Supplementary Appendix for Japan and UK Male/Female Data for Gaussian Process Models for Mortality Rates and Improvement Factors

Mike Ludkovski Jimmy Risk Howard Zail *

April 22, 2018

1 TABLES AND FIGURES FOR JAPAN AND UK MALE/FEMALE DATA

	DiceKriging			
	Japan M	Japan F	UK M	UK F
θ_{ag}	10.0969	24.1946	31.3212	16.9089
θ_{yr}	11.4233	13.6124	5.2799	25.8392
η^2	2.571	4.6069	3.0681	3.0043
σ^2	1.257e-03	2.239e-03	1.569e-03	2.135e-03
β_0	-4.9898	-5.5825	-4.2687	-4.2945

Table 1: Hyperparameter estimates based on maximum likelihood (`DiceKriging`). The GP is fitted to all data and uses squared-exponential covariance kernel (Equation (11) in main text) with prior mean $m(x) = \beta_0$.

*Ludkovski is with the Department of Statistics & Applied Probability, University of California, Santa Barbara CA 93106-3110; Risk is with the Department of Mathematical Sciences, Cal Poly Pomona. H. Zail is an Actuary and Founder of Elucidor, LLC ludkovski@pstat.ucsb.edu,risk@cpp.edu,hzail@elucidor.com

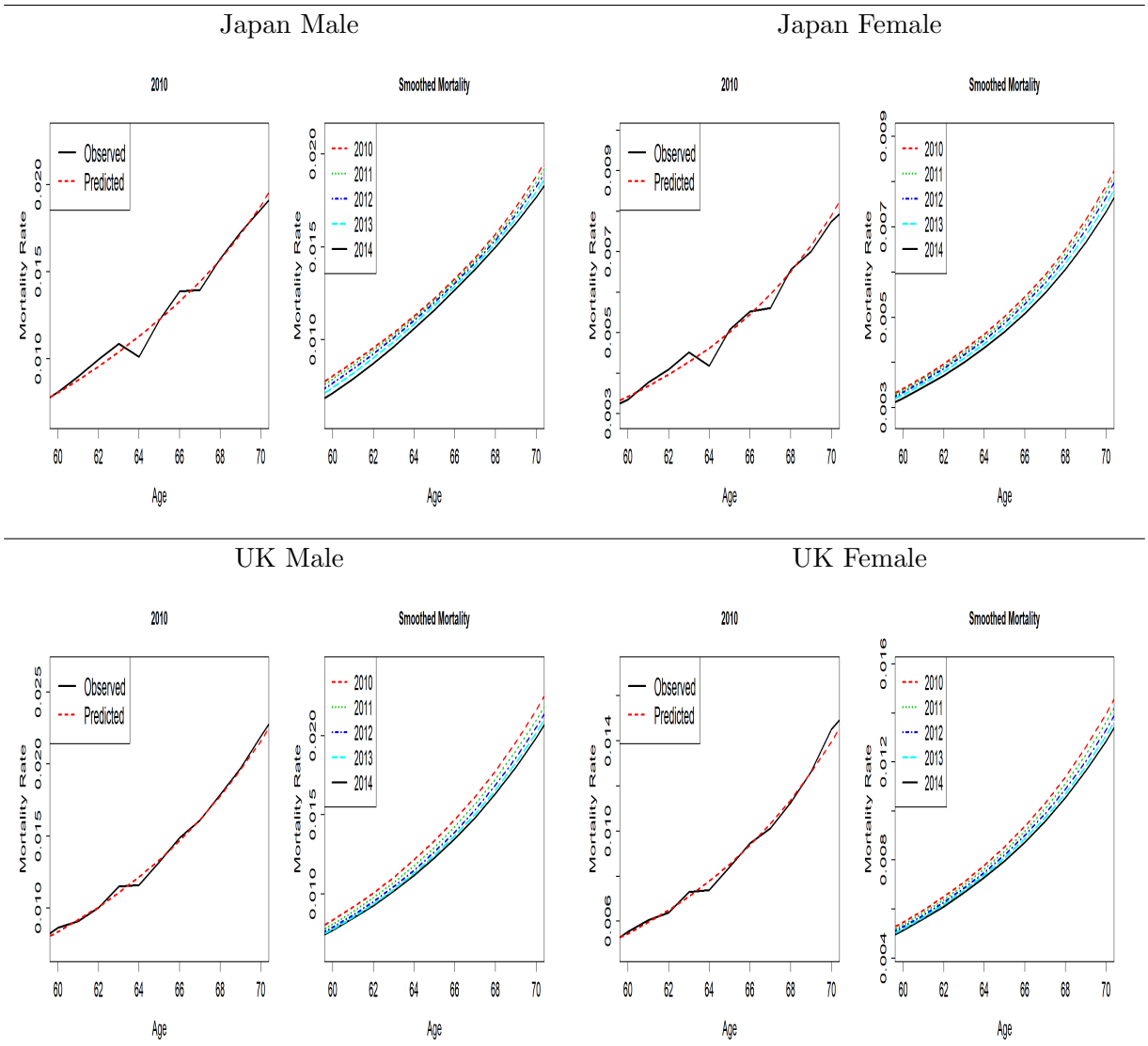

Figure 1: Mortality rates for individuals aged 60–70 during the year 2010. Raw (solid) vs. smoothed (dashed) mortality curves. Models are fit to 1999–2014 HMD data for Ages 50–84 (All data). Mean function $m(x)$ is intercept-only, $m(x) = \beta_0$.

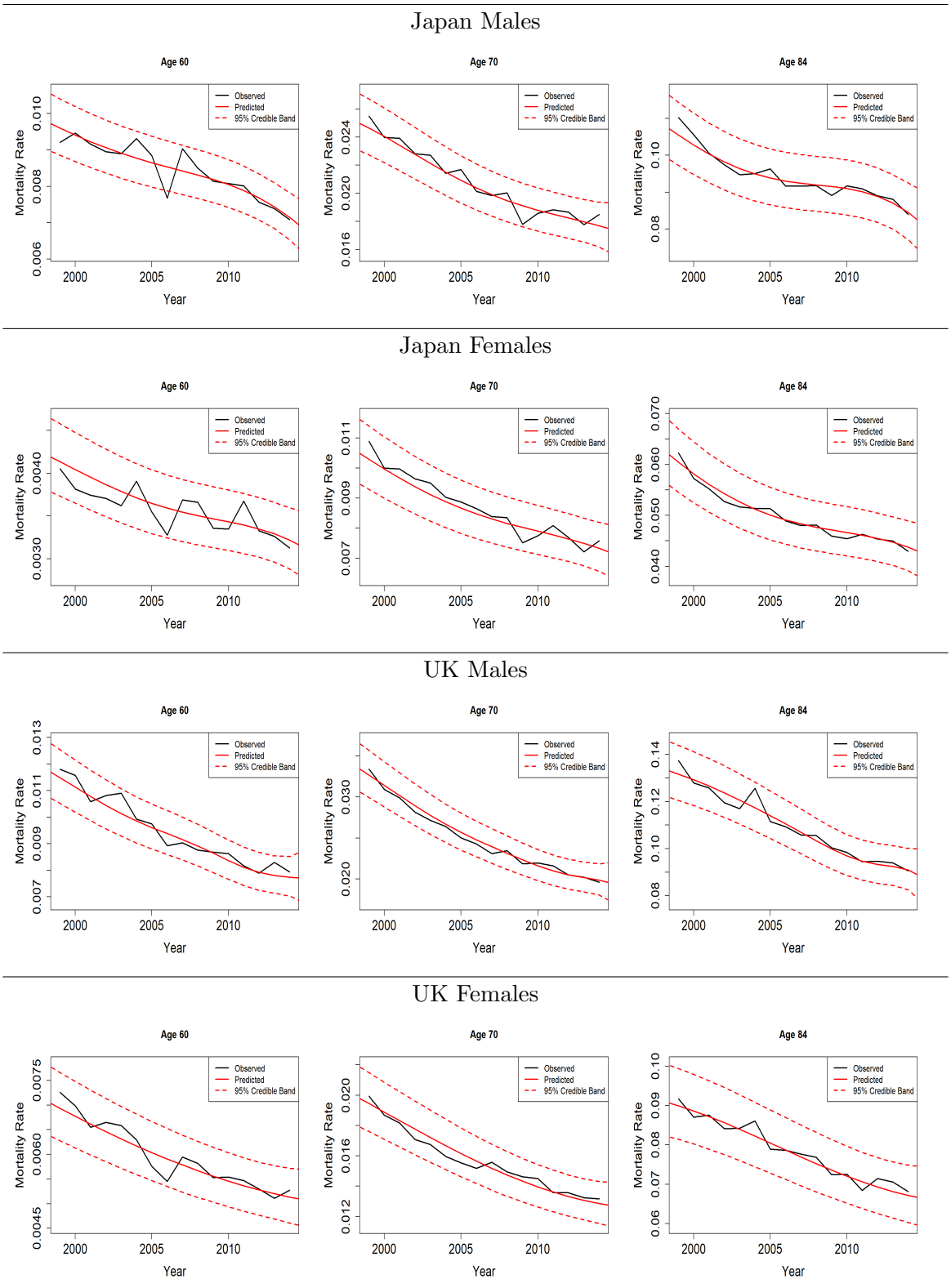

Figure 2: Mortality rates for individuals aged 60, 70 and 84 over time. The plots show raw mortality rates (solid black) for years 1999–2014, as well as predicted mean of the smoothed mortality surface (solid red) and its 95% credible band, for 1999–2016. Mean function is intercept-only, $m(x) = \beta_0$.

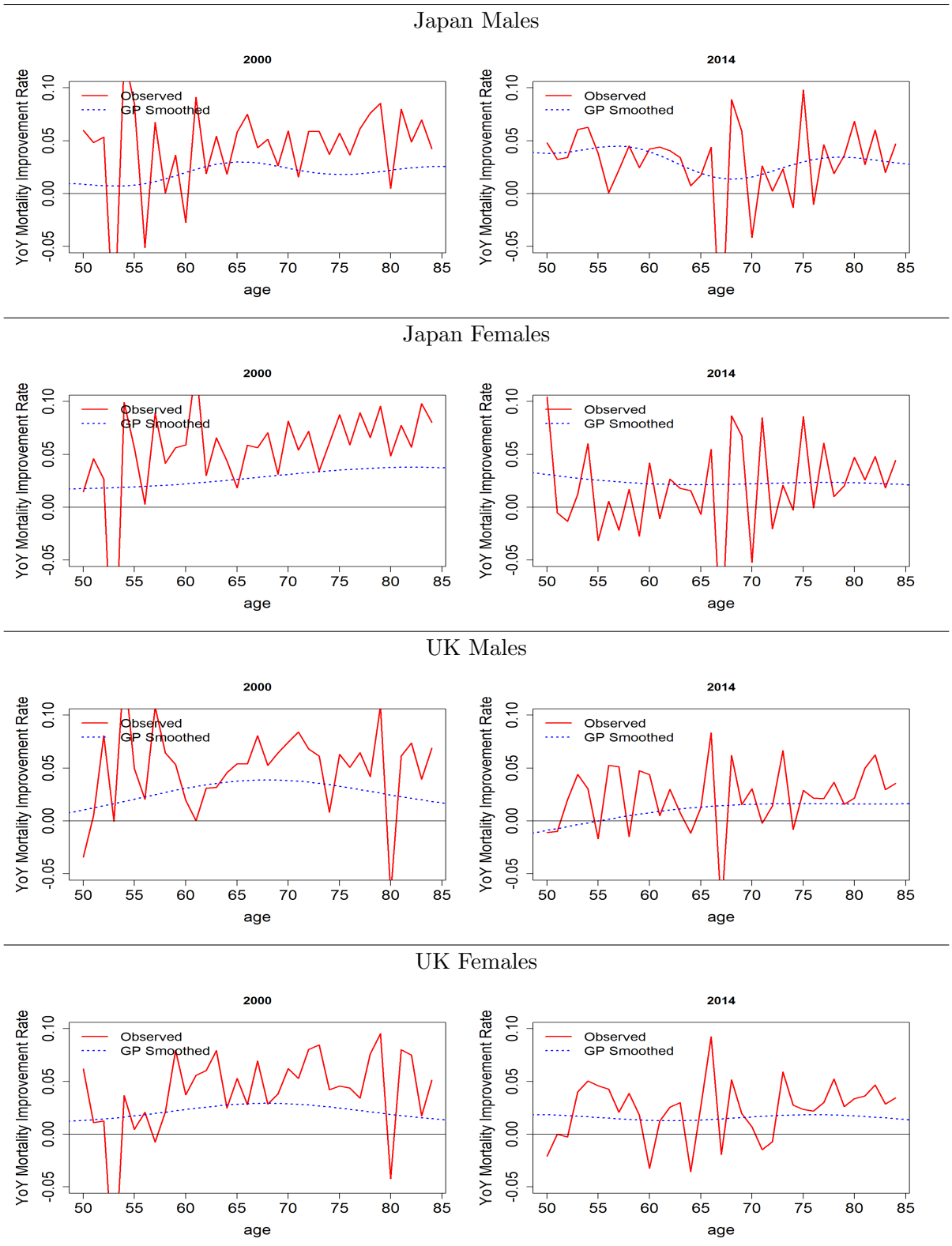

Figure 3: Mortality improvement factors for All Data. Solid lines indicate the empirical mortality experience $MI^{obs}_{back}(\cdot; yr)$ for years $yr \in \{2000, 2014\}$, the dotted lines are $\partial m^{GP}_{back}(\cdot; yr)$ (Equations (13) and (14) in the main text).

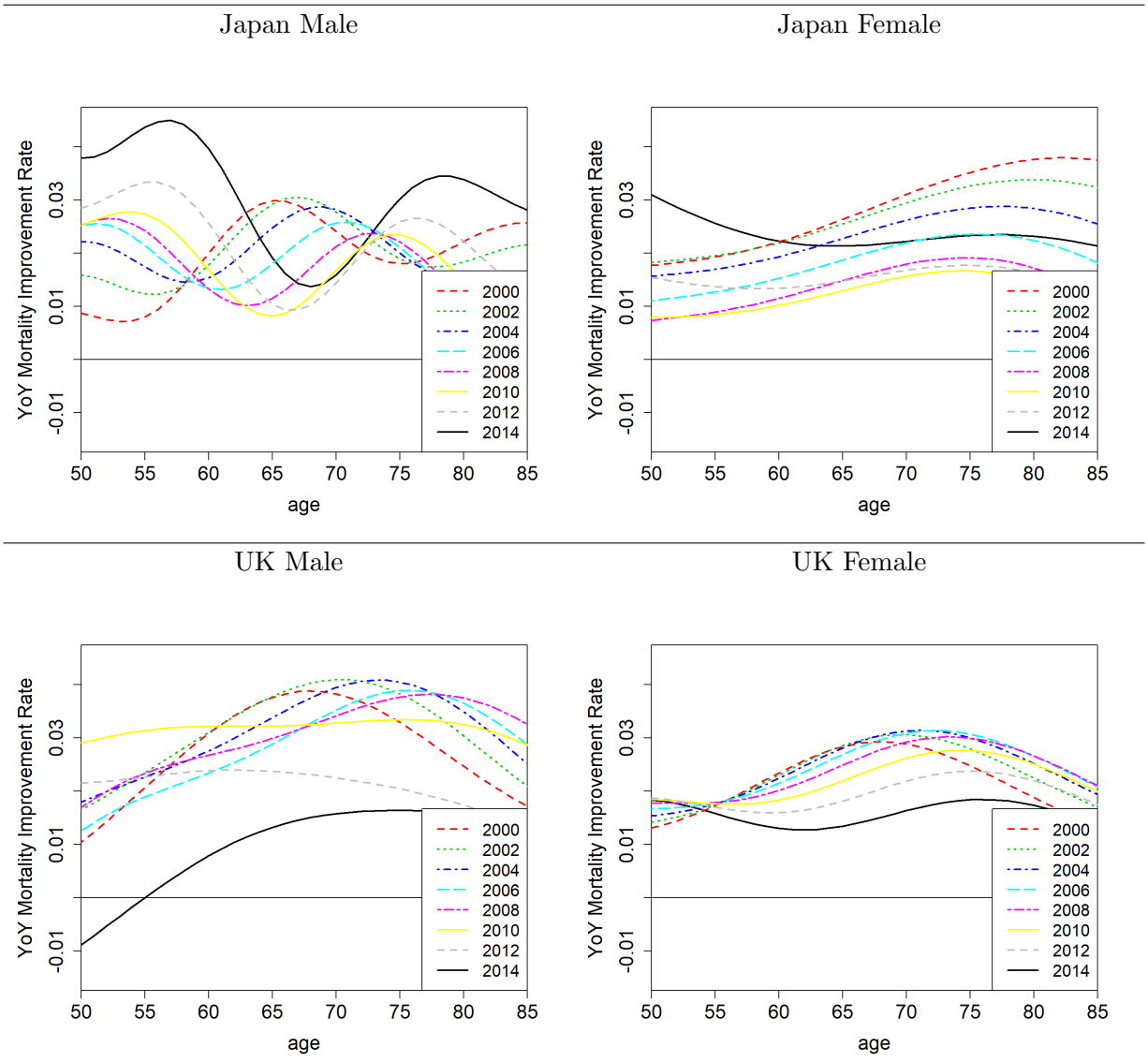

Figure 4: Comparison of smoothed yearly mortality improvement factors $\partial m^{GP}_{back}(x_{ag}; yr)$ from Equation (14) in the main text, for Males using All data and $yr = 2000, \ldots, 2014$. The curves for 2000 and 2014 are the same as in Figure 3.

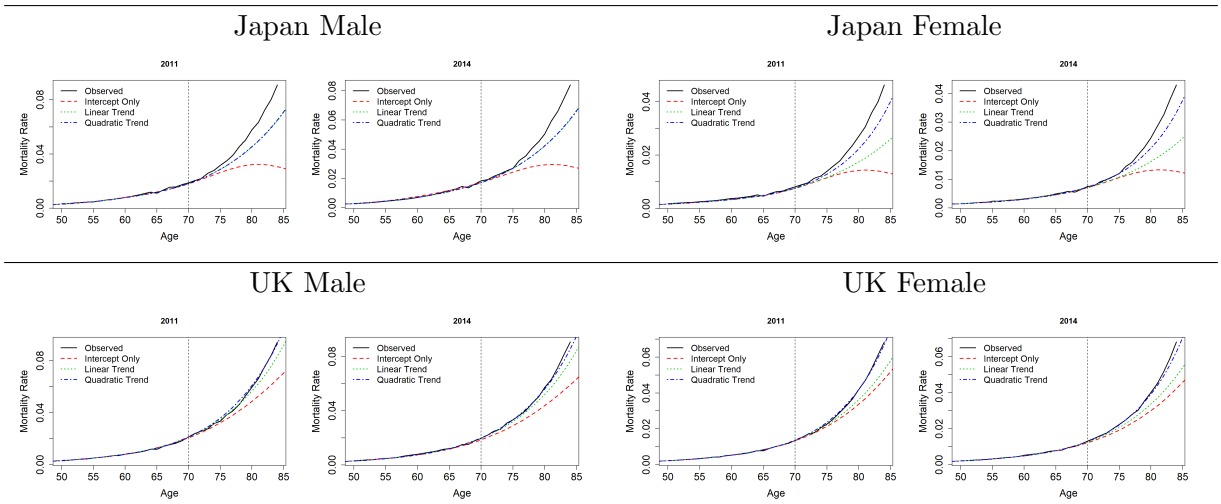

Figure 5: Comparison of mean function choices in extrapolating mortality rates at old ages. Models are fit to years 1999–2010 and ages 50–70 (Subset III), with estimates made for Age 50–85 in 2011 and 2014. The vertical line indicates the boundary of the training dataset in x_{ag}. The mean functions are given in Table 5 in the main text.

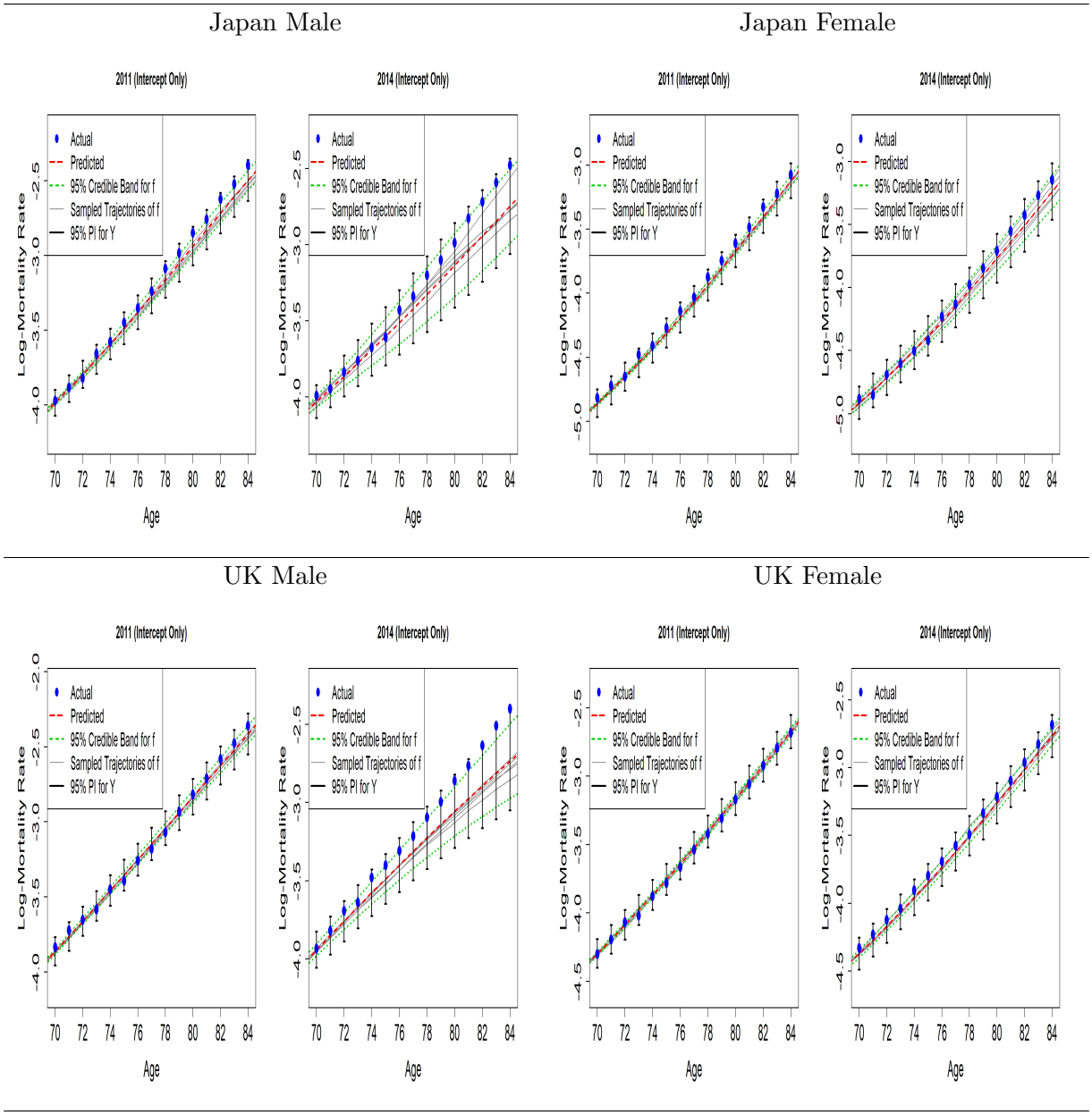

Figure 6: Mortality rate prediction for years 2011 and 2014 and ages 71–84. Model is fit with Subset II data with intercept-only mean functions and squared-exponential kernel. "Simulated paths of f" refers to simulated trajectories of the latent $\mathbf{f}_*$. Credible bands are for the mortality surface $\mathbf{f}_*$; vertical intervals are for predicted observable mortality experience $\mathbf{y}_*$.

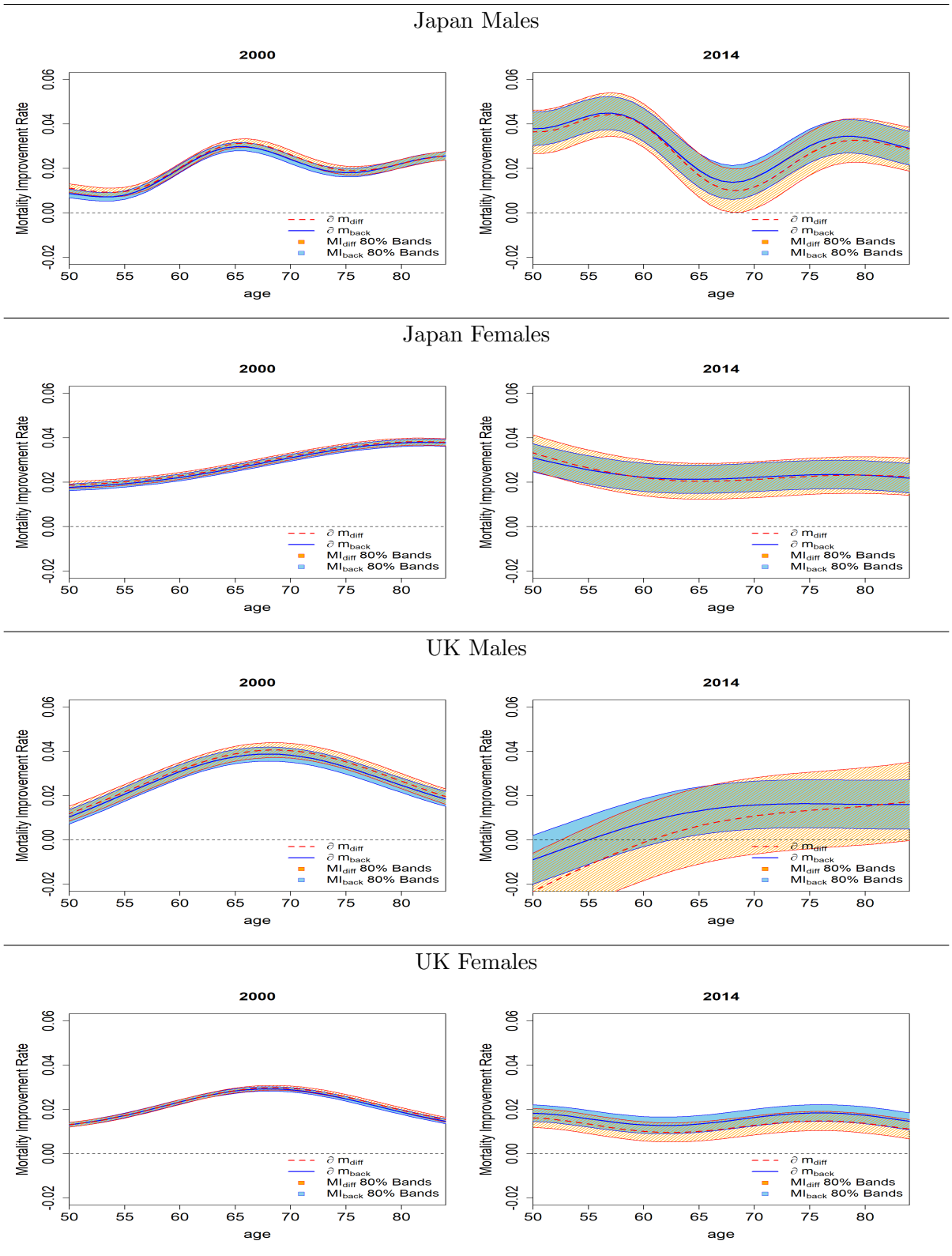

Figure 7: Estimated annualized mortality improvement using the differential GP model (instantaneous improvement) and the YoY improvement from the original GP model. We show the means and 80% credible bands for MI^{GP}_{diff} and MI^{GP}_{back} for ages 50–84 and years 2000 & 2014. Models used are fit to All Data with $m(x) = \beta_0$.