**APPENDIX B: An alternative derivation of the time dependent hardness based on the scalar loading conditions and the proposed creep mechanisms**

Equations (6) – (18) relate the measured indentation strain rate to the measured hardness through the mechanisms of Harper-Dorn creep and Nabarro-Herring creep. This appendix considers a complimentary approach to the rationalization of the Li indentation results presented here by utilizing the applied load-time history to analyze the applicability of the proposed creep mechanism models. This complimentary approach considers the calculation of $h\left(t\right)$ from an applied loading history using a specific creep model. The resulting time dependence of the indentation depth can then be used with the loading history to calculate the hardness for comparison with experimental observations. The Harper-Dorn and Nabarro-Herring creep processes are again considered as possible mechanisms to couple the observed loading rate to the observed hardness. The physics utilized in the appendix are similar to the development in the main text in that the indentation strain rate at the indentation depths of interest is considered to be dominated by the plastic (creep) strain rate. As justification, we consider that the Sneddon7 solution allows for the definition of a scalar indentation elastic strain that can be related to the measured hardness as ${σ}/{E}$ and, thus, the scalar elastic strain rate is

$$\frac{dϵ\_{elastic}}{dt}=\frac{1}{E}\frac{dσ}{dt} . (B1)$$

The total indentation strain rate is then given by the indentation depth relationship containing the contribution from the elastic strain rate and the creep (plastic) strain rate. Over the indentation depth range of interest here (approximately 100 to 350 nm), it is observed that the total indentation strain rate is approximately a factor of 10 or more than the elastic strain rate, or

$$\frac{1}{h\left(t\right)}\frac{dh\left(t\right)}{dt}\gg \frac{1}{E}\frac{dσ}{dt} , (B2)$$

which suggests the total indentation strain rate is dominated by the plastic (creep) strain rate. We take advantage of this situation by expressing the indentation strain rate as a function of the proposed creep mechanism. The main text has considered that Harper-Dorn creep is operating under the applied constant loading rate, $P\left(t\right)=γt$, so that the inequality expressed by Eq. (B2) allows the approximation

$$\frac{1}{h\left(t\right)}\frac{dh\left(t\right)}{dt}=\frac{βγt\sqrt{ρ}}{αh\left(t\right)^{3}} , (B3)$$

where $β$ and $ρ$ are as previously defined.

The solution to the differential Eq. (B3) gives

$$h\left(t\right)=\left(\frac{3γ\sqrt{ρ}β}{2α}\right)^{{1}/{3}}t^{{2}/{3}} . (B4)$$

Using Eq. (B4) to relate time to indentation depth in Eq. (B3) gives an expression for the indentation strain rate as a function of indentation depth,

$$\frac{1}{h\left(t\right)}\frac{dh\left(t\right)}{dt}=Ah^{{-3}/{2}} , (B5)$$

where

$$A=\frac{\left(6βγ\sqrt{ρ}\right)^{{1}/{2}}}{3\sqrt{α}} . (B6)$$

The result in Eq. (B5) reproduces the depth dependence of the experimentally observed indentation strain rate utilizing the experimentally determined indentation load history coupled with the proposed Harper-Dorn creep mechanism. Using Eq. (B4) to eliminate time in Eq. (B7) gives the experimentally measured form for the depth dependence of the hardness

$$σ=\frac{γt}{αh\left(t\right)^{2}}=\frac{B}{\sqrt{h\left(t\right)}} , (B7)$$

where

$$B=\left(\frac{6γ}{α\sqrt{ρ}β}\right)^{{1}/{2}} . (B8)$$

It is emphasized that the result in Eqs. (B5) and (B7) was also developed in Eq. (17), indicating both the results developed in the appendix and in the main body are fully consistent with the experimental observations. That is, both the experimentally determined loading history and experimentally determined strain rate history in the constant strain rate experiment can be coupled to the observed $h^{{-1}/{2}}$ hardness dependence using the Harper-Dorn creep mechanism.

An analogous analysis to Eqs. (B1) – (B8), but for the Nabarro-Herring creep mechanism at the slower strain rate (and lower loading rate) is developed below. The time dependence of the applied load for the targeted ${\dot{P}}/{P}$ of 0.05 s-1 was averaged over 56 indentations. The load-time relationship can be fit to the equation (R2 = 0.98)

$$P\left(t\right)=γt^{{5}/{3}}+b (B9)$$

where $b$ is the intercept at $t=0$. At time scales corresponding to 100 $\leq h\leq $ 350 nm, we note $b\ll γt^{{5}/{3}}$. The consideration from Eq. (B1) and (B2) for the Nabarro-Herring creep mechanism leads to

$$\frac{1}{h\left(t\right)}\left(\frac{dh\left(t\right)}{dt}\right)=\frac{β\left(γt^{{5}/{3}}+b\right)}{αch\left(t\right)^{4}} . (B10)$$

The solution to the differential Eq. (B10) then gives (neglecting the small term ($bt$) because $bt\ll γt^{{7}/{3}}$)

$$h=1.11\left(\frac{γβ}{αc}\right)^{{1}/{4}}t^{{2}/{3}} , (B11)$$

and thus the hardness written using Eq. (B11) to eliminate time and neglecting the small intercept, $b$,

$$σ\left(t\right)=\frac{γt^{{5}/{3}}+b}{αh\left(t\right)^{2}}=\frac{γ\sqrt{h\left(t\right)}}{αC^{{5}/{3}}} , (B12)$$

where

$$C=\left(\frac{γβ}{αc}\right)^{{1}/{4}} . (B13)$$

Eq. (B12) is consistent with the observed form of the experimental hardness-depth relationship for the slow loading rate experiment, namely $σ(t)∝\sqrt{h(t)}$. This same result was also developed using the Nabarro-Herring creep mechanism in Eq. (14). It is emphasized the approach in this appendix predicts essentially the same functional relationships reported in Eqs. (15) and (18) using the relations (B1) and (B2) as the starting point. The Appendix B result is complimentary to the result in the main text in that the Eq. (B7) and Eq. (B12) solutions are developed utilizing the known time dependence of the load while the Eq. (15) and Eq. (18) solutions utilize the known time dependence of the indentation strain rate.

**REFERENCES**

1. A.F. Bower, N.A. Fleck, A. Needleman and N. Ogbonna: Indentation of a Power Law Creeping Solid. *Proc. R. Soc. Lond. A,* **441**, 97 (1993).
2. R.S. Ginder, W.D. Nix and G.M. Pharr: A simple model for indentation creep. *J. Mech. Phys. Solids*, **112**, 552 (2018).
3. V. Pandey and S. Holm: Linking the fractional derivative and the Lomnitz creep law to non-Newtonian time-varying viscosity. *Phys. Rev. E*, **94**, 032606 (2016).
4. E.H. Lee and J.R.M. Radok: The contact problem for viscoelastic bodies. *J. App. Mech.,* **27**, 438 (1960).
5. T.C.T. Ting: The contact stresses between a rigid indenter and a viscoelastic half-space. *J. App. Mech.,* **33**, 845 (1966).
6. H. Li and A.H.W. Ngan: Size effects of nanoindentation creep. *J. Mater. Res.*, **19**, 513 (2004).
7. I.N. Sneddon: The relation between load and penetration in the axisymmetric boussinesq problem for a punch of arbitrary profile. *Int. J. Eng. Sci.,* **3**, 47 (1965).
8. R.S. Lakes: *Viscoelastic Solids*. CRC Press, Boca Raton, 21 (1999).