Supplementary Material to Massam et al. A comparison of annual layer thickness model estimates with observational measurements using the Berkner Island ice core, Antarctica.

Annual Layer Thickness Model

A model estimates how the annual layer thickness of a deposited layer, λ_M, varies with depth, z. This is calculated by reconstructing the past accumulation history for the ice-core site and accounting for the vertical compaction of the ice column as more ice is deposited above.

An initial accumulation profile, $A_i(z)$, is calculated by assuming snow deposition is proportional to the derivative of the mean saturation vapour pressure at the inversion layer with respect to the temperature at the inversion layer in the atmosphere, T_I (K) (Parrenin et al., 2001; 2004; Schwander et al., 2001). Inversion temperature is determined through an empirical relationship (Connolley, 1996) before accumulation is calculated using the following relationships (S1):

$$A_i(z) = A^{\theta} \cdot \frac{f(T_I)}{f(T_I^{\theta})}; \tag{S1}$$

$$f(T_I) = \frac{d}{dT}\left[\frac{P_S(T)}{T}\right] \tag{S2}$$

For which $T = T_I$, and similarly $f(T_I^{\theta})$ uses the modern mean inverse temperature. Ps(T), the saturation vapour pressure function of temperature, is calculated through an exponential relationship for which $A_S = 3.64149 \cdot 10^{12}$ Pa, and $B_S = 6148.3$ K (two constants taken from Smithsonian tables; a best fit to the empirical curve of P_S over ice in the temperature range -60 - 20°C).

$$P_S(T) = A_S \cdot \exp^{-\frac{B_S}{T}} \tag{S3}$$

As a result, a change in accumulation is assumed to be proportional to a change in temperature, as inferred by the meteoric water line (MWL) (Dansgaard 1953, 1964). Accumulation history is optimised as part of an inverse approach by estimating a perturbation profile, $\varepsilon(z)$, accounting for any inaccuracies in the assumptions made in the Clausius-Clapeyron relationship.

$$A(z) = A_i \cdot [1 + \varepsilon(z)] \tag{S4}$$

A thinning function is reconstructed to estimate the down-core compaction of each annual layer. Using Lliboutry's approximation of vertical strain as a one-dimensional model (Lliboutry 1979), a shape function is calculated in terms of ζ, a non-dimensional vertical coordinate defined as $\zeta = \hat{z}/H$, where $\hat{z}$ is the depth below the surface, and H is the total ice sheet thickness. The model incorporates the Shallow Ice Approximation (SIA) and Glen's flow law to estimate the vertical velocity of an ice particle (Parrenin et al. 2007).

$$\eta(\zeta) = s \cdot \zeta + (1 - s) \cdot \eta_D(\zeta); \tag{S5}$$

$\eta(\zeta)$ can be used as a simplistic shape function of vertical thinning, discounting temporal variations such as changes in ice sheet thickness as the ice sheet is assumed stable throughout the period of analysis. In eq. S5, *s* is the sliding ratio (1 for no sliding and 0 for full sliding) and $\eta_D(\zeta)$ is the vertical profile of deformation. $\eta_D(\zeta)$ is calculated using the p-parameter, $p = n - 1 + kG_\theta H$, where n is Glen's exponent, G_θ is the linear temperature profile of the ice sheet, and $k = \frac{Q}{RT_B^2}$ is an Arrhenius-like constant that uses the activation energy, Q, the gas molar constant, R, and the temperature at the bedrock, T_B (Parrenin & Hindmarsh 2007).

$$\eta_D(\zeta) = 1 - \frac{p+2}{p+1}(1-\zeta) + \frac{1}{p+1}(1-\zeta)^{p+2}; \tag{S6}$$

The age of a particle of ice at a specific depth is equal to the number of annual layers that lie above this depth. Net annual layer thickness is the product of the accumulation reconstruction with a thinning function applied (eq. S7). The optimised, *a posteriori* shape function is calculated by iterating the value of the p-parameter in order to find the best-fit profile.

$$\lambda_M(z) = A(z) \cdot \eta(z); \tag{S7}$$

An integration with respect to depth from the surface of the inverse of the annual layer thickness will give an age-depth profile for the ice core as the sum of the mean annual layers from a specified depth to the surface.

Translated mathematically, in a hypothetical situation where a model *G* is calculated using poorly-known parameters, an inverse approach can improve the accuracy of the estimations by searching within parameterized bounds of a model space to fit a set of observations. The inverse method will calculate the poorly-known parameters while the model is iterated in order for the output, *m*, to fit a set of observational parameters on the model space, *d*. Mathematically defined, $G: M \rightarrow D$, where M is a matrix for which each $m \in M$, and the independent information, *d*, can be ascribed to: $d = G(m) \in D$ (Tarantola, 1987; Parrenin et al., 2001).

In the construction of age-depth profiles, the aim of the inverse approach is to examine the difference between an empirically-calculated age-depth model and *a posteriori* age-depth profile. By examining the disagreement, any differences between the two profiles may reflect (i) simplifications adopted within the modelling process, or (ii) inadequacies in the distribution of the age constraints. To achieve this, a Monte-Carlo sampling technique has been prescribed to systematically explore the model manifold. Mathematically, the inverse method tries to infer the optimal information from i) the estimated values for observational datapoints, or *a priori* information, ρ^D, ii) the estimated values extracted from the model, ρ^M, iii) the relationship between ρ^D and ρ^M given by the model G (Parrenin et al., 2001). Optimal information, given by the model, is presented as *a posteriori* probability density, σ^D and σ^M for the data and model manifolds, respectively. Tarantola (1987) presents this relationship as:

$$\sigma^{\mathrm{M}}(\mathrm{m}) = \rho^{\mathrm{M}}(\mathrm{m}) \frac{\rho^{\mathrm{D}}(\mathrm{G}(\mathrm{m}))}{\mu^{\mathrm{D}}(\mathrm{G}(\mathrm{m}))} \tag{S8}$$

$$\sigma^{\mathrm{D}}(\mathrm{d}) = \frac{\rho^{\mathrm{D}}(\mathrm{d})}{\mu^{\mathrm{D}}(\mathrm{d})} \int_{\mathrm{M}} \delta\big(\mathrm{d} - \mathrm{G}(\mathrm{m})\big)\, \rho^{\mathrm{M}}(\mathrm{m})\, \mathrm{dm} \tag{S9}$$

For which $\mu^D(d)$ and $\mu^M(m)$ are the homogenous probability densities (or noninformative probability densities) on the data and model manifolds, respectively. The array *E(m)* analyses the difference between $\rho^D(d)$ and $\rho^M(m)$ and is calculated per iteration of the model G. In the inverse method, it is used quantify the deviation of each iterated model output, *m*, with respect to the observational parameters, *d*, for which low values that suggest a good-fit of model parameters are preferred over higher values. This is calculated using a root mean square error approach below:

$$\mathrm{E}(\mathrm{m}) = \frac{1}{n} \sum_{\mathrm{i}=1}^{\mathrm{n}} (\rho^D(d)_i - \rho^M(m)_i)^2 \tag{S10}$$

For the Berkner Island ice core, table S1 outlines the parameters used in the annual layer thickness model, with the known-age horizons outlined in table S2:

Table S1: List of parameters and the values used in calculating the annual layer thickness in the Berkner Island ice core, Weddell Sea.

Variable	Value
A^{θ}	0.185 m yr^{-1}
H	948 m
T_B	262 K
T_S	247 K
n	3
Q	60 kJ mol^{-1}

Table S2: **Known-age horizons along the Berkner Island ice core with associated error**

Depth (m)	Age (ka BP)	Error (ka BP)
631.744	13.50	0.3
637.778	14.75	0.5
650.328	17.60	0.5
699.878	38.10	0.5
727.058	46.50	1.0
756.488	57.16	1.0
773.818	64.75	1.0

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