition 3 has been met precisely. P1 (or Z), the observer's position prior to the Northerly displacement, must have been exactly 1° further south than P2, to meet condition 2. Therefore, the Northerly solution must place the initial position P1 at Lat = N 44·998°, Long = W 45·429°, and the Southerly solution for P1 at Lat = S 44·999°, Long = W 44·600°.

If we back-calculate from that Northern alternative for P1, then for the altitude of the star S1, which according to the specification of the problem should be at an altitude of 30°, we find instead that it is at 29·753°. Similarly, from the Southern alternative for P1, we find an altitude of 30·231°, instead of the specified 30°. These are very serious discrepancies. Condition 1 has not been met, by a long way.

The errors result from the incorrect assumption that $XZ = X^*Z^*$. That would be true in plane geometry, in which case, when the centre of a circle is displaced through a certain distance and direction, points on its periphery are displaced through the same distance and direction. On a sphere, however, that is not the case, except for circles that are small compared with the size of the sphere. On a sphere, if all points on a circle are displaced through the same distance and direction, the result is not a circle at all, but will have been distorted into a sort-of egg-shape. Therefore, the next step, calculating the intersections of two circles, cannot apply.

Section 5 states- "… *It is argued that the correct method* (*GHA-Dec Updating Technique; GD-UT*) *for transferring the position circle of an earlier sight is to transfer the coordinates of its GP* (*GHA and Dec*) *for the run data, i.e. distance* (d) *and course* (α)". However, that has not been argued; just asserted. Section 5 continues, strangely- "*The effect is that an observer's position on this position circle will not be transferred according to the distance d and course* α *….*" Exactly so! What is strange is that, in making that statement, Zevering fatally undermines his own proposal. The observer ***must*** move through a distance d with a course α; it's one of the pre-conditions of the problem.

7. CHALLENGE.

I challenge Zevering to apply his procedure to the problem as set out above, to provide us with initial and final positions for the observer, the initial altitude of star S1, and the final altitude of star S2.

This is not the first airing of Zevering's proposal, which has previously appeared, at considerable length, in other journals[2,3]. I have criticised that proposal, in similar terms to those above[4], but direct answers to my questions have been avoided. I hope they will fare better in this Journal.

REFERENCES

[1] K. H. Zevering, (2006). Dependability of Position Solutions in Celestial Sight-Run-Sight Cases – Part 1., The Journal of Navigation, **59**, 155–166.

[2] In European Journal of Navigation, (2003) "The K-Z Position Solution for the Double Sight". **1, 3**, 43–46. That paper was rendered unintelligible by a combination of mistakes, both typographical errors and author's errors. Typographical errors were noted in a later issue, vol.2 no.2, but not the substantial errors made by the author. Although he has acknowledged some of those errors elsewhere, (in "The Navigator's Newsletter", issue 84) they remain uncorrected in EJN.

[3] In "The Navigator's Newsletter", issue 81 (fall 2003) and subsequently in issues 82, 84, 85, 86, 87 and 88.

[4] "The Navigator's Newsletter", issues 83 and 87.

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The Running Fix Technique – Response To G. Huxtable's Comments

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1. THE FIX AS INTERSECTION OF POSITION CIRCLES.

A fix is determined by the intersection of at least two position circles. There is no argument when sights are simultaneous. Two transfer techniques are demonstrated in the ANM when sights are not simultaneous, the running fix technique (RFT) and the GHA-Dec updating technique (GD-UT). With RFT/LSQ*1 the transferred position line is thus supposed to represent a transferred position circle as the

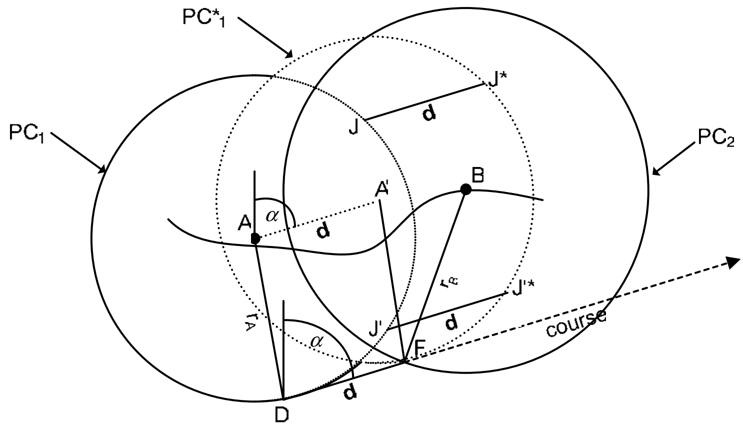

Figure 1. The GD-UT principle of terrestrial RFT.

mathematical locus of all points on the original position circle transferred for a given run. This condition is met with GD-UT but as we will see it is not with RFT/LSQ*.

2. THE TERRESTRIAL ANALOGY.

The model for celestial RFT is terrestrial RFT (e.g ANM Vol.II, p 62, p 191). Points A and B in Figure 1 represent landmarks with known height, but also the geographical positions (GP) of two celestial bodies with large altitudes. Assume in the terrestrial case the distance to A and to B is determined by vertical sextant angle; no bearings are used. In the terrestrial case just draw a transferred position circle with radius r_A from A′ and another position circle with radius r_B from B. In the celestial case, as r_A and r_B are short zenith distances, AA′ virtually equals d and the angle at A the course (α). The fix F is at the intersection of PC^*_1 and PC_2. Had only the bearings been used the fix would also have been in F, assuming that the celestial azimuths could be observed. In the celestial case fix F is therefore found with GD-UT by transferring the 1st sight's GP at A to A′ for the run data and plotting the relevant sections of PC^*_1 and PC_2 (with the *general equation* in ANM, Vol.III, p40).

In Figure 1 the observer could be at J or J′ and JJ* = J′J′* = AA′ = d. The RFT/LSQ* construction in this case is equivalent to moving a run track like JJ* until it coincides with DF, i.e the position *backward-projected* from the fix lies exactly on the original position circle. In this case, terrestrial RFT, celestial RFT and GD-UT are equivalent techniques. But in the general celestial case (large zenith distances) it is incorrectly presumed that DF remains sufficiently equivalent to AA′ and A′D parallel to A′F.

3. THE POSITION CIRCLE LOCUS OF TRANSFERRED POINTS.

Huxtable's present due-N2 double sight example is a variation on his earlier attempt at refuting the general validity of GD-UT3. Three vessels travel the same distance north from known points A, B, C on the same position circle (see Figure 2). The terrestrial analogy is clearly recognizable: the distances AA*, BB*,

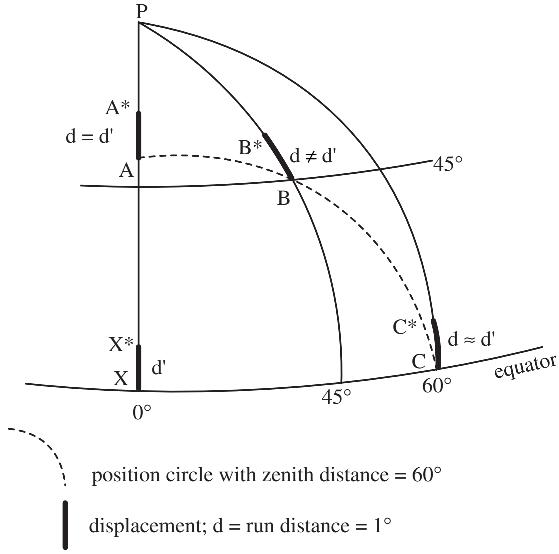

Figure 2. Sketch of cases A, B and C.

CC* are equivalent to JJ*, J′J′*, DF in Fig-2. It is evidently expected that the transferred position circle should pass through A*, B*, C*, like the transferred position circle in Figure 1 passes through J*, J′*, F, as the RFT precondition is that positions *backward-projected* from fixes like A*, B*, C* must lie on the original position circle. At the cardinal points A and C, GD-UT virtually achieves the correct transfer, but X*B* = 59°.7564, 15′ too short: "*..the Zevering procedure* (i.e GD-UT) … *assumes that the transferred position circle defines a locus which includes all such vessels and looks for its intersections with another circle. But that assumption is wrong*."4

But the argument remains inescapably about the correct transfer. If GD-UT is seen as failing to correctly define the transferred position circle as locus of all transferred points, can RFT/LSQ* do this? A*, B*, C* are seen as correctly transferred positions so that a transferred position circle through two such points should correctly account for any vessel's destination. B* and C* lie on a transferred position circle with GHA = 0/Dec′* = 0·7123/H'_o = 30·00685 but it will not pass through any other point with a backward-projected position 1° to the south on the original position circle; e.g AA* = 0°.7055, considerably short of 1°. As is seen, in the large-Zd case the run of 1° at B, C would imply a smaller GP transfer of 0°.7123 and a (negligible) contraction in Zd. The peculiarities of this transfer are nowhere supported in celestial navigation theory.

More revealing is the displacement at cardinal points A and C. The transferred position circle through A*, C* is GHA = 0/Dec′* = 1·0102/H'_o = 30·0102, practically the same as the position circle transferred with GD-UT (GHA = 0/Dec = 1/H_o = 30). The position circle transferred with RFT would be regarded as correct as the positions backward-projected for the run of 1° are exactly at respectively A and C. If the latitudes of the transferred positions were known, say Lat_{A*} = 61 and Lat_{C*} = 1, RFT would determine the positions at A* and C* as 61°N/0° and 1°N/60°E, GD-UT + K-Z as 61°N/0° and 1°N/60°.0101E.

Table 1. Transfer conformity between terrestrial RFT and RFT/LSQ* when Zd decreases.

H_o	30		60		70	
	B	C	B	C	B	C
Mer. Diff.	45	60	22·5	30	15	20
Zn	234·74	270·00	229·94	270·00	229·18	270·00
Lat	45	0·0000	20·3840	0·0000	13·3838	0·0000
Dec′*	0·7123		0·9268		0·9679	
H'_o	30·0068		60·0043		70·0030	
AA*	0°.7055		0°.9225		0°.9650	
H_o	80		85		88	
	B	C	B	C	B	C
Mer. Diff.	7·5	10	3·75	5	1·75	2
Zn	228·74	270·00	228·63	270·00	241·05	270·00
Lat	6·6334	0·0000	3·3096	0·0000	0·9684	0·0000
Dec′*	0·9925		0·9984		0·9997	
H'_o	80·0015		85·0008		88·0003	
AA*	0°.9909		0°.9976		0°.9994	

If the transfer with GD-UT at any position circle cardinal points is correct, would GD-UT not account in a general way for the correct transfer? The position circle transferred with RFT (GHA = 0, Dec′* = 1·0102 and H'_o = 30·0102) cannot account for the displacement from B to B* on Long = 45°E as BB* would be 1°.4197, contradicting the RFT precondition that the distance BB* must be equal to the run distance (1°).

In fact, a 2nd sight cannot confirm the correctness of any position circle transferred with RFT. A 2nd sight is say GHA = 280/Dec = 35/H_o = 58 with LSQ* fixes 49·6674N/40·7923E and 8·9502N/59·6785E. The transferred position circle through these two fixes is GHA = 0/Dec′* = 0·8218/H_o′* = 30·0586. If the transferred position circle were correct, the vessel going north along 45°E from B arrives at Lat_{B*} = 46·0607 and BB* = 1°.0607, not 1° as postulated. For the vessel at A, Lat_{A*} = 60·7632 and AA′* = 0°.7632. AA* was presumed to be 1° with either RFT or GD-UT, regardless of the properties of a 2nd sight.

Therefore, not only do RFT/LSQ* imply a transfer of the 1st sight's GP that is incompatible with the run data but the implied transferred position circle cannot account for any other positions presumed correctly transferred based on the backward-projected position criterion.

Already indicated is that at the cardinal points A and C the position circle transferred with either RFT or GD-UT is virtually equivalent. If the transferred position circle through A* and C* is run together with the same 2nd sight by means of K-Z, the position solution differences with GD-UT + K-Z are negligible. We would argue here that if the transfer for the run at the two cardinal points A and C is basically considered correct according to both RFT and GD-UT, the whole 1st sight's position circle should be accepted as correctly transferred with GD-UT.

To demonstrate that terrestrial RFT has in fact served as model for RFT/LSQ*, we trace the effect of decreasing Zd in Table 1 for the transferred position circle through

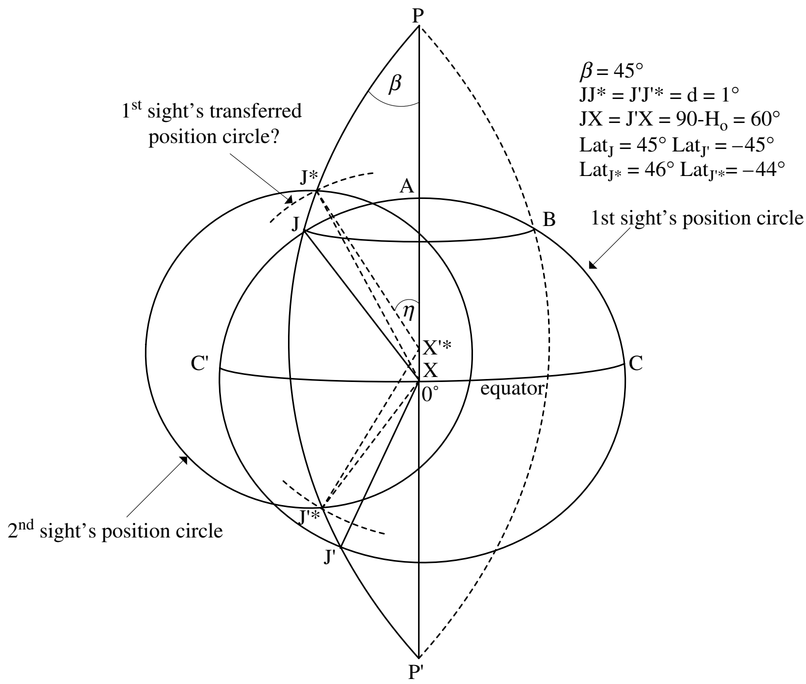

Figure 3. The due-N double sight thought experiment.

B* and C*. The mer.diff. of point B is consistently taken at 75% of Zd; cardinal point C is on the equator at zenith distance east of Greenwich. Point A is of course the other cardinal point. In each case the transferred position circle passes through B* and C* and BB* and CC* are exactly 1°.

As H_o increases from 60 to 88, Dec′* increases from 0·93 to almost 1·00. In other words, the GP transfer increasingly approaches the run distance as in the terrestrial analogy (Figure 1). When $H_o = 30$, the transferred position circle falls way short of passing through A*. As H_o increases the distance AA* begins to conform to the run distance of 1° too. Meanwhile the required adjustment to altitude remains practically insignificant, the difference between H'_o and H_o decreasing steadily, from 0·0068 when $H_o = 30$ to 0·0003 when $H_o = 88$.

4. HUXTABLE'S DUE-NORTH DOUBLE SIGHT. In Huxtable's present thought experiment the position at J (our Figure 3) is the western equivalent of position B in Figure 2, but also provided for is a 2nd sight to confirm the fix at J* so that its GP must lie on the same meridian as J and J*. The configuration is contrived but nonetheless relevant. Mindful of the requirements, Huxtable also provided for two possible position solutions, while each fix must meet the backward-projected position criterion of RFT. As the positions J and J′ on the same meridian are postulated as 45°N and 45°S, the 2nd sight is GHA = 45/Dec = 1/H_o = 45.

Huxtable relies on *simple geometry*, but the fixes at J* and J′* are obtainable with LSQ*, showing that the argument implicitly supports RFT. J* and J′* are obviously regarded as being on the correctly transferred position circle, which in fact is GHA = 0/Dec′* = 0·7071/H′$_o$ = 30·0025. AA* measured with the transferred position circle is 0°.7071. Thus again, the implied transfer with RFT/LSQ* cannot account for the correct transfer at cardinal point A! The correctness of positions J* and J′* based merely on the presumed certainty of the backward-projected positions J and J' respectively is illusory.

Also in this case will RFT/LSQ* begin to conform to terrestrial RFT when Zd decreases. This can be shown when the 1st sight's H$_o$ is allowed to increase, while the 2nd sight's Dec remains at 1°N and its GHA at 75% of the 1st sight's Zd. To obtain the 2nd sight's altitude compatible with the northern RFT/LSQ* fix at J* (so that JJ* = d = 1°), apply the sine formula followed by the Lat-finding quadratic to triangle JPX. As terrestrial RFT is essentially based on GD-UT we may compare the fix discrepancies. With Huxtable's case in Figure 3, the northern fix discrepancy with GD-UT is 17′.88. When the 1st sight's altitude increases from 30° to 60° this discrepancy decreases to 4′.35 and to a mere 0′.03 when altitude increases to 88°. In other words, RFT/LSQ* hold when Zd is small but not in the general case when Zd is large.

5. CONCLUSION. The inherent construction principle of RFT/LSQ* derives from terrestrial RFT, which guarantees that the position backward-projected from the fix will lie on the original position circle. In the celestial case this construction principle only conforms to GD-UT when Zd is small. RFT therefore merely represents a gimmick performed on the chart whereby the transferred position line does not actually represent a mathematically correctly transferred position circle. The properties of a transferred position circle passing through the RFT/LSQ* fixes depend on a given 2nd sight's properties and in the due-N case it cannot account consistently for the given run displacement at cardinal position points. The implied GP transfer matching a particular set of RFT/LSQ* fixes is in the general celestial case contrary to the run data. There is no theoretical explanation for this.

REFERENCES

[1] LSQ* stands for the Yallop-Hohenkerk Least-Squares solution program.

[2] A numerical due-North model first appeared in K.H. Zevering – "Position Solution Differences in the Sight-run-Sight Case", The Navigator's Newsletter (NN) – Foundation for the Promotion of Navigation, 86, p9–15.

[3] See NN 87, p2–4.

[4] Ibid, p4. In this example as well as in his present due-N double sight one, Huxtable is apparently not aware that his argument supports the correctness of RFT/LSQ*.

[5] TanDec′* = $-(a_1c_1 - a_2c_2)/(b_1 - b_2)$; SinH′$_o$ = a_1c_1CosDec′*+b_1SinDec′*; a_1 = Cos45; b_1 = Sin46; c_1 = Cos46; a_2 = Cos60; b_2 = Sin1; c_2 = Cos1.