Online Appendix: Not part of the paper

D Proof of Equation (2.5)

First note that

$$\begin{aligned}
&\sum_{j=1}^{d}\sum_{k=1}^{d}\binom{d}{k-1}^{-1}\frac{1}{d-k+1}\sum_{\mathcal{S}\in\mathbb{D}_j^{k-1}}\Big(g(\mathbf{y}^{\mathcal{S}\cup\{j\}})-g(\mathbf{y}^{\mathcal{S}})\Big)\\
&=\sum_{k=1}^{d}\frac{(k-1)!(d-k)!}{d!}\sum_{j=1}^{d}\sum_{\mathcal{S}\in\mathbb{D}_j^{k-1}}\Big(g(\mathbf{y}^{\mathcal{S}\cup\{j\}})-g(\mathbf{y}^{\mathcal{S}})\Big)\\
&=\frac{d}{d}g(\mathbf{y})+\sum_{k=1}^{d-1}\frac{(k-1)!(d-k)!}{d!}\sum_{j=1}^{d}\sum_{\mathcal{S}\in\mathbb{D}_j^{k-1}}g(\mathbf{y}^{\mathcal{S}\cup\{j\}})-\sum_{k=2}^{d}\frac{(k-1)!(d-k)!}{d!}\sum_{j=1}^{d}\sum_{\mathcal{S}\in\mathbb{D}_j^{k-1}}g(\mathbf{y}^{\mathcal{S}})-\frac{d}{d}g(\mathbf{0}).
\end{aligned}$$

Therefore, using the latter equation,

$$\begin{aligned}
&\sum_{j=1}^{d}\sum_{k=1}^{d}\binom{d}{k-1}^{-1}\frac{1}{d-k+1}\sum_{\mathcal{S}\in\mathbb{D}_j^{k-1}}\Big(g(\mathbf{y}^{\mathcal{S}\cup\{j\}})-g(\mathbf{y}^{\mathcal{S}})\Big)-[g(\mathbf{y})-g(\mathbf{0})]\\
=\;&\sum_{k=1}^{d-1}\frac{(k-1)!(d-k)!}{d!}\sum_{j=1}^{d}\sum_{\mathcal{S}\in\mathbb{D}_j^{k-1}}g(\mathbf{y}^{\mathcal{S}\cup\{j\}})-\sum_{k=2}^{d}\frac{(k-1)!(d-k)!}{d!}\sum_{j=1}^{d}\sum_{\mathcal{S}\in\mathbb{D}_j^{k-1}}g(\mathbf{y}^{\mathcal{S}})\\
=\;&\sum_{k=1}^{d-1}\frac{(k-1)!(d-k)!}{d!}\sum_{j=1}^{d}\sum_{\mathcal{S}\in\mathbb{D}_j^{k-1}}g(\mathbf{y}^{\mathcal{S}\cup\{j\}})-\sum_{k=1}^{d-1}\frac{k!(d-k-1)!}{d!}\sum_{j=1}^{d}\sum_{\mathcal{S}\in\mathbb{D}_j^{k}}g(\mathbf{y}^{\mathcal{S}})\\
=\;&\sum_{k=1}^{d-1}\frac{(k-1)!(d-k-1)!}{d!}\left[(d-k)\sum_{j=1}^{d}\sum_{\mathcal{S}\in\mathbb{D}_j^{k-1}}g(\mathbf{y}^{\mathcal{S}\cup\{j\}})-k\sum_{j=1}^{d}\sum_{\mathcal{S}\in\mathbb{D}_j^{k}}g(\mathbf{y}^{\mathcal{S}})\right]\\
=\;&\sum_{k=1}^{d-1}\frac{(k-1)!(d-k-1)!}{d!}\left[(d-k)k\sum_{\mathcal{S}\in\mathbb{D}^{k}}g(\mathbf{y}^{\mathcal{S}})-k(d-k)\sum_{\mathcal{S}\in\mathbb{D}^{k}}g(\mathbf{y}^{\mathcal{S}})\right]\\
=\;&0.
\end{aligned}$$

This implies

$$g(\mathbf{y})-g(\mathbf{0})=\sum_{j=1}^{d}\sum_{k=1}^{d}\binom{d}{k-1}^{-1}\frac{1}{d-k+1}\sum_{\mathcal{S}\in\mathbb{D}_j^{k-1}}\Big(g(\mathbf{y}^{\mathcal{S}\cup\{j\}})-g(\mathbf{y}^{\mathcal{S}})\Big).\quad\square$$

E Properties of risk allocation approaches

This work considers mappings $\mathcal{A}$ allocating the risk $\varrho(Z)$ to the various components $Z^{(j)}$, $j = 1, \ldots, J$. Denote such allocation by

$$\mathcal{A}(Z^{(1)}, \ldots, Z^{(J)}) = \left(\mathcal{A}_1\left(Z^{(1)}, \ldots, Z^{(J)}\right), \ldots, \mathcal{A}_J\left(Z^{(1)}, \ldots, Z^{(J)}\right)\right).$$

Allocation principles (2.13) and (2.14) satisfy the following properties defined in Schilling et al. (2020):

$$\varrho(Z) = \sum_{j=1}^{J} \quad \mathcal{A}_j\left(Z^{(1)}, \ldots, Z^{(J)}\right) \qquad \text{(Additive aggregation)}$$

and for any permutation $\pi(1), \ldots \pi(J)$ of $\{1, \ldots, J\}$ and any $j = 1, \ldots, J$,

$$\mathcal{A}_{\pi(j)}\left(Z^{(1)}, \ldots, Z^{(J)}\right) = \mathcal{A}_j\left(Z^{(\pi(1))}, \ldots, Z^{(\pi(J))}\right). \qquad \text{(Order invariance)}$$

F Futures price calculation

Define

$$\tilde{P}_{t,t+n} \equiv \mathbb{E}^{\mathbb{Q}}\left[\exp\left(\Delta \sum_{j=t}^{t+n-1} r_j\right) \middle| \mathcal{F}_t\right].$$

It can be shown first that

$$\mathbf{Fut}_{t,t+n}^{(j)} = S_t^{(j)} \tilde{P}_{t,t+n}.$$

Indeed, assuming the relationship holds for $t+1$, the Tower Law implies

$$
\begin{aligned}
\mathbf{Fut}^{(j)}_{t,t+n} &= \mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}}\left[S^{(j)}_{t+n}|\mathcal{F}_{t+1}\right]|\mathcal{F}_t\right] \\
&= \mathbb{E}^{\mathbb{Q}}\left[S^{(j)}_{t+1}\tilde{P}_{t+1,t+n}\;\middle|\;\mathcal{F}_t\right] \\
&= \mathbb{E}^{\mathbb{Q}}\left[S^{(j)}_t e^{R^{(S)}_{t+1,j}-r_t\Delta}e^{r_t\Delta}\mathbb{E}^{\mathbb{Q}}\left[\exp\left(\Delta\sum_{j=t+1}^{t+n-1} r_j\right)\;\middle|\;\mathcal{F}_{t+1}\right]\;\middle|\;\mathcal{F}_t\right] \\
&= S^{(j)}_t\mathbb{E}^{\mathbb{Q}}\left[e^{R^{(S)}_{t+1,j}-r_t\Delta}\mathbb{E}^{\mathbb{Q}}\left[\exp\left(\Delta\sum_{j=t}^{t+n-1} r_j\right)\;\middle|\;\mathcal{F}_{t+1}\right]\;\middle|\;\mathcal{F}_t\right] &\text{(F.1)} \\
&= S^{(j)}_t\underbrace{\mathbb{E}^{\mathbb{Q}}\left[e^{R^{(S)}_{t+1,j}-r_t\Delta}\;\middle|\;\mathcal{F}_t\right]}_{=1}\mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}}\left[\exp\left(\Delta\sum_{j=t}^{t+n-1} r_j\right)\;\middle|\;\mathcal{F}_{t+1}\right]\;\middle|\;\mathcal{F}_t\right] &\text{(F.2)} \\
&= S^{(j)}_t\tilde{P}_{t,t+n}
\end{aligned}
$$

where the passage from (F.1) to (F.2) stems from the conditional independence with respect to $\mathcal{F}_t$ as can be seen from (B.2). Moreover, define $\mathcal{X}^{(i)}_{n,t} \equiv \sum_{\ell=0}^{n-1} x^{(i)}_{t+\ell}$. Augustyniak et al. (2021) show that under $\mathbb{Q}$ and conditional on $\mathcal{F}_t$, $\mathcal{X}_{n,t} \equiv [\mathcal{X}^{(1)}_{n,t}\cdots\mathcal{X}^{(3)}_{n,t}]^\top$ is multivariate Gaussian mean vector $m_{n,t} \equiv \left[m^{(1)}_{n,t}\cdots m^{(3)}_{n,t}\right]^\top$ and covariance matrix $v_n \equiv \left[v^{(i,\ell)}_n\right]^3_{i,\ell=1}$, where

$$
m^{(i)}_{n,t} = \left(x^{(i)}_t - \tilde{\mu}_i\right)\left[\frac{1-(1-\tilde{\kappa}_i)^n}{\tilde{\kappa}_i}\right] + \tilde{\mu}_i n
$$

and

$$
v^{(i,\ell)}_n = \frac{\sigma_i\sigma_\ell}{\tilde{\kappa}_i\tilde{\kappa}_\ell}\Gamma_{i,\ell}\left[n - \frac{1-(1-\tilde{\kappa}_i)^n}{\tilde{\kappa}_i} - \frac{1-(1-\tilde{\kappa}_\ell)^n}{\tilde{\kappa}_\ell} + \frac{1-(1-\tilde{\kappa}_i)^n(1-\tilde{\kappa}_\ell)^n}{1-(1-\tilde{\kappa}_i)(1-\tilde{\kappa}_\ell)}\right].
$$

Thus

$$
\begin{aligned}
\tilde{P}_{t,t+n} &= \mathbb{E}^{\mathbb{Q}}\left[\exp\left(\Delta\sum_{i=1}^{3}\sum_{j=0}^{n-1} x^{(i)}_{t+j}\right)\;\middle|\;\mathcal{F}_t\right] \\
&= \mathbb{E}^{\mathbb{Q}}\left[\exp\left(\Delta\sum_{i=1}^{3}\mathcal{X}^{(i)}_{n,t}\right)\;\middle|\;\mathcal{F}_t\right] \\
&= \exp\left\{\Delta\sum_{i=1}^{3} m^{(i)}_{n,t} + \frac{\Delta^2}{2}\sum_{i=1}^{3}\sum_{\ell=1}^{3} v^{(i,\ell)}_n\right\}.
\end{aligned}
$$

G Guarantee equity Greek letters

To show the last equality in (3.16), the chain rule entails

$$\frac{\partial \Pi_t}{\partial S_t^{(j)}} = \frac{\partial \Pi_t}{\partial A_t}\frac{\partial A_t}{\partial F_t}\frac{\partial F_t}{\partial S_t^{(j)}} = \Delta_t^{(guar,A)}\frac{A_t}{F_t}\frac{\partial F_t}{\partial S_t^{(j)}} \tag{G.1}$$

where $\frac{\partial A_t}{\partial F_t} = \frac{A_t}{F_t}$ comes from the fact that the policy account A is fully invested in the mutual fund F. Furthermore, the following result translates the Greek with respect to the underlying fund into Greeks with respect to equity indices.

Lemma G.1. *For any $j = 1, 2$,*

$$\frac{\partial F_t}{\partial S_t^{(j)}} = \frac{\theta_j^{(S)} F_t}{S_t^{(j)}}. \tag{G.2}$$

Proof of Lemma G.1: Equation (3.7) implies

$$\begin{aligned}
F_t &= F_0 \exp\left(\Delta \sum_{k=0}^{t-1} r_k + \theta_0 t + \sum_{k=0}^{t-1}\sum_{i=1}^{3} \theta_i \left(x_{k+1}^{(i)} - (1-\tilde{\kappa}_i) x_k^{(i)} \right) + \sum_{k=0}^{t-1}\sum_{j'=1}^{2} \theta_{j'}^{(S)} R_{k+1,j}^{(S)} + \sum_{k=0}^{t-1} \sqrt{h_k^{(F)}} z_{k+1}^{(F)} \right) \\
&= \underbrace{F_0 \exp\left(\Delta \sum_{k=0}^{t-1} r_k + \theta_0 t + \sum_{k=0}^{t-1}\sum_{i=1}^{3} \theta_i \left(x_{k+1}^{(i)} - (1-\tilde{\kappa}_i) x_k^{(i)} \right) + \sum_{k=0}^{t-1}\sum_{j'=1, j'\neq j}^{2} \theta_{j'}^{(S)} R_{k+1,j'}^{(S)} + \sum_{k=0}^{t-1} \sqrt{h_k^{(F)}} z_{k+1}^{(F)} \right)}_{=:\mathcal{E}_{t,j}} \\
&\quad \times \exp\left(\sum_{k=0}^{t-1} \theta_j^{(S)} R_{k+1,j}^{(S)} \right) \\
&= \mathcal{E}_{t,j} \left(\frac{S_t^{(j)}}{S_0} \right)^{\theta_j^{(S)}}.
\end{aligned}$$

As a consequence,

$$\frac{\partial F_t}{\partial S_t^{(j)}} = \mathcal{E}_{t,j} \frac{\theta_j^{(S)}}{S_0^{(j)}} \left(\frac{S_t^{(j)}}{S_0^{(j)}} \right)^{\theta_j^{(S)}-1} = \mathcal{E}_{t,j} \frac{\theta_j^{(S)}}{S_t^{(j)}} \left(\frac{S_t^{(j)}}{S_0^{(j)}} \right)^{\theta_j^{(S)}} = \frac{\theta_j^{(S)} F_t}{S_t^{(j)}}. \quad \square$$

Substituting (G.2) into (G.1) thereby shows (3.16).

H Testing the stability of simulations

The experiment presented in the fifth column of Table 4, i.e. the 20-year GMMB policy without ratchets applied on the Assumption mixed fund is absence of hedging, is repeated five times with different random seeds to assess the stability of the simulation framework. Results are presented in Table 6. Although allocation values exhibit non-negligible fluctuations across runs, they produce qualitatively similar results.

Table 6: Impact of the seed on risk allocation results

	Assumption mixed fund 20-year maturity — No ratchets				
Run	1	2	3	4	5
Total expected loss decomposition					
Expected loss; $\mathbb{E}\left(-GL_{tot}\right)$	− 0.049	− 0.052	− 0.052	− 0.049	− 0.048
Equity $\mathcal{A}_{EQ_{tot}}$	− 4.093	− 3.385	− 3.938	− 4.082	− 3.216
Interest rate $\mathcal{A}_{IR_{tot}}$	− 4.984	− 4.491	− 4.723	− 4.886	− 4.238
Mortality $\mathcal{A}_{MO_{tot}}$	− 5.559	− 5.165	− 5.367	− 5.578	− 5.076
Time $\mathcal{A}_{\tilde{\Theta}_{tot}}$	14.587	12.990	13.977	14.497	12.482
Total variance decomposition					
Total risk; $\mathrm{Var}\left(-GL_{tot}\right)$	0.0055	0.0052	0.0049	0.0060	0.0063
Equity $\mathcal{A}_{EQ_{tot}}$	0.4582	0.5945	0.4493	0.6394	0.5924
Interest rate $\mathcal{A}_{IR_{tot}}$	0.0839	0.1778	0.1056	0.1626	0.1640
Mortality $\mathcal{A}_{MO_{tot}}$	0.0711	0.1368	0.0844	0.1280	0.1260
Time $\mathcal{A}_{\tilde{\Theta}_{tot}}$	−0.6076	−0.9039	−0.6345	−0.9240	−0.8761
Total $CVaR_{95\%}$ decomposition					
Total risk; $\mathrm{CVaR}_{95\%}\left(-GL_{tot}\right)$	0.16	0.16	0.15	0.18	0.19
Equity $\mathcal{A}_{EQ_{tot}}$	14.00	27.51	16.10	21.94	22.82
Interest rate $\mathcal{A}_{IR_{tot}}$	− 3.15	9.21	4.13	4.73	7.55
Mortality $\mathcal{A}_{MO_{tot}}$	− 0.24	4.39	0.43	0.73	2.53
Time $\mathcal{A}_{\tilde{\Theta}_{tot}}$	−16.74	−40.95	−20.53	−27.22	−32.71

Notes: For 5 different runs based on different random seeds, risk allocations $\mathcal{A}$ to time decay $(-\tilde{\Theta}_{tot})$, interest rate risk $(-\widetilde{\mathcal{C}}_{tot}^{(IR)})$, equity risk $(-\widetilde{\mathcal{C}}_{tot}^{(EQ)})$ and mortality risk $(-\widetilde{\mathcal{C}}_{tot}^{(MO)})$ applied on a 20-year maturity GMMB policy without ratchets on the Assumption mixed fund described in Section 3.3. Three risk measures are considered for the allocation: the expectation, the variance and the $\mathrm{CVaR}_{95\%}$; allocation principles for such measures are presented in Section 2.4. No hedging is applied.

I Impact of moneyness on time decay contribution

The magnitude and direction of contributions from the sources of risk vary not only with respect to time, but also with respect to the moneyness of the contract. Fig. 3 illustrates such phenomenon by showing realized values of the discounted time decay contribution $\tilde{\Theta}_t$ versus the account value A_{t-1} for two time points $t = 2$ and $t = 110$, this once again for the case of the 20-year GMMB without ratchet on the Assumption mixed fund (no hedging applied). The figure clearly shows that when the contract is in-the-money, i.e. the account value is lower than the strike, the decrease of time-to-maturity tends to improve profitability (positive $\tilde{\Theta}_t$) due to smaller likelihood of having to pay large benefits. Conversely, the passage of time for an out-of-the-money contract reduces possibilities for profitable increases in fees, thereby leading to negative values for $\tilde{\Theta}_t$.

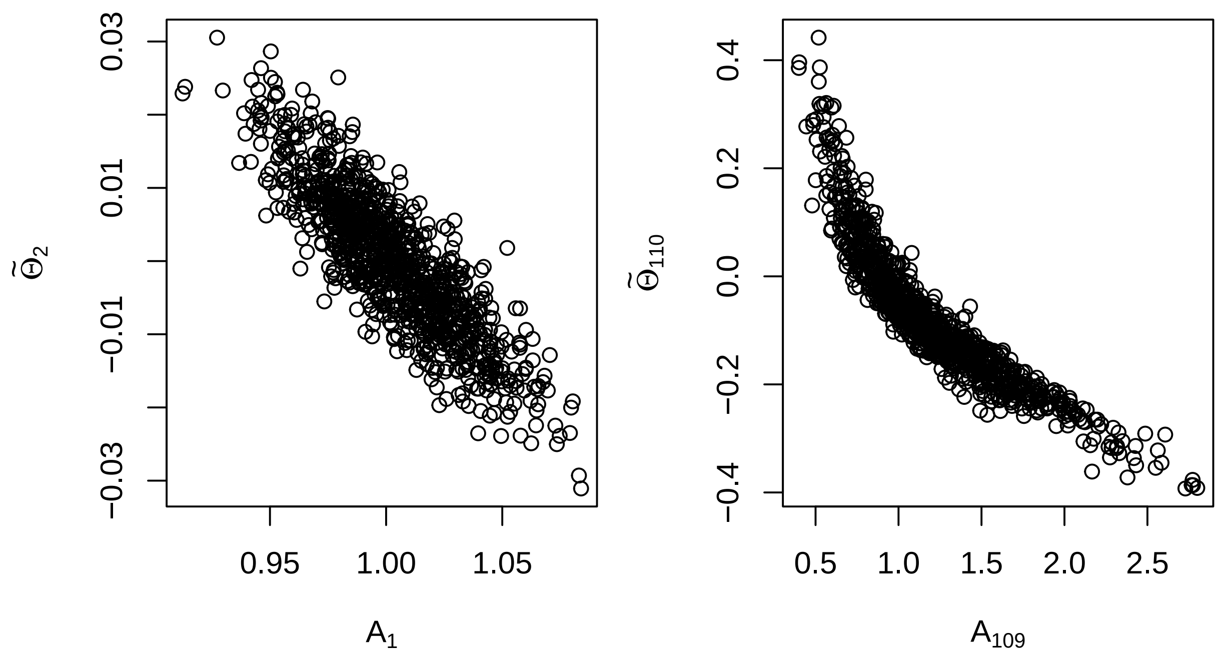

Figure 3: Realized values of the discounted time decay contribution $\tilde{\Theta}_t$ versus the account value A_{t-1} for two time points $t = 2$ and $t = 110$ in the simulation for the 20-year GMMB without ratchet on the Assumption mixed fund.

J Decomposing interest rate risk

To further shed light on the contribution of interest rate risk, an additional analysis separating the contribution of each of the three interest rate factors is provided. Define

$$\left(\mathcal{A}_{\tilde{\Theta}_{tot}}, \mathcal{A}_{IR^{(1)}_{tot}}, \mathcal{A}_{IR^{(2)}_{tot}}, \mathcal{A}_{IR^{(3)}_{tot}}, \mathcal{A}_{EQ_{tot}}, \mathcal{A}_{MO_{tot}}\right) \equiv \mathcal{A}\left(-\tilde{\Theta}_{tot}, -\widetilde{\mathcal{C}}^{(1)}_{tot}, -\widetilde{\mathcal{C}}^{(2)}_{tot}, -\widetilde{\mathcal{C}}^{(3)}_{tot}, -\widetilde{\mathcal{C}}^{(EQ)}_{tot}, -\widetilde{\mathcal{C}}^{(MO)}_{tot}\right) \tag{J.1}$$

where $\widetilde{\mathcal{C}}^{(i)}_{tot}$ is the component related to $\{z^{(r)}_{t,i}\}^T_{t=1}$, $i = 1, 2, 3$, i.e. assuming that the first element of the sources of risk vector (4.1) is split into three separate components, each of which is associated to one of the three term structure factors. For the same set of simulations presented in Table 4, the breakdown of risk according to (J.1) is presented in Table 7. Such results provide interesting additional explanations about why the allocation of risk to interest rates is low for the RBC bond fund. For instance, when considering the $\text{CVaR}_{95\%}$ risk measure, each term structure factor has a high associated allocation in absolute value, but some of the allocations are of opposite sign, creating a nullifying effect reducing the net total allocation to interest rate risk. This can partially be explained for instance by the negative correlation between innovations of the first interest rate factor, and those of factors 2 and 3, see the matrix Γ in Table 1. Interestingly, when comparing risk allocation to equity and mortality in Table 4 and Table 7, it can be seen they are of roughly of similar magnitude regardless of whether interest rate contributions are grouped or separated by factor. However, they are not strictly equal, which highlights that the choice of risk groups does impact the risk allocation results. Conversely, the total risk and the portion of risk allocated to the passage of time are strictly identical regardless of the grouping.

Table 7: Total risk allocation for GMMB policies in absence of hedging, with interest rate contributions split between each term structure factor

	RBC bond fund			Assumption mixed fund		
Maturity	10 years	20 years	20 years	10 years	20 years	20 years
With ratchet?	No	No	Yes	No	No	Yes
Total expected loss decomposition						
Expected loss; $\mathbb{E}\left(-GL_{tot}\right)$	0.029	0.029	0.030	−0.056	− 0.046	−0.078
Equity $\mathcal{A}_{EQ_{tot}}$	0.104	−1.109	−0.510	1.522	− 0.858	2.700
Factor 1 interest rate risk $\mathcal{A}_{IR^{(1)}_{tot}}$	−1.120	−5.303	−0.575	−0.072	− 2.790	−1.819
Factor 2 interest rate risk $\mathcal{A}_{IR^{(2)}_{tot}}$	0.122	−1.216	−0.748	−0.214	− 3.070	−2.040
Factor 3 interest rate risk $\mathcal{A}_{IR^{(3)}_{tot}}$	−0.554	−0.774	−2.258	−0.065	− 2.587	−1.652
Mortality $\mathcal{A}_{MO_{tot}}$	−0.064	−1.405	−0.879	−0.220	− 3.068	−2.023
Time $\mathcal{A}_{\tilde{\Theta}_{tot}}$	1.540	9.836	4.999	−1.006	12.328	4.757
Total variance decomposition						
Total risk; $\text{Var}\left(-GL_{tot}\right)$	0.0027	0.0013	0.0022	0.0095	0.0062	0.010
Equity $\mathcal{A}_{EQ_{tot}}$	0.0515	0.0477	0.0856	0.5405	0.5778	1.080
Factor 1 interest rate risk $\mathcal{A}_{IR^{(1)}_{tot}}$	0.1135	0.0619	0.0205	0.0758	0.0932	0.246
Factor 2 interest rate risk $\mathcal{A}_{IR^{(2)}_{tot}}$	0.0388	0.0282	0.0682	0.0515	0.0794	0.205
Factor 3 interest rate risk $\mathcal{A}_{IR^{(3)}_{tot}}$	−0.0809	0.0210	0.0895	0.0936	0.1239	0.297
Mortality $\mathcal{A}_{MO_{tot}}$	0.0106	0.0113	0.0438	0.0527	0.0808	0.204
Time $\mathcal{A}_{\tilde{\Theta}_{tot}}$	−0.1307	−0.1688	−0.3054	−0.8047	−0.9490	−2.023
Total $CVaR_{95\%}$ decomposition						
Total risk; $\text{CVaR}_{95\%}\left(-GL_{tot}\right)$	0.16	0.11	0.16	0.23	0.19	0.20
Equity $\mathcal{A}_{EQ_{tot}}$	3.35	3.34	5.37	17.00	24.13	31.94
Factor 1 interest rate risk $\mathcal{A}_{IR^{(1)}_{tot}}$	1.64	−7.69	−2.54	2.05	3.60	6.17
Factor 2 interest rate risk $\mathcal{A}_{IR^{(2)}_{tot}}$	1.76	1.02	4.57	1.19	1.91	4.36
Factor 3 interest rate risk $\mathcal{A}_{IR^{(3)}_{tot}}$	−3.32	4.14	4.82	2.66	5.55	8.20
Mortality $\mathcal{A}_{MO_{tot}}$	0.45	−0.02	3.13	1.18	2.00	4.19
Time $\mathcal{A}_{\tilde{\Theta}_{tot}}$	−3.73	−0.69	−15.18	−23.86	−37.00	−54.66

Notes: Risk allocations defined by (J.1) applied on GMMB policies on the RBC bond fund or the Assumption mixed fund mentioned in Section 3.3 in absence of hedging. Three risk measures are considered for the allocation: the expected value, the variance and the $\text{CVaR}_{95\%}$; allocation principles for such measures are presented in Section 2.4. Two policy maturities are considered: 10 years and 20 years. For 20-year policies, results with and without the inclusion of ratchets in accordance with the specification (3.1) are presented.

K List of symbols

- T: number of time periods at the onset until the contract maturity.
- Δ: length of a monthly time period in years (1/12),
- $(\Omega, \mathcal{F}, \mathbb{P})$: probability space composed of the sample set Ω, the event space $\mathcal{F}$ and the physical probability measure $\mathbb{P}$,
- $\mathbb{Q}$: risk-neutral probability measure,
- $\mathcal{F}_t$: sigma-algebra giving available information at time t,
- CF_t: time-t contract inflow to the insurer,
- r_t: annualized risk free short rate from t to $t+1$,
- Π_t: time-t contract fair value,
- D_{t_1,t_2} stochastic risk-free discount factor to discount between times t_1 and t_2,
- $\mathbf{X}$: risk factors stochastic process,
- $\mathbf{Y}$: risk factor shocks stochastic process,
- $\mathcal{U}_t$: function mapping previous risk factors and risk factor shocks into next-period risk factors,
- f: function mapping current time and risk factors into the contract fair value,
- V_t^{ϕ}: hedging portfolio value at time t,
- H_t^{ϕ}: hedging gains between $t-1$ and t,
- GL_t: total gains by the insurer between $t-1$ and t, including hedging gains,
- GL_{tot}: sum of discounted gains by the insurer over the entire contract duration,
- $\widetilde{r}$, $\widetilde{CF}_t$, $\tilde{H}_t^{\phi}$, $\widetilde{GL}_t$: functions mapping the risk factors into the risk-free rate, or into the time-t insurer inflow, hedging gains or gain and loss,
- $\mathbb{D}$: the set of subsets of $\{1, \ldots, d\}$,
- $\mathbf{Y}_t^{\mathcal{S}} \equiv \mathbb{1}_{\mathcal{S}} \circ \mathbf{Y}_t$ where $\mathbb{1}_{\mathcal{S}}$ is a dummy vector with ones in elements of indices in $\mathcal{S} \subseteq \mathbb{D}$,
- Θ_t: time-t time decay contribution to the gain and loss,
- $\mathcal{C}_t^{(j)}$: time-t source of risk j's contribution to the gain and loss,
- $\tilde{\Theta}_t$ and $\tilde{\mathcal{C}}_t^{(j)}$: discounted version of contributions Θ_t and $\mathcal{C}_t^{(j)}$,
- $\tilde{\Theta}_{tot}$ and $\tilde{\mathcal{C}}_{tot}^{(j)}$: discounted contributions summed over all time steps,
- VaR_α and CVaR_α: Value-at-Risk and Conditional Value-at-Risk of level α,
- ϱ: risk measure,
- $\mathcal{A}_j$: allocation of total risk to source of risk j,
- A_t: time-t variable annuity underlying account value,
- F_t: time-t value of underlying fund F,
- ω: periodic fee rate charged to the policyholder,
- ${}_ta_x$: probability of the policy remaining active t months given the policyholder was aged x months at the onset,
- ${}_tp_x$ probability that the policyholder survives t months given he is aged x months,
- $\mathcal{L}$: function mapping the contract moneyness level into a one-period lapse probability,
- $\mathcal{L}^{ann}$: annualized version of $\mathcal{L}$,
- m_t: time-t moneyness of the policy,

- Fee_t: period-t fee charged to the policyholder,
- Lapse penalty$_t$: lapse penalty for a lapse on period t,
- $\mathcal{P}(t)$: lapse penalty rate for a lapse on period t,
- $q^{\mathcal{L}}_{x,t-1}$: proportion of policyholders active at $t-1$ who lapse their policy in the next period
- $\lambda^{\mathcal{L}}_t$ and $\lambda^{\mathcal{L}}_t$: forces of lapsation and mortality on period t,
- G_t: GMMB guaranteed amount at time t,
- ζ_{t+1}: number of times in the current year that the guaranteed amount was increased prior to time $t+1$
- Benefit: GMMB benefit paid to the policyholder at maturity if he is active,
- $x^{(i)}_t$: time-t value of the i^{th} term structure factor,
- $\kappa_i, \mu_i, \sigma_i$ speed of reversion, long-term mean and volatility of $x^{(i)}_t$ under measure $\mathbb{P}$,
- $\tilde{\kappa}_i, \tilde{\mu}_i$ speed of reversion and long-term mean of $x^{(i)}_t$ under measure $\mathbb{Q}$,
- $z^{(r)}_{t,i}$, $z^{(S)}_{t,j}$, $z^{(F)}_t$: time-t $\mathbb{P}$-innovations for term structure factor i, equity index j and the underlying fund,
- $\tilde{z}^{(r)}_{t,i}$, $\tilde{z}^{(S)}_{t,j}$, $\tilde{z}^{(F)}_t$: time-t $\mathbb{Q}$-innovations for term structure factor i, equity index j and the underlying fund,
- Γ: contemporaneous correlation matrix for innovations $z^{(r)}_{t,i}$, $i = 1, 2, 3$,
- ρ: contemporaneous correlation matrix for innovations $z^{(S)}_{t,j}$, $j = 1, 2$,
- $R^{(S)}_{t+1,j}$, $R^{(F)}_{t+1}$ log-returns for equity index j and the underlying fund between t and $t+1$,
- $S^{(j)}_t$ time-t value of equity index j,
- $\lambda^{(S)}_j$, $\omega^{(S)}_j$, $\alpha^{(S)}_j$, $\beta^{(S)}_j$, $\gamma^{(S)}_j$: GARCH parameters for the equity index j model,
- $\omega^{(F)}$, $\alpha^{(F)}$, $\beta^{(F)}$, $\gamma^{(F)}$: GARCH parameters for the underlying fund basis risk model,
- $(\theta_0, \theta_1, \theta_2, \theta_3)$, $(\theta^{(S)}_1, \theta^{(S)}_2)$: linear loading parameters for the fund returns on term structure factor shocks and equity returns,
- $h^{(S)}_{t,j}$, $h^{(F)}_t$: time-t volatility for equity index j and the basis risk part of the underlying fund,
- $\mathcal{Y}(x)$: age in years (integer) of a policyholder aged x months,
- a_x, b_x: Lee-Carter mortality model parameters,
- σ_ε: function mapping policyholder age into Lee-Carter innovations ε volatility,
- $\gamma_1, \gamma_2, \delta_1, \delta_2$: dynamic lapse model parameters,
- $u_t, \varepsilon_t, \nu_t$: random variables driving the mortality process,
- $\phi^{(i)}_t$ number of long positions on traded instrument i in the hedging portfolio between $t-1$ and t,
- $\delta^{(i)}_t$: price variation of a unitary position in instrument i between $t-1$ and t
- $\mathbf{Fut}^{(j)}_{t,t+n}$: time-$t$ price for the futures on equity index j maturing at $t+n$,
- $\Delta^{(guar,A)}_t$: time-t guarantee value sensitivity (Delta) to the account value,
- $\Delta^{(fut,S,j,n)}_t$: time-t price sensitivity (Delta) of the futures on index j with time-to-maturity n to its underlying asset value.