Online Appendix: Treating Time With All Due Seriousness*

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*Thanks to David Nickerson for help coding the simulations in R.

In our paper, "Treating Time with All Due Seriousness," we presented results from two sets of experimental results. The first set of experiments is designed to evaluate the bias in estimates of the long run multiplier from the ADL and GECM and to demonstrate the size of the approximate standard errors for a 95% confidence level using the long run multiplier calculated from the Bewley ECM. The second set of experiments evaluates the exact maximum likelihood estimator in the context of a number of potentially fractionally integrated models. In this appendix we present details of the simulations and the fuller set of results from the experiments.

Comparing Inferences about the Long Run Multiplier from the ADL and GECM

In section 2 of "Treating Time with All Due Seriousness" we examine both the equivalence of estimates of the LRM from three different regression models and the size of the test on the LRM from the Bewley ECM. To do so we simulate two unrelated autoregressive series and estimate an ADL, GECM, and Bewley error correction model to derive and compare estimates of the long run multiplier (LRM). The Bewley error correction model is used to derive the standard error of the LRM. Simulation and estimation were conducted in `R`. Specifically, we generated:

$$Y_t = \phi_y Y_{t-1} + e_{1t} \tag{1}$$

$$X_t = \phi_x X_{t-1} + e_{2t} \tag{2}$$

where e_{1t}, e_{2t} are two unrelated white noise processes with mean zero and variance 1.0. Values of ϕ_y and ϕ_x= 0.50, 0.70, 0.90, 0.95, 0.99.

We estimate the following three models: the ADL, GECM, and Bewley ECM:

$$
\begin{aligned}
ADL \quad & Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \beta_0 X_t + \beta_1 X_{t-1} + \varepsilon_{1t} && (3) \\
GECM \quad & \Delta Y_t = \alpha_0^* + \alpha_1^* Y_{t-1} + \beta_0^* \Delta X_t + \beta_1^* X_{t-1} + \varepsilon_{2t} && (4) \\
\text{Bewley } ECM \quad & Y_t = \pi_0 - \pi_1 \Delta Y_t + \psi_0 X_t - \psi_1 \Delta X_t + \varepsilon_{3t}. && (5)
\end{aligned}
$$

Estimation of the Bewley ECM is by instrumental variables where ΔY_t is instrumented with X_t, X_{t-1}, and Y_{t-1}.

The estimates of the LRM are derived equivalently from the ADL as $\frac{\beta_0+\beta_1}{1-\alpha_1}$, the GECM as $\frac{\beta_1^*}{-\alpha_1^*}$, and the Bewley ECM as ψ_0.[1] The standard errors for the LRM are given by the coefficient ψ_0 in the Bewley ECM.

As reported in the paper, the estimates of the LRM from the three models were identical. As such, although we report mean biases and rejection rates on the LRM from the Bewley estimates, the results apply equally to LRMs estimated from any of the three models. The paper presents results for sample sizes of 100 and 250. Here we report results for sample sizes of 500 and 1,000.

Uncertainty in Estimates of Fractional Integration

In Section 3.3 of the paper we simulated and estimated a number of ARFIMA models. We provide additional details and further results here. The data was simulated and estimation was conducted in `R` using the ARFIMA package.

The data is generated following the ARFIMA(p, d, q) model given by:

$$
\left(1 - \sum_{i=1}^{p} \phi_i L^i\right) (1-L)^d \, Y_t = \left(1 + \sum_{i=1}^{q} \theta_i L^i\right) \varepsilon_t \tag{6}
$$

where ε_t is a white noise process with mean zero and variance 1.0.

[1]For details of the Bewley ECM see De Boef and Keele (2008).

Table 1: Average bias and rejection rates for LRM when X_t and Y_t are unrelated.

ρ_y	ρ_x				
$T = 500$	0.50	0.70	0.90	0.95	0.99
0.50	$-0.003 \mid 0.045$	$0.000 \mid 0.050$	$-0.001 \mid 0.066$	$-0.001 \mid 0.049$	$0.000 \mid 0.055$
0.70	$0.002 \mid 0.052$	$-0.003 \mid 0.048$	$0.001 \mid 0.067$	$0.001 \mid 0.060$	$0.001 \mid 0.059$
0.90	$-0.025 \mid 0.032$	$-0.012 \mid 0.056$	$0.007 \mid 0.061$	$0.006 \mid 0.070$	$-0.002 \mid 0.066$
0.95	$0.026 \mid 0.021$	$0.014 \mid 0.035$	$0.032 \mid 0.089$	$0.003 \mid 0.107$	$-0.001 \mid 0.087$
0.99	$0.042 \mid 0.003$	$0.024 \mid 0.009$	$-0.000 \mid 0.031$	$0.428 \mid 0.046$	$-0.812 \mid 0.137$
$T = 1000$	0.50	0.70	0.90	0.95	0.99
0.50	$-0.002 \mid 0.057$	$-0.000 \mid 0.056$	$0.000 \mid 0.060$	$0.000 \mid 0.061$	$0.000 \mid 0.057$
0.70	$-0.005 \mid 0.057$	$-0.002 \mid 0.062$	$-0.002 \mid 0.044$	$0.000 \mid 0.050$	$0.000 \mid 0.057$
0.90	$-0.012 \mid 0.047$	$-0.010 \mid 0.060$	$0.000 \mid 0.049$	$-0.005 \mid 0.060$	$-0.003 \mid 0.065$
0.95	$0.028 \mid 0.027$	$0.008 \mid 0.046$	$-0.000 \mid 0.056$	$-0.004 \mid 0.067$	$-0.003 \mid 0.059$
0.99	$-0.066 \mid 0.006$	$0.148 \mid 0.008$	$-0.051 \mid 0.019$	$-0.008 \mid 0.040$	$0.024 \mid 0.097$

The data generating processes are given by $Y_t = \phi_y Y_{t-1} + e_{1t}$; $X_t = \phi_x X_{t-1} + e_{2t}$; and e_{1t},$e_{2t} \sim IN(0,1)$. Estimates and standard errors are from the Bewley ECM (see above). Values are the (average bias for ψ_0 | rejection rate ψ_0). Results are for 1,000 simulations.

We allow for a range of dynamics, including ARFIMA(0,d,0), ARFIMA(1,d,0), ARFIMA(0,d,1) and ARFIMA(1,d,1) processes. The autoregressive parameter, ϕ, is set to 0.60, the moving average parameter, θ, is set to 0.60 in the AR and MA models, respectively, while $\phi = 0.50$ and $\theta = 0.30$ in the combined ARMA models. In the simulations, d takes on the values 0 (no fractional integration), 0.20, 0.40, 0.45, and 0.80. In the latter case, the data is integer differenced before simulation and estimation so that d=-0.20 in the transformed data. The mean of the white noise error is zero and the variance is 1.0. We estimate the ARFIMA process under the optimal, but unrealistic assumption that the order of the short run dynamics is known. We conducted simulations for samples of size 50, 100, 250, 500, 1000, and 1500.

The sample mean is used as the estimate of the true mean (which is zero) and the models are estimated with the number of starting values set to twice the number of estimated parameters (other than the constant). The AIC is used to select the estimate when the likelihood surface has multiple modes.

Table 2 presents results for the ARFIMA(0,d,0) case, Table 3 presents results from the ARFIMA(1,d,0) case, Table 4, present results from the ARFIMA(0,d,1) case, and finally Table 5 presents results for the ARFIMA(1,d,1) case. The tables present the true value of d in the first column, the sample size in the second column, the mean estimate of $\hat{d}$ in the third column, the 95% confidence interval in the fourth column, and the minimum, 25%, median, 75% and maximum values of $\hat{d}$ in columns five through nine.

References

De Boef, Suzanna and Luke Keele. 2008. "Taking time seriously." *American Journal of Political Science* 52(1):184–200.

Table 2: ARFIMA(0,d,0) Simulation Results

d	T	$\bar{\hat{d}}$	95 % CI	Min	25 %	Med	75%	Max
.0	50	−.075	[−.297,.145]	−.530	−.156	−.077	.030	.197
.0	100	−.037	[−.191,.117]	−.344	−.109	−.037	.029	.160
.0	250	−.021	[−.117,.075]	−.134	−.064	−.022	.023	.098
.0	500	−.012	[−.081,.055]	−.101	−.041	−.011	.010	.087
.0	1,000	−.005	[−.053,.042]	−.076	−.021	−.005	.008	.049
.0	1,500	−.003	[−.042,.035]	−.038	−.016	−.003	.008	.063
.1	50	−.000	[−.221,.221]	−.545	−.068	.005	.079	.307
.1	100	.055	[−.099,.210]	−.189	.007	.052	.117	.270
.1	250	.071	[−.025,.167]	−.091	.039	.078	.111	.204
.1	500	.083	[.015,.152]	−.000	.055	.082	.839	.156
.1	1,000	.095	[.047,.144]	.037	.083	.094	.110	.161
.1	1,500	.095	[.056,.135]	.055	.081	.097	.111	.138
.2	50	.090	[−.130,.312]	−.213	−.002	.097	.211	.361
.2	100	.156	[.001,.311]	−.081	.098	.162	.218	.366
.2	250	.191	[.094,.288]	.043	.161	.184	.229	.309
.2	500	.186	[.118,.255]	.080	.168	.187	.210	.285
.2	1,000	.191	[.143,.239]	.112	.177	.194	.210	.238
.2	1,500	.196	[.157,.236]	.139	.183	.195	.210	.239
.3	50	.187	[−.033,.409]	−.192	.107	.190	.282	.424
.3	100	.244	[.098,.399]	.003	.187	.248	.311	.410
.3	250	.283	[.186,.379]	.123	.256	.286	.317	.419
.3	500	.284	[.215,.352]	.189	.257	.288	.308	.398
.3	1,000	.293	[.245,.342]	.239	.274	.291	.310	.350
.3	1,500	.291	[.252,.330]	.247	.278	.292	.303	.355
.4	50	.263	[.042,.485]	−.051	.193	.273	.347	.453
.4	100	.336	[.182,.491]	.011	.287	.352	.398	.475
.4	250	.382	[.285,.478]	.258	.352	.389	.415	.460
.4	500	.384	[.316,.452]	.322	.369	.386	.402	.446
.4	1,000	.387	[.338,.435]	.319	.371	.391	.404	.432
.4	1,500	.395	[.355,.434]	.350	.383	.396	.409	.438
.45	50	.315	[.093,.536]	.009	.262	.333	.394	.466
.45	100	.360	[.205,.515]	.126	.310	.369	.416	.481
.45	250	.413	[.317,.510]	.275	.388	.417	.446	.483
.45	500	.431	[.362,.499]	.343	.412	.436	.453	.485
.45	1,000	.442	[.394,.491]	.386	.428	.444	.461	.482
.45	1,500	.444	[.405,.484]	.391	.431	.446	.455	.484

The data generating process is ARFIMA(0,d,0).The true values of d are: 0, .1, .2, .3, .4. and .45. Results are for 100 simulations.

Table 3: ARFIMA(1,d,0) Simulation Results

d	T	$\bar{\hat{d}}$	95 % CI	Min	25 %	Med	75%	Max
.0	50	−.195	[−.630,.239]	−.546	−.325	−.203	−.089	.428
.0	100	−.199	[−.508,.109]	−.530	−.316	−.207	−.089	.174
.0	250	−.121	[−.371,.127]	−.470	−.236	−.122	.002	.232
.0	500	−.102	[−.300,.096]	−.436	−.176	−.099	−.016	.143
.0	1,000	−.073	[−.231,.084]	−.390	−.139	−.064	−.010	.140
.0	1,500	−.042	[−.176,.091]	−.313	−.093	−.045	.011	.169
.1	50	−.093	[−.490,.302]	−.497	−.250	−.095	.033	.356
.1	100	−.096	[−.401,.207]	−.442	−.192	−.088	−.004	.369
.1	250	−.025	[−.278,.228]	−.376	−.130	−.017	.074	.362
.1	500	.023	[−.176,.223]	−.233	−.052	.036	.089	.250
.1	1,000	.064	[−.103,.231]	−.222	−.013	.058	.127	.243
.1	1,500	.052	[−.081,.186]	−.110	−.003	.052	.108	.228
.2	50	−.073	[−.410,.263]	−.446	−.166	−.059	.038	.385
.2	100	.000	[−.281,.282]	−.465	−.089	.021	.100	.335
.2	250	.069	[−.156,.296]	−.243	−.045	.057	.159	.371
.2	500	.102	[−.098,.303]	−.194	.013	.110	.200	.340
.2	1,000	.149	[−.018,.316]	−.110	.096	.151	.214	.327
.2	1,500	.176	[.041,.310]	−.030	.125	.186	.228	.315
.3	50	.010	[−.311,.332]	−.360	−.096	−.002	.152	.345
.3	100	.054	[−.210,.318]	−.301	−.010	.046	.146	.296
.3	250	.141	[−.079,.363]	−.134	.037	.143	.232	.423
.3	500	.169	[−.006,.345]	−.111	.069	.173	.266	.435
.3	1,000	.243	[.086,.401]	.008	.200	.242	.311	.441
.3	1,500	.268	[.139,.398]	.048	.221	.273	.324	.417
.4	50	.097	[−.214,.409]	−.330	−.002	.094	.218	.402
.4	100	.137	[−.127,.401]	−.182	.039	.134	.227	.420
.4	250	.196	[−.006,.398]	−.078	.113	.202	.283	.431
.4	500	.275	[.099,.451]	−.051	.190	.298	.366	.472
.4	1,000	.331	[.187,.475]	−.001	.296	.344	.389	.468
.4	1,500	.333	[.206,.460]	.144	.282	.333	.385	.468
.45	50	.141	[−.155,.437]	−.153	.044	.146	.256	.400
.45	100	.167	[−.069,.404]	−.174	.080	.182	.262	.422
.45	250	.245	[.043,.446]	−.018	.175	.244	.337	.455
.45	500	.310	[.138,.481]	.011	.240	.319	.396	.473
.45	1,000	.365	[.228,.502]	.072	.319	.380	.424	.486
.45	1,500	.387	[.271,.503]	.180	.342	.394	.437	.480

The data generating process is ARFIMA(1,d,0) with $\phi = 0.6$. The true values of d are: 0, .1, .2, .3, .4. and .45. Results are for 100 simulations.

Table 4: ARFIMA(0,d,1) Simulation Results

d	T	$\bar{\hat{d}}$	95 % CI	Min	25 %	Med	75%	Max
.0	50	−.379	[−.972,.212]	−.999	−.586	−.358	−.189	.183
.0	100	−.227	[−.659,.204]	−.844	−.360	−.227	−.076	.140
.0	250	−.132	[−.425,.160]	−.613	−.283	−.151	.002	.311
.0	500	−.041	[−.263,.179]	−.416	−.143	−.039	.036	.296
.0	1,000	−.019	[−.193,.154]	−.246	−.091	−.037	.048	.230
.0	1,500	−.024	[−.157,.108]	−.233	−.069	−.023	.022	.174
.1	50	−.325	[−.877,.225]	−1.00	−.499	−.293	−.117	.261
.1	100	−.173	[−.621,.273]	−.700	−.323	−.173	−.026	.344
.1	250	−.046	[−.336,.243]	−.386	−.167	−.048	.048	.396
.1	500	.022	[−.197,.243]	−.285	−.064	.022	.094	.367
.1	1,000	.061	[−.106,.229]	−.126	.003	.071	.116	.241
.1	1,500	.060	[−.078,.198]	−.108	.001	.067	.107	.256
.2	50	−.323	[−.852,.204]	−.958	−.532	−.302	−.062	.282
.2	100	−.095	[−.519,.327]	−.800	−.229	−.100	.071	.308
.2	250	.069	[−.202,.342]	−.333	−.047	.059	.188	.451
.2	500	.106	[−.114,.327]	−.167	.022	.111	.173	.398
.2	1,000	.151	[−.011,.315]	−.089	.098	.152	.193	.440
.2	1,500	.156	[.020,.293]	−.023	.109	.163	.211	.334
.3	50	−.191	[−.723,.339]	−.999	−.327	−.172	.000	.299
.3	100	−.009	[−.425,.406]	−.526	−.139	−.001	102.	.332
.3	250	.125	[−.164,.416]	−.205	.004	.145	.247	.391
.3	500	.195	[−.024,.415]	−.062	.119	.199	.275	.435
.3	1,000	.251	[.096,.251]	.035	.199	.268	.313	.434
.3	1,500	.272	[.142,.402]	.074	.223	.272	.314	.468
.4	50	−.127	[−.639,.385]	−.884	−.293	−.115	.063	.410
.4	100	.071	[−.324,.467]	−.464	−.028	.077	.206	.414
.4	250	.237	[−.019,.494]	−.214	.138	.269	.338	.434
.4	500	.301	[.106,.496]	−.015	.224	.322	.374	.465
.4	1,000	.342	[.197,.487]	.113	.302	.343	.388	.469
.4	1,500	.355	[.233,.478]	.213	.315	.361	.396	.481
.45	50	−.069	[−.577,.439]	−.633	−.232	−.050	.078	.385
.45	100	.128	[−.262,.519]	−.465	.007	.168	.272	.398
.45	250	.246	[−.014,.506]	−.086	.143	.266	.366	.454
.45	500	.345	[.159,.530]	.112	.289	.366	.408	.480
.45	1,000	.382	[.248,.516]	.204	.357	.397	.435	.482
.45	1,500	.398	[.282,.514]	.249	.373	.401	.431	.484

The data generating process is ARFIMA(0,d,1) with $\theta = 0.6$ .The true values of d are: 0, .1, .2, .3, .4. and .45. Results are for 100 simulations.

Table 5: ARFIMA(1,d,1) Simulation Results

d	T	$\bar{\hat{d}}$	95 % CI	Min	25 %	Med	75%	Max
.0	50	−.490	[−1.04,.063]	−.999	−.777	−.490	−.310	.355
.0	100	−.530	[−.938,−.121]	−.999	−.772	−.530	−.354	.271
.0	250	−.316	[−.627,−.005]	−.926	−.659	−.316	−.059	.378
.0	500	−.131	[−.443,.179]	−.838	−.192	−.131	.001	.213
.0	1,000	−.056	[−.291,.178]	−.776	−.108	−.056	.025	.287
.0	1,500	−.064	[−.258,.128]	−.842	−.074	−.064	.014	.345
.1	50	−.382	[−.993,.167]	−.999	−.635	−.437	−.078	.412
.1	100	−.486	[−.908,−.063]	−.999	−.701	−.545	−.380	.267
.1	250	−.265	[−.560,.029]	−.865	−.619	−.210	.066	.390
.1	500	−.059	[−.372,.253]	−.800	−.171	.017	.130	.411
.1	1,000	−.023	[−.245,.198]	−.835	−.053	.024	.095	.381
.1	1,500	.025	[−.159,.211]	−.822	.007	.055	.111	.349
.2	50	−.350	[−.871,.170]	−.999	−.617	−.392	−.130	.392
.2	100	−.442	[−.882,−.002]	−.999	−.632	−.495	−.293	.293
.2	250	−.354	[−.628,−.081]	−.847	−.611	−.493	−.114	.407
.2	500	−.002	[−.278,.273]	−.691	−.076	.069	.146	.355
.2	1,000	−.127	[−.294,.040]	−.754	−.620	.063	.150	.435
.2	1,500	.041	[−.159,.242]	−.685	.093	.148	.206	.425
.3	50	−.345	[−.893,.202]	−.999	−.544	−.361	−.187	.297
.3	100	−.379	[−.770,.011]	−.772	−.545	−.437	−.272	.423
.3	250	−.339	[−.600,−.079]	−.722	−.557	−.467	−.196	.420
.3	500	.112	[−.185,.410]	−.569	.037	.187	.286	.469
.3	1,000	−.135	[−.287,.017]	−.630	−.559	.058	.232	.468
.3	1,500	.035	[−.140,.212]	−.669	−.510	.241	.294	.424
.4	50	−.274	[−.802,.253]	−.733	−.452	−.322	−.125	.428
.4	100	−.328	[−.716,.059]	−.824	−.443	−.347	−.239	.449
.4	250	−.328	[−.580,−.076]	−.607	−.487	−.402	−.234	.462
.4	500	.093	[−.163,.350]	−.489	−.348	.272	.360	.482
.4	1,000	−.203	[−.335,−.071]	−.567	−.475	−.417	.275	.455
.4	1,500	.001	[−.138,.140]	−.544	−.437	.262	.001	.452
.45	50	−.260	[−.804,.282]	−.857	−.403	−.270	−.117	.402
.45	100	−.250	[−.666,.165]	−.655	−.384	−.289	−.118	.435
.45	250	−.309	[−.556,−.061]	−.566	−.432	−.363	−.237	.471
.45	500	.123	[−.123,.371]	−.453	−.242	.258	.384	.466
.45	1,000	−.160	[−.290,−.031]	−.529	−.440	−.372	.271	.470
.45	1,500	.001	[−.120,.123]	−.504	−.428	.270	.385	.482

The data generating process is ARFIMA(1,d,1) with $\phi = 0.5$ and $\theta = 0.3$.The true values of d are: 0, .1, .2, .3, .4. and .45. Results are for 100 simulations.