On-line Appendix for "Influence without Bribes: A Non-Contracting Model of Campaign Giving and Policymaking"

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ABSTRACT

Section A establishes Properties 1 through 4 and Propositions 2, 3, and 5 of "Influence without Bribes." Section B identifies sufficient conditions for the existence of a unique equilibrium. Section C numerically solves our model over a range of parameters. Finally, Section D considers a variant of our theoretical setting in which the policy space is continuous.

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A. PROOFS OF RESULTS FROM MAIN TEXT

Upon formalizing our solution concept, we proceed to characterize the interest group's equilibrium behavior, the incumbent's equilibrium behavior, and the interest group's equilibrium beliefs. We then prove Properties 1 through 4. Finally, proofs are provided for the propositions not proven in the main text.

A.1. *Strategies and Solution Concept*

We begin with some notation. Let $P \equiv \{x, y\}$, let $D \equiv \mathbb{R}^2_+$, and let $T \equiv \mathbb{R}$, where P is the set of policies from which the office holder can select, D is the set of possible donation pairs that the group can offer, and T is the set of types from which each politician's type is drawn.

Given that the game ends in the second period, it is obvious that the election winner will select her preferred policy in the second period. We denote the policy choice of an election winner with type t as $\sigma_2^*(t)$, where

$$\sigma_2^*(t) = \begin{cases} x & \text{if } t \geq 0 \\ y & \text{otherwise} \end{cases} .$$

From here on, we take as given that the election winner picks her preferred policy. As such, in analyzing the model, we will focus on the strategic interaction between the incumbent and the interest group.

A strategy for the incumbent is a function, $\sigma : T \rightarrow P$, that specifies a first-period policy for each incumbent type. A strategy for the interest group is a function $(\gamma_i, \gamma_c) : P \rightarrow D$. For each first-period policy $p_1 \in P$, $\gamma_i(p_1)$ is the interest group's donation to the incumbent and $\gamma_c(p_1)$ specifies its donation to the challenger. Since the interest group is uncertain of the incumbent's type at the time it decides how to allocate its resources, this model constitutes an extensive-form game of incomplete information. Consequently, our solution concept is perfect Bayesian equilibrium (PBE). A candidate for a PBE is a strategy for the incumbent, a strategy for the interest group, and a belief function. A belief function for this model is a mapping, $\mu : P \rightarrow \Delta(T)$, where $\Delta(T)$ is the set of density functions with domain T. For each first-period policy $p_1 \in P$, $\mu(p_1)$ specifies the interest group's belief about which incumbent types may have selected p_1; we interpret $\mu(t|p_1)$ to be the weight that the interest group attaches to the incumbent's type being t when the first-period policy is p_1.

To define a PBE to our game formally, one additional piece of notation is needed: Fixing the incumbent's type, the first-period policy, and the interest group's donation pair, write $V_j(t_i, p_1, d_i, d_c)$, $j \in \{i, g\}$, for j's expected payoff given that uncertainty remains about the challenger's type and the election's outcome. That is,

$$\begin{aligned} V_j(t_i, p_1, d_i, d_c) \equiv r(d_i, d_c) \left[U_j(p_1, d_i, d_c, i, \sigma_2^*(t_i); t_j)\right] + \\ (1 - r(d_i, d_c)) \left[\int U_j(p_1, d_i, d_c, c, \sigma_2^*(t_c); t_j) f_c(t_c) dt_c\right], \end{aligned}$$

where U_j is as defined in the main text. The first bracketed term is j's payoff conditional on the incumbent winning reelection, while the second bracketed term is j's expected payoff conditional on the challenger winning the election. Note that uncertainty regarding the challenger's type enters the second bracketed term only. The weight attached to each bracketed term is determined by the incumbent's reelection probability. We can now formally define a PBE as follows.

Definition 1 *A PBE is a strategy profile (σ^*, γ^*) and a belief function μ^* in which the following three conditions are satisfied:*

a. for each $t \in T$, $\sigma^*(t)$ is a solution to

$$\max_{p_1 \in P} V_i(t, p_1, \gamma_i^*(p_1), \gamma_c^*(p_1));$$

b. for each $p_1 \in P$, $(\gamma_i^*(p_1), \gamma_c^*(p_1))$ is a solution to

$$\max_{(d_i, d_c) \in D} \int V_g(t, p_1, d_i, d_c) \mu^*(t|p_1) dt;$$

and

c. for each $p_1 \in P$, $\mu^*(p_1)$ is derived from σ^* through Bayes's rule when possible.

A.2. *The Interest Group's Donation Problem*

Given a first-period policy of p_1 and beliefs μ, write $W(d_i, d_c; p_1, \mu)$ for the interest group's expected payoff from donation pair (d_i, d_c):

$$W(d_i, d_c; p_1, \mu) \equiv \int V_g(t, p_1, d_i, d_c)\mu(t|p_1)dt.$$

The interest group's *donation problem* is

$$\max_{(d_i,d_c)\in D} W(d_i, d_c; p_1, \mu). \tag{A1}$$

As a prelude to characterizing the solution to the interest group's donation problem, let

$$\pi_c \equiv \int_0^\infty f_c(t)dt,$$

and let

$$\pi_i(p_1, \mu) \equiv \int_0^\infty \mu(t|p_1)dt.$$

Thus, π_c is the group's assessment of its ideological congruence with the challenger, and for a given first-period policy p_1 and belief function μ, $\pi_i(p_1, \mu)$ is the group's updated assessment of its ideological congruence with the incumbent. With this notation, one can establish the useful equivalence:

$$W(d_i, d_c; p_1, \mu) \equiv z(p_1; t_g) + r(d_i, d_c)[\pi_i(p_1, \mu) - \pi_c]t_g + \pi_c t_g - m(d_i, d_c).$$

This expression reveals that the interest group's marginal return from a campaign contribution (i.e., the derivative of W with respect to either d_i or d_c) depends on the difference in each politician's probability of pursuing x in the second period.

We can now more formally characterize the solution to the interest group's donation problem by stating the *Kuhn-Tucker first-order necessary conditions* that such a solution must satisfy.

Lemma 1 *Suppose that (d_i^*, d_c^*) is a solution to (A1). Then there exists a vector $(\lambda_i^*, \lambda_c^*)$:*

$$\frac{\partial r(d_i^*, d_c^*)}{\partial d_i}[\pi_i(p_1, \mu) - \pi_c]t_g - \frac{\partial m(d_i^*, d_c^*)}{\partial d_i} + \lambda_i^* = 0 \tag{A2}$$

$$\frac{\partial r(d_i^*, d_c^*)}{\partial d_c}[\pi_i(p_1, \mu) - \pi_c]t_g - \frac{\partial m(d_i^*, d_c^*)}{\partial d_c} + \lambda_c^* = 0 \tag{A3}$$

$$\lambda_i^* \geq 0 \quad d_i^* \lambda_i^* = 0 \tag{A4}$$

$$\lambda_c^* \geq 0 \quad d_c^* \lambda_c^* = 0. \tag{A5}$$

Proof: As the constraint qualification holds at any $(d_i, d_c) \in D$, by the Kuhn-Tucker Theorem, the result follows. ■

We now show that in any solution to the group's donation problem, only the politician most likely to pursue x in the second period is ever offered a positive donation.

Lemma 2 *Let (d_i^*, d_c^*) denote a solution to (A1). If $d_i^* > 0$, then $\pi_i(p_1, \mu) > \pi_c$. If $d_c^* > 0$, then $\pi_i(p_1, \mu) < \pi_c$.*

Proof: Suppose that (d_i^*, d_c^*) is a solution to (A1), where $d_i^* > 0$. We need to show that $\pi_i(p_1, \mu) > \pi_c$. To do so, we invoke the Kuhn-Tucker first-order necessary conditions for a maximum to the interest group's donation problem. When $d_i^* > 0$, (A2) and (A4) imply that

$$\frac{\partial r(d_i^*, d_c^*)}{\partial d_i}[\pi_i(p_1, \mu) - \pi_c]t_g - \frac{\partial m(d_i^*, d_c^*)}{\partial d_i} = 0.$$

By assumption, $\partial r(d_i^*, d_c^*)/\partial d_i > 0$, $\partial m(d_i^*, d_c^*)/\partial d_i > 0$, and $t_g > 0$. Consequently, if the preceding equality is to hold, $\pi_i(p_1, \mu) > \pi_c$. A similar argument applied to (A3) and (A5) shows that $d_c^* > 0$ implies that $\pi_i(p_1, \mu) < \pi_c$. ■

Finally, we can show that, as the incumbent's probability of pursuing x in the second period increases, the magnitude of the interest group's optimal donation to the incumbent (challenger) is non-decreasing (non-increasing).

Lemma 3 *Fix belief functions μ' and μ''. Suppose that (d_i^*, d_c^*) is a solution to*

$$\max_{(d_i, d_c) \in D} W(d_i, d_c; p_1', \mu'),$$

and that (d_i^{**}, d_c^{**}) *is a solution to*

$$\max_{(d_i,d_c)\in D} W(d_i, d_c; p_1'', \mu'').$$

i. If $\pi_i(p_1', \mu') \geq \pi_c \geq \pi_i(p_1'', \mu'')$*, then* $d_i^* \geq d_i^{**} = 0$ *and* $d_c^{**} \geq d_c^* = 0$*.*

ii. If $\pi_i(p_1', \mu') > \pi_i(p_1'', \mu'') > \pi_c$*, then* $d_i^* \geq d_i^{**}$ *and* $d_c^{**} = d_c^* = 0$*, where the former inequality is strict if* $d_i^{**} > 0$*.*

iii. If $\pi_c > \pi_i(p_1', \mu') > \pi_i(p_1'', \mu'')$*, then* $d_i^* = d_i^{**} = 0$ *and* $d_c^{**} \geq d_c^*$*, where the latter inequality is strict if* $d_c^* > 0$*.*

Proof:

Case (i): Since $(d_i^*, d_c^*) \in \arg\max W(d_i, d_c; p_1', \mu')$, by Lemma 2, $\pi_i(p_1', \mu') \geq \pi_c$ implies that $d_c^* = 0$. Since $(d_i^{**}, d_c^{**}) \in \arg\max W(d_i, d_c; p_1'', \mu'')$, by Lemma 2, $\pi_c \geq \pi_i(p_1'', \mu'')$ implies that $d_i^{**} = 0$. Consequently, $d_i^* \geq d_i^{**} = 0$ and $d_c^{**} \geq d_c^* = 0$.

Case (ii): Since $(d_i^*, d_c^*) \in \arg\max W(d_i, d_c; p_1', \mu')$, by Lemma 2, $\pi_i(p_1', \mu') > \pi_c$ implies that $d_c^* = 0$. Since $(d_i^{**}, d_c^{**}) \in \arg\max W(d_i, d_c; p_1'', \mu'')$, by Lemma 2, $\pi_i(p_1'', \mu'') > \pi_c$ implies that $d_c^{**} = 0$.

All that remains to establish is that $d_i^* \geq d_i^{**}$, where this inequality is strict if $d_i^{**} > 0$. Since

$$\frac{\partial^2 W}{\partial d_i \partial \pi_i} = \frac{\partial r(d_i, 0)}{\partial d_i} t_g > 0,$$

the group's marginal return on a campaign donation to the incumbent is increasing in the group's perception of its ideological affinity with the incumbent. Consequently, the Strict Monotonicity Theorem of Edlin and Shannon (1998, 205) applies, so our desired conclusion follows.

Case (iii): This case is established by employing arguments similar to those employed to establish Case (ii). ■

A.3. *Incumbent's First-Period Policy Problem*

Let $d^{p_1} = (d_i^{p_1}, d_c^{p_1})$ denote the donation pair offered when the first-period policy is p_1. Given (d^x, d^y), the incumbent's *first-period policy problem* is

$$\max_{p_1 \in P} V_i(t_i, p_1, d_i^{p_1}, d_c^{p_1}). \tag{A6}$$

In this section, we establish that, for any (d^x, d^y), the set of incumbent types that find it optimal to choose policy p_1 in the first period is convex.

We begin by expressing $V_i(t_i, p_1, d_i, d_c)$ in terms of the model's parameters:

$$V_i(t_i, p_1, d_i, d_c) = \begin{cases} t_i + \rho + r(d_i, d_c)[t_i + \rho] + [1 - r(d_i, d_c)]\pi_c t_i & \text{if } p_1 = x \text{ and } t_i \geq 0 \\ \rho + r(d_i, d_c)[t_i + \rho] + [1 - r(d_i, d_c)]\pi_c t_i & \text{if } p_1 = y \text{ and } t_i \geq 0 \\ t_i + \rho + r(d_i, d_c)\rho + [1 - r(d_i, d_c)]\pi_c t_i & \text{if } p_1 = x \text{ and } t_i < 0 \\ \rho + r(d_i, d_c)\rho + [1 - r(d_i, d_c)]\pi_c t_i & \text{if } p_1 = y \text{ and } t_i < 0 \end{cases}.$$

Given (d^x, d^y), write $T^{p_1}(d^x, d^y)$ for the set of incumbent types for whom p_1 is a solution to (A6):

$$T^x(d^x, d^y) \equiv \{t_i \in T : V_i(t_i, x, d_i^x, d_c^x) \geq V_i(t_i, y, d_i^y, d_c^y)\},$$

and

$$T^y(d^x, d^y) \equiv \{t_i \in T : V_i(t_i, y, d_i^y, d_c^y) \geq V_i(t_i, x, d_i^x, d_c^x)\}.$$

The following lemma, which involves only simple algebra, characterizes the sets $T^x(d^x, d^y)$ and $T^y(d^x, d^y)$.

Lemma 4 *Let*

$$c(d^x, d^y) \equiv \begin{cases} \frac{\rho[r(d^y) - r(d^x)]}{1 + [r(d^y) - r(d^x)]\pi_c} & \text{if } r(d^x) > r(d^y) \\ 0 & \text{if } r(d^x) = r(d^y) \\ \frac{\rho[r(d^y) - r(d^x)]}{1 + [r(d^x) - r(d^y)](1 - \pi_c)} & \text{if } r(d^x) < r(d^y) \end{cases}. \tag{A7}$$

$T^x(d^x, d^y) = [c(d^x, d^y), +\infty)$ *and* $T^y(d^x, d^y) = (-\infty, c(d^x, d^y)]$.

Inspection of (A7) reveals the following two facts: First, for any (d^x, d^y) in which the incumbent's reelection prospects are maximized by selecting x (y), then $c(d^x, d^y)$ negative (positive). Second, as the net electoral benefit of selecting policy x increases, the set of incumbent types that find it optimal to do so increases as well. The following lemma summarizes these facts.

Lemma 5 *For any (d^x, d^y) and $(d^{x'}, d^{y'})$ we have: (a) If $r(d^x) > r(d^y)$, then $c(d^x, d^y) < 0$; (b) If $r(d^x) < r(d^y)$, then $c(d^x, d^y) > 0$; and (c) If $r(d^x) - r(d^y) > r(d^{x'}) - r(d^{y'})$, then $c(d^x, d^y) < c(d^{x'}, d^{y'})$.*

A.4. *Equilibrium Cutpoint and Equilibrium Beliefs*

Say that σ *is a cutpoint strategy with cutpoint* $c \in \mathbb{R}$ if

$$\sigma(t) = \begin{cases} x & \text{if } t > c \\ y & \text{if } t < c \end{cases} .$$

Lemma 6 *If $(\sigma^*, \gamma^*, \mu^*)$ is a PBE of the incomplete information matching model, then we have the following:*

a. σ^* is a cutpoint strategy with cutpoint $c(\gamma^*(x), \gamma^*(y))$;

b.

$$\mu^*(t|x) = \begin{cases} \frac{f_i(t)}{\int_{c(\gamma^*(x),\gamma^*(y))}^{\infty} f_i(t)dt} & \text{if } t > c(\gamma^*(x), \gamma^*(y)) \\ \frac{f_i(t)}{\int_{c(\gamma^*(x),\gamma^*(y))}^{\infty} f_i(t)dt} & \text{if } t = c(\gamma^*(x), \gamma^*(y)) \text{ and } \sigma^*(t) = x \\ 0 & \text{if } t = c(\gamma^*(x), \gamma^*(y)) \text{ and } \sigma^*(t) = y \\ 0 & \text{if } t < c(\gamma^*(x), \gamma^*(y)) \end{cases} , \tag{A8}$$

and

$$\mu^*(t|y) = \begin{cases} 0 & \text{if } t > c(\gamma^*(x), \gamma^*(y)) \\ 0 & \text{if } t = c(\gamma^*(x), \gamma^*(y)) \text{ and } \sigma^*(t) = x \\ \frac{f_i(t)}{\int_{-\infty}^{c(\gamma^*(x),\gamma^*(y))} f_i(t)dt} & \text{if } t = c(\gamma^*(x), \gamma^*(y)) \text{ and } \sigma^*(t) = y \\ \frac{f_i(t)}{\int_{-\infty}^{c(\gamma^*(x),\gamma^*(y))} f_i(t)dt} & \text{if } t < c(\gamma^*(x), \gamma^*(y)) \end{cases} . \tag{A9}$$

Proof: Suppose that $(\sigma^*, \gamma^*, \mu^*)$ is a PBE. Part (a) is an immediate consequence of Lemma 4. Part (b) immediately follows from part (a) taken together with Bayes's rule. ■

A.5. *Proofs of Properties*

Proof of Property 1: This result immediately follows from part (a) of Lemma 6 and parts (a) and (b) of Lemma 5. ■

Proof of Property 2: Suppose that $(\sigma^*, \gamma^*, \mu^*)$ is an equilibrium of our model. We need to show that $\pi_i^*(x, \mu^*) > \pi_c > \pi_i^*(y, \mu^*)$. In words, upon observing the incumbent select policy x (y), the interest group's updated assessment of its ideological affinity with the incumbent is greater (less) than its initial assessment.

There are two cases to consider: that in which the cutpoint of the incumbent's strategy is greater than or equal to zero and that in which the cutpoint of the incumbent's strategy is negative.

Letting t^* denote the cutpoint of the incumbent's strategy, we begin by considering the case in which $t^* < 0$. By (A9), for each $t > 0$, $\mu^*(t|y) = 0$. Hence, $\pi_i^*(y, \mu^*) \equiv \int_0^\infty \mu^*(t|y)dt = 0 < \pi_c$. And by (A8), for each $t > 0$,

$$\mu^*(t|x) = \frac{f_i(t)}{\int_{t^*}^\infty f_i(t)dt}.$$

Thus,

$$\pi_i^*(x, \mu^*) \equiv \int_0^\infty \mu^*(t|x)dt = \frac{\int_0^\infty f_i(t)dt}{\int_{t^*}^\infty f_i(t)dt} = \frac{\pi_i}{\int_{t^*}^\infty f_i(t)dt}.$$

In words, upon observing x selected in the first period, the probability that the group assigns to the incumbent sharing its preference for x is simply the probability that a randomly drawn incumbent type prefers policy x divided by the probability that a randomly drawn incumbent type selects policy x. As the denominator of this expression is less than one (this follows from our assumption that the support of f_i is the real line), the group's updated assessment of its ideological affinity with an incumbent who chooses x is greater than its initial assessment, so $\pi_i^*(x, \mu^*) > \pi_i$. A similar argument establishes our claim for the case in which $t^* \geq 0$. ■

Proof of Property 3: This result is an immediate consequence of Lemma 2. ■

Proof of Property 4: From Property 2, we know that, in any equilibrium $(\sigma^*, \gamma^*, \mu^*)$, $\pi_i^*(x, \mu^*) > \pi_i^*(y, \mu^*)$. Hence, Property 4 is an immediate consequence of this fact taken together with Lemma

3. ■

A.6. *Proofs of Propositions*

Before turning to the proofs of Propositions 2, 3, and 5, we establish conditions for a unique equilibrium to exist.

Lemma 7 *Suppose that f_i is continuous. Further, suppose that r is strictly concave in d_i and strictly convex in d_c. Finally, suppose that $m(d_i, 0)$ is convex in d_i and $m(0, d_c)$ is convex in d_c. Then a unique equilibrium exists.*

Proof: See Section B. ■

Proof of Proposition 2: Assume f_i is continuous. In addition, impose the convexity conditions on r and m specified in Lemma 7. Thus, by Lemma 7, a unique equilibrium exists. In addition, assume that $\pi_c = 1$ and

$$\lim_{d_c \to 0} \frac{\partial m(0, d_c)}{\partial d_c} = 0. \tag{A10}$$

That $\pi_c = 1$ guarantees that, in an equilibrium, the interest group never donates to the incumbent. And (A10), together with the strict convexity of r in d_c, ensures that, in an equilibrium, the interest group always offers a positive donation to the challenger when $p_1 = y$. Hence, the equilibrium is non-trivial and Proposition 1 therefore applies, so $t^* < 0$. Thus, under the specified assumptions, the group never donates to the incumbent, yet its giving biases incumbent behavior. ■

Proof of Proposition 3: Suppose that the incumbent's type is known. Since the incumbent picks her preferred policy when donations are banned, in order to show that group giving has no effect on her behavior, we need to show that she picks her preferred policy when donations are allowed. This will occur so long as the group's giving is independent of the incumbent's policy choice.

So suppose that a subgame perfect equilibrium exists, and denote it by (σ^*, γ^*). If $\gamma^*(x) = \gamma^*(y)$, then we are done. So suppose that $\gamma^*(x) \neq \gamma^*(y)$. Then it is sufficient to show that there exists another subgame perfect equilibrium $(\sigma^{**}, \gamma^{**})$ in which $\gamma^{**}(x) = \gamma^{**}(y) = \gamma^*(x)$. That this is so follows from the fact that, since the incumbent's type is known, the interest group's marginal return to a donation to either politician is independent of the incumbent's first-period policy choice. Hence, the set of solutions to the interest group's donation problem is independent of the incumbent's first-period policy choice. Accordingly, a subgame perfect equilibrium exists in which $\gamma^{**}(x) = \gamma^{**}(y) = \gamma^{*}(x)$. ■

Proof of Proposition 5: As h is separable in campaign spending and the probability η the incumbent's constituents assign to her sharing the group's preference for policy x, Properties 1 through 4 continue to hold even when $\alpha = \kappa$.

Notice, however, that the cutpoint that defines the incumbent's best-response when $\alpha = \kappa$ now depends on both the group's spending and the probability the incumbent's constituents assign to her sharing the group's preference for policy x. Let η^{p_1} denote the probability assigned to the incumbent preferring policy x when her first-period policy is p_1. Given that the group's anticipated spending is (d^x, d^y) and the anticipated beliefs of the incumbent's constituents are (η^x, η^y), it is easily verified that the cutpoint that characterizes the incumbent's best-response is:

$$c(d^x, d^y, \eta^x, \eta^y, \kappa) \equiv \begin{cases} \frac{\rho[h(d^y,\eta^y;\kappa)-h(d^x,\eta^x;\kappa)]}{1+[h(d^y,\eta^y;\kappa)-h(d^x,\eta^x;\kappa)]\pi_c} & \text{if } h(d^x,\eta^x;\kappa) > h(d^y,\eta^y;\kappa) \\ 0 & \text{if } h(d^x,\eta^x;\kappa) = h(d^y,\eta^y;\kappa) \\ \frac{\rho[h(d^y,\eta^y;\kappa)-h(d^x,\eta^x;\kappa)]}{1+[h(d^x,\eta^x;\kappa)-h(d^y,\eta^y;\kappa)](1-\pi_c)} & \text{if } h(d^x,\eta^x;\kappa) < h(d^y,\eta^y;\kappa) \end{cases}.$$

With this fact in hand, we proceed in two steps: First, we show that the cutpoint of the incumbent's strategy is positive when $\alpha = \kappa$ and donations are banned. We then show that campaign spending increases the fraction of incumbent types that cater to the group.

Step 1: Let $t^{\circ}(\kappa)$ denote the cutpoint of the incumbent's strategy in an equilibrium in which salience is positive $(\alpha = \kappa)$ and donations are prohibited. In any such equilibrium, $t^{\circ}(\kappa) > 0$.[1]

Suppose that $\alpha = \kappa$ and donations are banned. Consequently, the incumbent's probability of reelection is given by $h(0, 0, \eta; \kappa) = (1-\kappa)r(0, 0)+\kappa g(\eta)$. And since Property 2 continues to hold, in any equilibrium in which donations are banned, the probability assigned to the incumbent sharing the group's preference for x when her first-period policy is p_1—denoted $\pi_i^{\circ}(p_1)$—is maximized when the incumbent selects policy x: $\pi_i^{\circ}(x) > \pi_i^{\circ}(y)$. As g is decreasing η, so is h; therefore, $h(0, 0, \pi_i^{\circ}(y); \kappa) > h(0, 0, \pi_i^{\circ}(x); \kappa)$. Thus, in any equilibrium in which campaign donations are prohibited, the net electoral benefit of catering to the group is negative. As such, the equilibrium

[1] If g is continuous, it is easily verified that when $\alpha = \kappa$ and donations are banned an equilibrium exists and is unique.

cutpoint of the incumbent's strategy when donations are prohibited must be positive.

Step 2: When donations are allowed, in any non-trivial equilibrium, the cutpoint of the incumbent's equilibrium strategy $t^*(\kappa)$ is less than that when donations are prohibited $t^\circ(\kappa)$.

Suppose that donations are allowed and a non-trivial equilibrium exists. And, by way of contradiction, suppose that $t^*(\kappa) \geq t^\circ(\kappa)$. As $t^*(\kappa) \geq t^\circ(\kappa)$, the net electoral benefit of catering to the group when donations are allowed is less than or equal to the net electoral benefit of catering to the group when donations are prohibited:

$$h(0,0,\pi_i^\circ(x);\kappa) - h(0,0,\pi_i^\circ(y),\kappa) \geq h(\gamma_i^*(x),\gamma_c^*(x),\pi_i^*(x);\kappa) - h(\gamma_i^*(y),\gamma_c^*(y),\pi_i^*(y);\kappa). \quad \text{(A11)}$$

Since $t^*(\kappa) \geq t^\circ(\kappa) > 0$, we have:

$$\pi_i^*(x) = \pi_i^\circ(x) = 1.$$

Using this fact along with the definition of h, we can rewrite (A11) as:

$$\kappa[g(\pi_i^*(y)) - g(\pi_i^\circ(y))] - (1-\kappa)[r(\gamma_i^*(x),\gamma_c^*(x)) - r(\gamma_i^*(y),\gamma_c^*(y))] \geq 0. \quad \text{(A12)}$$

That $t^*(\kappa) \geq t^\circ(\kappa) > 0$ implies that $\pi_i^*(y) \geq \pi_i^\circ(y)$. This, together with the fact that g is a decreasing function, implies that the first term of the above inequality is non-positive. This fact, together with (A12), implies that $r(\gamma_i^*(x),\gamma_c^*(x)) \leq r(\gamma_i^*(y),\gamma_c^*(y))$. However, given that Property 4 continues to apply, $r(\gamma_i^*(x),\gamma_c^*(x)) > r(\gamma_i^*(y),\gamma_c^*(y))$, which yields a contradiction. ■

B. EXISTENCE AND UNIQUENESS

Throughout this section, we assume that f_i is continuous. In addition, we impose the following convexity conditions on r and m:

Assumption 1 *r is strictly concave in d_i and strictly convex in d_c.*

Assumption 2 *$m(\cdot,0)$ is convex in d_i and $m(0,\cdot)$ is convex in d_c.*

Assumption 1 implies that the marginal productivity of campaign spending is decreasing, while assumption 2 implies that the marginal cost to campaign spending is non-decreasing. We will establish that the specified convexity conditions, together with the continuity of f_i, are sufficient to enure that a unique equilibrium exists. Upon characterizing the properties of the interest group's best-response function (Section A.1) and the incumbent's best-response function (Section A.2), we prove existence and uniqueness (Section A.3).

B.1. *Interest Group's Best-Response Function*

We begin by identifying the interest group's optimal campaign spending as a function of the cutpoint of the incumbent's strategy (c) and first-period policy choice (p_1).

Holding c fixed, denote $\tilde{\pi}_i^{p_1}(c)$ as the probability that the interest group assigns to the incumbent sharing its preference for x when the first-period policy is p_1. By Bayes's rule,

$$\tilde{\pi}_i^x(c) \equiv \begin{cases} 1 & \text{if } c \geq 0 \\ \frac{\int_0^\infty f_i(t)dt}{\int_c^\infty f_i(t)dt} & \text{otherwise} \end{cases},$$

and

$$\tilde{\pi}_i^y(c) \equiv \begin{cases} \frac{\int_0^c f_i(t)dt}{\int_{-\infty}^c f_i(t)dt} & \text{if } c \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

Denote $\widetilde{W}^{p_1}(d_i, d_c; c)$ for the interest group's expected payoff from donation pair (d_i, d_c) when the first-period policy is equal to p_1 and the cutpoint of the incumbent's strategy is equal to c. Formally,

$$\widetilde{W}^{p_1}(d_i, d_c; c) \equiv u_g(p_1; t_g) + r(d_i, d_c)[\tilde{\pi}_i^{p_1}(c) - \pi_c]t_g + \pi_c t_g - m(d_i, d_c).$$

As a result, for any cutpoint c and first-period policy p_1, the interest group must solve the subsequent optimization problem:

$$\max_{(d_i,d_c)\in D} \widetilde{W}^{p_1}(d_i, d_c; c). \tag{A13}$$

Let

$$D^{p_1*}(c) \equiv \arg\max\{\widetilde{W}^{p_1}(d_i, d_c; c) : (d_i, d_c) \in D\}. \tag{A14}$$

Hence, $D^{p_1*}(c)$ is the set of the interest group's best-responses to a first-period policy choice of p_1 when the incumbent's strategy is characterized by a cutpoint of c. In what follows, we denote an arbitrary element of $D^{p_1*}(c)$ as $(\tilde{d}_i^{p_1}(c), \tilde{d}_c^{p_1}(c))$.

B.1.1. Necessary Conditions that the Interest Group's Best-Response Must Satisfy

Lemma 8 *Suppose that $(d_i^*, d_c^*) \in D^{p_1*}(c)$. Then there exists a vector $(\lambda_i^*, \lambda_c^*)$:*

$$\frac{\partial r(d_i^*, d_c^*)}{\partial d_i}[\tilde{\pi}_i^{p_1}(c) - \pi_c]t_g - \frac{\partial m(d_i^*, d_c^*)}{\partial d_i} + \lambda_i^* = 0 \tag{A15}$$

$$\frac{\partial r(d_i^*, d_c^*)}{\partial d_c}[\tilde{\pi}_i^{p_1}(c) - \pi_c]t_g - \frac{\partial m(d_i^*, d_c^*)}{\partial d_c} + \lambda_c^* = 0 \tag{A16}$$

$$\lambda_i^* \geq 0 \quad d_i^* \lambda_i^* = 0 \tag{A17}$$

$$\lambda_c^* \geq 0 \quad d_c^* \lambda_c^* = 0. \tag{A18}$$

Proof: As the constraint qualification holds at any $(d_i, d_c) \in D$, by the Kuhn-Tucker Theorem, the result follows. ■

The subsequent lemma is an immediate consequence of Lemma 8.

Lemma 9 *Suppose that $(d_i^*, d_c^*) \in D^{p_1*}(c)$. If $d_i^* > 0$, then $\tilde{\pi}_i^{p_1}(c) > \pi_c$. If $d_c^* > 0$, then $\pi_c > \tilde{\pi}_i^{p_1}(c)$.*

B.1.2. Existence of a Best-Response

Lemma 10 *For any first-period policy choice of the incumbent, the interest group's best-response is well defined. That is, $D^{p_1*}(c) \neq \emptyset$.*

Proof: Fix p_1. To establish that the interest group's best-response is well defined, we need to show that a solution to (A13) exists. Given that, as d_i increases, $m(\cdot, 0)$ increases, eventually approaching infinity (due to the convexity of m), the equation $m(d_i, 0) - m(0, 0) = 4t_g$ has a unique solution in d_i, say $\bar{d}_i$. Also, given that, as d_c increases, $m(0, \cdot)$ increases, eventually approaching infinity, the equation $m(0, d_c) - m(0, 0) = 4t_g$ has a unique solution in d_c, say $\bar{d}_c$. Let

$$\overline{D} \equiv \{(d_i, d_c) \in D : d_i \leq \bar{d}_i \text{ and } d_c \leq \bar{d}_c\}. \tag{A19}$$

Take any $(d_i', d_c') \notin \overline{D}$. Notice that

$$\widetilde{W}^{p_1}(d_i', d_c'; c) - \widetilde{W}^{p_1}(0, 0; c) = [\tilde{\pi}_i^{p_1}(c) - \pi_c][r(d_i', d_c') - r(0, 0)]t_g - m(d_i', d_c') + m(0, 0).$$

Because the π's and r's are probabilities,

$$[\tilde{\pi}_i^{p_1}(c) - \pi_c][r(d_i', d_c') - r(0, 0]]t_g < t_g.$$

Because m is increasing in both of its arguments and $(d_i', d_c') \notin \overline{D}$,

$$m(d_i', d_c') - m(0, 0) > 4t_g.$$

From these inequalities, it follows that

$$\widetilde{W}^{p_1}(d_i', d_c'; c) - \widetilde{W}^{p_1}(0, 0; c) < t_g - 4t_g = -3t_g < 0.$$

As such, $(0, 0)$ yields the interest group a greater expected payoff than (d_i', d_c'). Hence, a solution to (A13) must be an element of $\overline{D}$. As this is the case, a solution to

$$\max_{(d_i, d_c) \in \overline{D}} \widetilde{W}^{p_1}(d_i, d_c; c) \tag{A20}$$

is a solution to (A13). Since $\overline{D}$ is compact and $\widetilde{W}^{p_1}(d_i, d_c; c)$ is continuous in d_i and d_c, the Weierstrass Theorem yields a solution to (A20). ■

B.1.3. Uniqueness of Interest Group's Best-Response

Lemma 11 *For any first-period policy p_1, the interest group has a unique best-response.*

Proof:. Fix p_1. We need to show that the solution to (A13) is unique. There are three cases to consider: *(i)* $\tilde{\pi}_i^{p_1}(c) = \pi_c$, *(ii)* $\tilde{\pi}_i^{p_1}(c) > \pi_c$, and *(iii)* $\tilde{\pi}_i^{p_1}(c) < \pi_c$.

Consider case *(i)*. Since $\tilde{\pi}_i^{p_1}(c) = \pi_c$, by Lemma 9, if (d_i^*, d_c^*) is a solution to (A13), then $(d_i^*, d_c^*) = (0, 0)$.

Consider case (ii). Since $\tilde{\pi}_i^{p_1}(c) > \pi_c$, by Lemma 9, if (d_i^*, d_c^*) is a solution to (A13), then $d_c^* = 0$. We now identify the interest group's optimal donation to the incumbent. By assumption, we have: $\tilde{\pi}_i^{p_1}(c) > \pi_c$, r is strictly concave in d_i, and m is convex in d_i when $d_c = 0$. Consequently, the second derivative of $\widetilde{W}^{p_1}$ with respect to d_i at $(d_i, 0)$—

$$\frac{\partial^2 \widetilde{W}^{p_1}(d_i, 0; c)}{\partial d_i^2} \equiv \frac{\partial^2 r(d_i, 0)}{\partial d_i^2}[\tilde{\pi}_i^{p_1}(c) - \pi_c]t_g - \frac{\partial^2 m(d_i, 0)}{\partial d_i^2}$$

—is negative. In words, when $d_c = 0$, the interest group's expected payoff is strictly concave in d_i.

Suppose that $\partial \widetilde{W}^{p_1}(0, 0; c)/\partial d_i \leq 0$. Then, as $\widetilde{W}^{p_1}$ is strictly concave in d_i when $d_c = 0$, for all $d_i > 0$, $\partial W^{p_1}(d_i, 0; c, \pi)/\partial d_i < 0$. Consequently, if (d_i^*, d_c^*) is a solution to (A13), as $d_c^* = 0$, first-order conditions (A15) and (A17) imply that $d_i^* = 0$.

Next, suppose that $\partial \widetilde{W}^{p_1}(0, 0; c)/\partial d_i > 0$. Then if (d_i^*, d_c^*) is a solution to (A13), as $d_c^* = 0$, first-order conditions (A15) and (A17) imply that $d_i^* > 0$, where d_i^* is a solution to

$$\frac{\partial \widetilde{W}^{p_1}(d_i, 0; c)}{\partial d_i} = 0$$

in d_i. As a solution to (A13) exists (see Lemma 10), it follows that a solution to above equation in d_i must exist. Further, since W is strictly concave in d_i when $d_c = 0$, this solution is unique. Consequently, when $\partial \widetilde{W}^{p_1}(0, 0; c)/\partial d_i > 0$, there is a unique solution to (A13).

A similar argument establishes uniqueness for case (iii). ■

B.1.4. Continuity of the Interest Group's Best-Response in the Cutpoint of the Incumbent's Strategy

Lemma 12 *$D^{p_1 *}(\cdot)$ is a continuous function of c.*

Proof: Fix p_1. The proof of Lemma 10 established that a solution to

$$\max_{(d_i, d_c) \in \overline{D}} \widetilde{W}^{p_1}(d_i, d_c; c),$$

where

$$\overline{D} \equiv \{(d_i, d_c) \in D : d_i \leq \bar{d}_i \text{ and } d_c \leq \bar{d}_c\},$$

is a solution to (A13). As $\widetilde{W}^{p_1}$ is continuous in its arguments and $\overline{D}$ is compact, by the Theorem of Maximum, $D^{p_1*}(\cdot)$ is an upper-semicontinuous correspondence in c. Given assumptions 1–2, by Lemma 11, $D^{p_1*}(\cdot)$ is single valued. Hence, $D^{p_1*}(\cdot)$ is a single-valued upper-semicontinuous correspondence in c. In other words, $D^{p_1*}(\cdot)$ is a continuous function of c. ■

B.1.5. Effect of a Change in the Cutpoint of the Incumbent's Strategy on the Group's Best-Response

Let $(d_i^{p_1}(c), d_c^{p_1}(c))$ denote the interest group's best-response when p_1 is selected and the incumbent's strategy is characterized by cutpoint c. The next result establishes that, upon observing that the incumbent selects x, the interest group's spending on behalf of the incumbent (challenger) is non-decreasing (non-increasing) in the cutpoint of the incumbent's strategy.

Lemma 13 *Assume that $c' < c'' \leq 0$. Then:*

1. $\tilde{d}_i^x(c') \leq \tilde{d}_i^x(c'')$, *where this inequality is strict if* $\tilde{d}_i^x(c') > 0$.

2. $\tilde{d}_c^x(c') \geq \tilde{d}_c^x(c'')$, *where this inequality is strict if* $\tilde{d}_c^x(c') > 0$.

Proof: We begin with item (1). Suppose that $c' < c'' \leq 0$. Since the density f_i has full support on $\mathbb{R}$, $\tilde{\pi}_i^x(c') < \tilde{\pi}_i^x(c'')$. Accordingly, one of three possible orderings of beliefs can arise: (i) $\tilde{\pi}_i^x(c') < \tilde{\pi}_i^x(c'') < \pi_c$, (ii) $\tilde{\pi}_i^x(c') \leq \pi_c \leq \tilde{\pi}_i^x(c'')$, or (iii) $\pi_c < \tilde{\pi}_i^x(c') < \tilde{\pi}_i^x(c'')$.

In both case (i) and case (ii), at c', when x is selected, the interest group weakly prefers the challenger's election to the incumbent's reelection. Consequently, $\tilde{d}_i^x(c') = 0$, and our desired conclusion follows.

Now consider case (iii). At both c' and c'', when x is selected, the interest group prefers the incumbent's reelection to the challenger's election. If it is not optimal for the group to spend money on behalf of the incumbent at c', then our desired conclusion immediately follows. Consequently, all that remains to be established is that if $\tilde{d}_i^x(c') > 0$, then $\tilde{d}_i^x(c') < \tilde{d}_i^x(c'')$. By Edlin and Shannon's

(1998) Strict Monotonicity Theorem, this follows as long as

$$\frac{\partial^2 \widetilde{W}^{p_1}(d_i, 0; c)}{\partial d_i \partial c} \equiv \frac{\partial r(d_i, 0)}{\partial d_i} \frac{\partial \tilde{\pi}_i^x(c)}{\partial c} t_g$$

is positive; this cross-partial is positive because r is increasing in d_i, $\tilde{\pi}_i^x$ is increasing in c on $(-\infty, 0]$, and the constant t_g is positive.

Item (2) of this lemma is proven in an analogous manner. ■

We now show that, upon observing y selected, the group's spending is independent of the cutpoint of the incumbent's strategy.

Lemma 14 *Assume $c' < c'' \leq 0$. Then $\tilde{d}_i^y(c') = \tilde{d}_i^y(c'') = 0$ and $\tilde{d}_c^y(c') = \tilde{d}_c^y(c'') = \{d_c^*\}$.*

Proof: Begin by considering the interest group's incentive to fund the incumbent when y is selected. For all $c \leq 0$, upon observing policy y, the group knows that the incumbent does not share its preference for x: $\tilde{\pi}_i^y(c) = 0 \leq \pi_c$. Hence, whenever $c < 0$, by Lemma 9, $\tilde{d}_i^y(c) = 0$.

Turning to the group's incentive to fund the challenger when y is selected: Suppose that $c' < c'' \leq 0$. Since $\tilde{\pi}_i^y(c') = \tilde{\pi}_i^y(c'') = 0$, $\widetilde{W}^y(d_i, d_c; c') = \widetilde{W}^y(d_i, d_c; c'')$. As such, the group's set of best-responses when y is selected is independent of the magnitude of the incumbent's cutpoint. Given that the group has a unique best-response under assumptions 1–2 (see Lemma 11), the group's spending on behalf of the challenger at c' and c'' is identical. ■

B.2. *Incumbent's Best-Response*

Let $d^x \in D$ denote the interest group's spending when $p_1 = x$; let $d^y \in D$ denote the interest group's spending when $p_1 = y$.

Lemma 15 *Given (d^x, d^y), x is a best-response of the incumbent if and only if $t \geq \tilde{c}(d^x, d^y)$ and y is a best-response of the incumbent if and only if $t \leq \tilde{c}(d^x, d^y)$, where*

$$\tilde{c}(d^x, d^y) \equiv \begin{cases} \frac{\rho[r(d^y) - r(d^x)]}{1 + [r(d^y) - r(d^x)]\pi_c} & \text{if } r(d^x) > r(d^y) \\ 0 & \text{if } r(d^x) = r(d^y) \\ \frac{\rho[r(d^y) - r(d^x)]}{1 + [r(d^x) - r(d^y)](1 - \pi_c)} & \text{if } r(d^x) < r(d^y) \end{cases} . \tag{A21}$$

Proof: This lemma is an immediate consequence of the discussion of the incumbent's first-period policy problem in Section A. ■

B.3. *Proof of Existence and Uniqueness*

Proposition 1 *A unique equilibrium exists.*

Proof: Let $q : (-\infty, 0] \to \mathbb{R}$, where

$$q(c) \equiv \frac{\rho[r(\tilde{d}_i^y(c), \tilde{d}_c^y(c)) - r(\tilde{d}_i^x(c), \tilde{d}_c^x(c)]}{1 + [r(\tilde{d}_i^y(c), \tilde{d}_c^y(c)) - r(\tilde{d}_i^x(c), \tilde{d}_c^x(c))]\pi_c}. \tag{A22}$$

In words, $q(c)$ is the incumbent type that is exactly indifferent between choosing x and y in the first period when the interest group best-responds to an incumbent strategy with a cutpoint of c (i.e., $q(c) = \tilde{c}(\tilde{d}^x(c), \tilde{d}^y(c))$).

We will proceed to establish the following: (1) if c^* is a fixed point of q, then there exists an equilibrium of our model in which the incumbent's strategy has a cutpoint of c^*; (2) if c' is not a fixed point of q, then there does not exist an equilibrium of our model in which the incumbent's strategy has a cutpoint of c'; (3) there is a unique fixed point of q. Together, these claims imply that there is a unique equilibrium to our model.

Step 1: If c^* is a fixed point of q, then there exists an equilibrium of our model in which the incumbent's strategy has a cutpoint of c^*

Suppose that c^* is a fixed point of q. We claim that there exists an equilibrium $(\sigma^*, \gamma^*, \mu^*)$ in which σ^* has a cutpoint of c^*, $(\gamma_i^*(x), \gamma_c^*(x)) = (\tilde{d}_i^x(c^*), \tilde{d}_c^x(c^*))$, $(\gamma_i^*(y), \gamma_c^*(y)) = (\tilde{d}_i^y(c^*), \tilde{d}_c^y(c^*))$, and μ^* is derived via Bayes's rule from the incumbent's strategy. To prove this claim, given that the incumbent's strategy is characterized by a cutpoint of c^*, we need to establish that, for each policy choice p_1 of the incumbent, the interest group's spending profile $(\gamma_i^*(p_1), \gamma_c^*(p_1))$ is a best-response. We also need to show that, given the interest group's strategy, each incumbent type is best-responding under σ^*. This is equivalent to showing that incumbent-type c^* is indifferent between choosing x and y in first period. That the former is the case follows from the construction of γ^*. That the latter is the case follows from the fact that

$$\tilde{c}(\gamma^*(x), \gamma^*(y)) = q(c^*) = c^*,$$

where the last equality is a consequence of c^* being a fixed point of q. Thus, incumbent-type c^* is indifferent between choosing x and y in the first period, which is what we needed to show.

Step 2: If c' is not a fixed point of q, then there does not exist an equilibrium of our model in which the incumbent's strategy has a cutpoint of c'.

To see why this is true, suppose that an equilibrium $(\sigma^*, \gamma^*, \mu^*)$ exists in which the incumbent's strategy is characterized by a cutpoint of c'. However, suppose that c' is not a fixed point of q.

Given that the incumbent's strategy has a cutpoint of c', $(\gamma_i^*(x), \gamma_c^*(x)) = (\tilde{d}_i^x(c'), \tilde{d}_c^x(c'))$ and $(\gamma_i^*(y), \gamma_c^*(y)) = (\tilde{d}_i^y(c'), \tilde{d}_c^y(c'))$. Given γ^*, the incumbent-type that is indifferent between x and y is $q(c')$. However, since c' is not a fixed point of q, $c' \neq q(c')$. As such, some incumbent-types are not maximizing their respective expected payoffs under σ^*; this yields a contradiction with our supposition that an equilibrium exists that has a cutpoint of c'.

Step 3: Establishing that q has a unique fixed point.

To prove that q has a unique fixed point, it is sufficient to show the following: (i) q maps non-positive cutpoints into non-positive cutpoints, (ii) q is continuous in c, and (iii) q is non-increasing in c.

We begin by establishing that q maps non-positive cutpoints into non-positive cutpoints. As the denominator of q is positive at each $c \in (-\infty, 0]$, the sign of q is determined by the sign of the numerator. As such, to show that $q(c) \leq 0$, we need to show that $r(\tilde{d}_i^x(c), \tilde{d}_c^x(c)) \geq r(\tilde{d}_i^y(c), \tilde{d}_c^y(c))$. To prove that $r(\tilde{d}_i^x(c), \tilde{d}_c^x(c)) \geq r(\tilde{d}_i^y(c), \tilde{d}_c^y(c))$, begin by noticing that $\tilde{\pi}_i^x(c) > \tilde{\pi}_i^y(c)$. Hence, by Lemma 3, if the interest group is best-responding, then $\tilde{d}_i^x(c) \geq \tilde{d}_i^y(c)$ and $\tilde{d}_c^y(c) \geq \tilde{d}_c^x(c)$. Due to the fact that r is increasing in d_i and decreasing in d_c, it thus follows that $r(\tilde{d}_i^x(c), \tilde{d}_c^x(c)) \geq r(\tilde{d}_i^y(c), \tilde{d}_c^y(c))$.

We now prove that q is continuous in c. To do so, it is sufficient to demonstrate that the component functions of q are continuous. We know that r is continuous in d_i and d_c. And, given assumptions 1–2, it follows from Lemma 12 that $\tilde{d}_i^{p_1}$ and $\tilde{d}_c^{p_1}$ are both continuous in c. As such, q is continuous in c.

Finally, to see that q is non-increasing in c, it is sufficient to show that, for any $c' < c'' \leq 0$,

$$r(\tilde{d}_i^x(c''), \tilde{d}_c^x(c'') - r(\tilde{d}_i^y(c''), \tilde{d}_c^y(c'')) \geq r(\tilde{d}_i^x(c'), \tilde{d}_c^x(c') - r(\tilde{d}_i^y(c'), \tilde{d}_c^y(c')).$$

It follows from Lemma 14 that $(\tilde{d}_i^y(c''), \tilde{d}_c^y(c'')) = (\tilde{d}_i^y(c'), \tilde{d}_c^y(c'))$. Hence, all we need to show is

$$r(\tilde{d}_i^x(c''), \tilde{d}_c^x(c'')) \geq r(\tilde{d}_i^x(c'), \tilde{d}_c^x(c')).$$

To demonstrate this, it is sufficient to show that $\tilde{d}_i^x(c'') \geq \tilde{d}_i^x(c')$ and $\tilde{d}_c^x(c') \geq \tilde{d}_c^x(c'')$. That this is so is an immediate consequence of Lemma 13. ■

C. SIMULATIONS

We now numerically solve our baseline model over a range of parameters.[2] In doing so, we learn three things:

1. The group's impact on the probability the incumbent caters to it can be quite large even when the group's influence on the incumbent's probability of winning is not large.

2. In equilibrium, the group's expected campaign spending is often small relative to size of the benefit the group receives from getting it preferred policy. Hence, the model can generate correlations between benefits received and campaign outlays consistent with Gordon Tullock's "missing money puzzle" (Tullock 1972).

3. Even when there is an "incumbency advantage," the group's attempt to elect an ideological ally can have a non-trivial effect on the equilibrium probability the incumbent caters to the interest group.

In what follows, we compute four different equilibrium quantities: (1) the incumbent's equilibrium cutpoint when donations are allowed; (2) the ex ante probability the group's giving affects the incumbent's policy choice; (3) the ratio of the group's benefit from getting its preferred policy to the group's equilibrium expected campaign expenditure; and (4) the group's net impact on the

[2] Mathematica code available upon request.

incumbent's probability of reelection from selecting a given policy.

1. Let t^* denote the equilibrium cutpoint of the incumbent's strategy when donations are allowed. Recall, all incumbents for whom $t > t^*$ select policy x, all incumbents for whom $t < t^*$ select policy y, and in any non-trivial equilibrium—i.e., an equilibrium in which the group's spending is positive—$t^* < 0$.

2. Let χ^* denote the ex ante probability that the group's giving affects the incumbent's policy choice. Recall, when donations are prohibited, the equilibrium cutpoint of the incumbent's strategy $t^\circ = 0$. Thus, the group's giving affects the behavior of each incumbent whose type $t \in (t^*, 0)$. Consequently, $\chi^* \equiv \int_{t^*}^{t^\circ} f_i(t)dt = \int_{t^*}^{0} f_i(t)dt$.

3. Let ψ^* denote the ratio of the group's benefits from securing its preferred policy to its expected campaign expenditures. Denoting the interest group's equilibrium campaign donation to candidate j when policy p_1 is chosen as $d_j^{p_1 *}$, the interest group's expected equilibrium spending is $\Pr(y \text{ is chosen})(d_i^{y*} + d_c^{y*}) + \Pr(x \text{ is chosen})(d_i^{x*} + d_c^{x*})$, which is equivalent to $(\int_{-\infty}^{t^*} f_i(t)dt)(d_i^{y*} + d_c^{y*}) + (\int_{t^*}^{\infty} f_i(t)dt)(d_i^{x*} + d_c^{x*})$. Interpreting t_g as the monetary benefit the group receives from securing its preferred policy, we define

$$\psi^* \equiv \frac{t_g}{(\int_{-\infty}^{t^*} f_i(t)dt)(d_i^{y*} + d_c^{y*}) + (\int_{t^*}^{\infty} f_i(t)dt)(d_i^{x*} + d_c^{x*})}.$$ [3]

4. Let $n^{p_1 *}$ denote the interest group's net impact on the incumbent's probability of reelection when she selects policy p_1. Formally, $n^{x*} \equiv r(d_i^{x*}, d_c^{x*}) - r(0,0)$ and $n^{y*} \equiv r(d_i^{y*}, d_c^{y*}) - r(0,0)$. Hence, $n^{p_1 *}$ provides a measure of how much better (or worse) the incumbent's reelection prospect are in a world in which campaign contributions are allowed given that she selects policy p_1.

To solve our model numerically, it is necessary to impose additional structure on r (the reelection function), m (the interest group's cost function), and f_i (the density function which describes the distribution from which the incumbent's type is drawn). In what follows, we take $r(d_i, d_c) = \frac{d_i + k_i}{d_i + d_c + k_i + k_c}$, where $k_i > 0$ and $k_c > 0$ are exogenous parameters that influence both the

[3] As discussed by Ansolabehere et al. (2003, 110-112), for several industries (e.g., defense, oil and gas, dairy), the benefits received from the government swamp industry-wide campaign giving by an order of 5000 or more.

responsiveness of election outcomes to campaign spending[4] and influence the incumbent's baseline electoral advantage. Notice that the incumbent's probability of reelection is increasing and strictly concave in d_i and decreasing and strictly convex in d_c. In addition, we take the cost to the interest group of campaign outlays (d_i, d_c) to be given by $m(d_i, d_c) = d_i + d_c$. Finally, we assume that the incumbent's type is drawn from a normal distribution with mean θ and variance ν. For the purposes of computation, we fix the mean of the distribution from which the incumbent's type is drawn $\theta = 0$, the value the incumbent attaches to reelection $\rho = 5$, the value the group attaches to getting its preferred policy $t_g = 10$, and the probability that the challenger shares the group's policy preference $\pi_c = .5$.

The results of our simulations are presented graphically in Figures A.1, A.2, and A.3. In particular, we depict each of our equilibrium quantities of interest as a function of the variance ν of the distribution from which the incumbent's type is drawn. Thus, along the horizontal axis of each plot, we vary the variance ν;[5] the vertical axis gives the units of the equilibrium quantity of interest. What differs from one figure to the next is marginal responsiveness of election outcomes to campaign spending (i.e., the exogenous parameters k_i and k_c are varied). In Figure A.1, we depict the results of numerically solving our model when $k_i = k_c = .5$. In Figure A.2, we fix $k_i = k_c = 1$. Finally, in Figure A.3, we fix $k_i = 1$ and $k_c = .5$.

Let us begin with Figure A.1. Increasing the variance of the distribution from which the incumbent's type is drawn has the effect of decreasing the cutpoint of the incumbent's equilibrium strategy (panel i), decreasing the ex ante probability the group's giving affects the incumbent's policy choice (panel ii), and decreasing the ratio of group benefit to group spending (panel iii). We see that as the variance of the distribution from which the incumbent's type is drawn approaches .25, the probability the group's giving affects the incumbent's policy choice approaches .5 (the maximal possible impact given our distributional assumptions) and the ratio of group benefit to group spending approaches a very large number.[6]

[4]Letting $j \in \{i, c\}$, increasing k_j has the effect of decreasing the marginal responsiveness of the election outcome to spending on behalf of j. Formally, $\frac{\partial^2 r}{\partial k_i \partial d_i} < 0$ and $\frac{\partial^2 r}{\partial k_c \partial d_c} > 0$. While the signs of $\frac{\partial^2 r}{\partial k_i \partial d_c}$ and $\frac{\partial^2 r}{\partial k_c \partial d_i}$ are ambiguous, $\frac{\partial^2 r(0,0)}{\partial k_i \partial d_c} = \frac{\partial^2 r(0,0)}{\partial k_c \partial d_i} = \frac{k_i - k_c}{(k_i + k_c)^2}$. Thus, when $k_i > k_c$, increasing k_i has the effect of dampening the sensitivity of the election outcome to spending on behalf of the challenger at $(0, 0)$.

[5]The range of the variance in each plot is .25 to 10. Letting the variance get too close to zero results in computational difficulties.

[6]One can show numerically that as $\nu \to .25$, ψ^* exceeds 600,000.

The reason for the latter results is the following: When the variance of the distribution of incumbent types is sufficiently small, the group only spends resources when the incumbent selects policy y. In addition, when the variance is small, the probability that the incumbent is a type that selects policy y (in period one) is small. These two facts, taken together, imply that, when the variance is small, the group's expected campaign spending is small. Consequently, as the variance of the distribution of incumbent types approaches zero, the ratio of group benefit to group spending approaches a very large number.

Finally, panel iv illustrates that when $k_i = k_c = .5$, the group's equilibrium spending has a substantial impact on the incumbent's reelection prospects. For instance, the group's campaign giving reduces the incumbent's probability of reelection when $p_1 = y$ from .5 to .3.[7] Given that the group has a relatively large impact on the incumbent's electoral prospects when $k_i = k_c = .5$, it should not be much of a surprise that the group can have a relatively large impact on the incumbent's behavior.

Figure A.2 depicts the results of numerically solving our model when $k_i = k_c = 1$. Hence, relative to the case analyzed in Figure A.1, the marginal responsiveness of election outcomes to campaign spending is now lower. While, not surprisingly, the group's equilibrium impact on incumbent's reelection prospects (panel iv) and the group's ex ante influence over incumbent behavior (panel ii) are lower than when $k_i = k_c = .5$, the group's influence remains non-trivial. Indeed, one can show numerically that as $\nu \to .25$, χ^* exceeds .35.

Figure A.3, which considers the case in which $1 = k_i > k_c = .5$, highlights that our results continue to hold even when there is an "incumbency advantage."[8] Interestingly, when $1 = k_i > k_c = .5$, the interest group fails to donate to the incumbent even when the incumbent selects the group's preferred policy (panel iv). This is because the marginal impact of spending on behalf of the incumbent on the incumbent's probability of reelection at (0,0) is now less than that when $k_i = k_c = .5$ or $k_i = k_c = 1$.

Finally, note that in each figure, the ratio ψ^* of group benefit to expected group spending never falls below twenty and is often much, much, higher. Hence, the current theoretical framework can

[7] While this is undoubtedly a large effect, it is not outside the realm of empirical plausibility, especially if the group's giving is instrumental in facilitating a serious challenge to an existing incumbent. For instance, some attribute Arlen Specter's close call in the 2004 Republican senate primary to the Club for Growth's financial backing of Pat Toomey.

[8] When $k_i > k_c$, there is an incumbency advantage in the sense that the challenger must spend more than the incumbent in order for the challenger's probability of reelection to be greater than one half.

generate ratios of group benefits to group spending similar to those identified by Ansolabehere et al. (2003, 110-112) in elaborating upon Tullock's missing money puzzle.

D. CONTINUOUS POLICY SPACE

In what follows, we provide a simple example that illustrates that an electoral contributor's attempts to elect an ideological ally can bias an incumbent's policy choice even in a world in which there is a continuous policy space. Relative to our baseline model, everything is held constant except that the policy space is continuous instead of discrete and the type space is discrete instead of continuous.

Let $X = [0, +\infty)$ denote the policy space and let $x \in X$ denote a generic policy. The interest group's preferences over policies is represented by v, where v is increasing in x. Hence, the larger the value of x, the greater the interest group's policy payoff. The policy preferences of politicians, in contrast, are represented by $u(x,t) = -(x-t)^2$. Thus, a politician's policy payoff is maximized when $x = t$. We will refer to t as a politician's type. We will assume that politicians come in one of two types, t_l and t_r, where $t_l < t_r$. Ex ante, the interest group is uncertain of both the incumbent's type and the challenger's type. Let π_i denote the prior probability the incumbent is type t_r. And let π_c denote the prior probability the challenger is type t_r. In what follows, we will say that a politician *selects her ideal point* when she sets policy equal to her type.

In the absence of campaign giving, each incumbent type will select her preferred policy in period one, as the incumbent's policy choice does not have a direct effect on her reelection prospects. In addition, in the second period, the election winner will set policy equal to her type.

Now consider when campaign giving is allowed. The interest group anticipates that the election winner will set policy equal to her type. Hence, if the interest group knew that the incumbent's type was t_r, the group would prefer the incumbent's reelection to the challenger's election. In contrast, if the interest group knew that the incumbent's type was t_l, the group would prefer the challenger's election to the incumbent's reelection. We assume that when the interest group knows the incumbent is type t_r it offers $d_i^* > 0$ to the incumbent. In addition, we assume that when the interest group knows the incumbent is type t_l if offers $d_c^* > 0$ to the challenger.[9]

[9] Assuming that r is strictly concave in d_i and strictly convex in d_c and that the marginal cost of the first unit of campaign spending is approximately zero (i.e., for each $j \in \{i, c\}$, $\lim_{d_j \to 0} \frac{\partial m(0,0)}{\partial d_j} = 0$) ensures that the group will offer a positive donation to its preferred candidate in the event that it knows the incumbent's type.

We will construct a separating equilibrium in which type t_l selects her ideal point and type t_r selects a policy to the right of her ideal point. Hence, in the constructed equilibrium, the group's giving will bias the behavior of type t_r, inducing her to select a more rightward policy than she would select when campaign giving is prohibited.

Incumbent type t_l's payoff from selecting her ideal point when doing so reveals her type is:

$$\underbrace{u(t_l,t_l)}_{\text{first-period payoff}} + \underbrace{r(0,d_c^*)[\rho+u(t_l,t_l)]+[1-r(0,d_c^*)][(1-\pi_c)u(t_l,t_l)+\pi_c u(t_r,t_l)]}_{\text{expected second-period payoff}}.^{10} \tag{A23}$$

Suppose that by selecting policy $x' \neq t_l$, type t_l could induce the interest group to believe that she is type t_r. Type t_l's payoff from selecting x' would then be:

$$\underbrace{u(x',t_l)}_{\text{first-period payoff}} + \underbrace{r(d_i^*,0)[\rho+u(t_l,t_l)]+[1-r(d_i^*,0)][(1-\pi_c)u(t_l,t_l)+\pi_c u(t_r,t_l)]}_{\text{expected second-period payoff}}. \tag{A24}$$

Thus, relative to setting $x=t_l$, setting $x=x'$ lowers type t_l's first-period policy payoff but enhances her expected second-period payoff. Let x^* denote the maximal value of x' such that (A24) is at least as large as (A23). One can show that

$$x^* \equiv t_l + \sqrt{(r(d_i^*,0)-r(0,d_c^*))(\rho-\pi_c u(t_r,t_l))}.$$

Notice that x^* is increasing in ρ. Further, notice that $x^* > t_r$ if and only if

$$\rho > \bar{\rho} \equiv -\frac{u(t_r,t_l)[1-\pi_c(r(d_i^*,0)-r(0,d_c^*))]}{r(d_i^*,0)-r(0,d_c^*)}.$$

Before proceeding to the main result of this section, we note that our model has a multiplicity of perfect Bayesian equilibria. This is because perfect Bayesian equilibrium fails to pin down the interest group's beliefs following off-path policy proposals. Hence, "unnatural" equilibria can be supported by specifying "unnatural" off-path beliefs. Thus, we will be interested in equilibria in

[10] To derive type t_l's expected second-period payoff, begin by noting that type t_l's second-period payoff in the event that she wins reelection is the value she attaches to holding office plus the policy payoff that she receives when her ideal point is implemented: $\rho+u(t_l,t_l)$. In contrast, when type t_l loses, her expected second-period payoff is a convex combination of the payoffs associated with her ideal point being selected and type t_r's ideal point being selected, where the weight attached to the latter event is the probability that the challenger is of type t_r. Thus, in the event that type t_l loses, her expected second-period payoff is $(1-\pi_c)u(t_l,t_l)+\pi_c u(t_r,t_l)$. Letting δ denote the probability with which the incumbent is reelected, her expected second-period payoff is thus $\delta[\rho+u(t_l,t_l)]+(1-\delta)[(1-\pi_c)u(t_l,t_l)+\pi_c u(t_r,t_l)]$.

which off-path beliefs satisfy criterion D1 of Cho and Kreps 1987, which requires the interest group to consider the incentives of each incumbent type when forming beliefs following off-path policy proposals.

To formalize this refinement in the context of our setting, let δ denote the incumbent's probability of reelection. Regardless of the group's belief following a policy proposal of x, we know that in an equilibrium δ must be an element of the set $\Delta \equiv [r(d_c^*, 0), r(0, d_i^*)]$. Further, let $U^*(t_l)$ denote type t_l's equilibrium payoff and let $U^*(t_r)$ denote type t_r's equilibrium payoff. Finally, let

$$\Delta(t_l, x) \equiv \{\delta \in \Delta : u(x, t_l) + \delta[\rho + u(t_l, t_l)] + (1 - \delta)[(1 - \pi_c)u(t_l, t_l) + \pi_c u(t_r, t_l)] \geq U^*(t_l)\},$$

and

$$\Delta(t_r, x) \equiv \{\delta \in \Delta : u(x, t_r) + \delta[\rho + u(t_r, t_r)] + (1 - \delta)[(1 - \pi_c)u(t_l, t_r) + \pi_c u(t_r, t_r)] \geq U^*(t_r)\}.$$

In words, $\Delta(t_j, x)$ is the set of reelection probabilities that would result in policy x yielding incumbent-type t_j a higher payoff than she in fact receives from her equilibrium payoff. A perfect Bayesian equilibrium *satisfies criterion D1* if, for any $x \in X$ that is not selected with positive probability by either incumbent type, the interest group's belief assigns probability zero to incumbent type t_j having chosen x whenever $\Delta(t_j, x)$ is a strict subset of $\Delta(t_{-j}, x)$.

The subsequent proposition establishes conditions under which group campaign giving induces a type t_r incumbent to select a policy more favorable to the interest group than the policy that she selects when donations are prohibited.

Proposition 2 *Suppose $\rho > \bar{\rho}$. Then there exists a separating equilibrium in which incumbent type t_l sets $x = t_l$ and type t_r sets $x = x^* > t_r$; the interest group offers donation pair $(0, d_c^*)$ upon observing $x < x^*$ and offers donation pair $(d_i^*, 0)$ upon observing $x \geq x^*$; the group places probability one on the incumbent being type t_l upon observing $x < x^*$ and places probability one on the incumbent being type t_r upon observing $x \geq x^*$. A sufficient condition for the specified equilibrium to satisfy criterion D1 of Cho and Kreps 1987 is that $\pi_c \leq \frac{1}{2}$.*

Proof: Assume that $\rho > \bar{\rho}$. It is immediate that the interest group's beliefs are consistent with the requirements of perfect Bayesian equilibrium. It is also immediate that given its beliefs, the interest group is optimizing following each possible policy choice.

Turning to the incumbent's incentives, given the interest group's strategy, the best incumbent type t_l can do by choosing $x \neq t_l$ is to select $x = x^*$. However, by construction, type t_l has no incentive to do so. All that remains to show is that type t_r cannot do better with some $x \neq x^*$. The best type t_r can do by selecting $x \neq x^*$ is to set $x = t_r$. Her payoff from doing so is

$$\underbrace{u(t_r, t_r)}_{\text{first-period payoff}} + \underbrace{r(0, d_c^*)[\rho + u(t_r, t_r)] + [1 - r(0, d_c^*)][(1 - \pi_c)u(t_l, t_r) + \pi_c u(t_r, t_r)]}_{\text{expected second-period payoff}}, \tag{A25}$$

whereas her payoff from selecting $x = x^*$ is

$$\underbrace{u(x^*, t_r)}_{\text{first-period payoff}} + \underbrace{r(d_i^*, 0)[\rho + u(t_r, t_r)] + [1 - r(d_i^*, 0)][(1 - \pi_c)u(t_l, t_r) + \pi_c u(t_r, t_r)]}_{\text{expected second-period payoff}}. \tag{A26}$$

As $t_l < t_r < x^*$, it is considerably less costly policy-wise for type t_r to select policy x^* than it is for type t_l. This fact, taken together with the fact that type t_l's payoff to the game is the same regardless of whether she chooses t_l or x^*, essentially implies that type t_r's payoff to the game is greater when she selects x^* than when she selects her ideal point: (A26) is greater than (A25).[11] Consequently, the specified strategies and beliefs constitute a perfect Bayesian equilibrium.

[11] To establish this formally, begin by noticing that due to the construction of x^*,

$$u(t_l, t_l) + r(0, d_c^*)[\rho + u(t_l, t_l)] + [1 - r(0, d_c^*)][(1 - \pi_c)u(t_l, t_l) + \pi_c u(t_r, t_l)] =$$
$$u(x^*, t_l) + r(d_i^*, 0)[\rho + u(t_l, t_l)] + [1 - r(d_i^*, 0))][(1 - \pi_c)u(t_l, t_l) + \pi_c u(t_r, t_l)].$$

Rearranging terms, we have

$$u(t_l, t_l) - u(x^*, t_l) = (r(d_i^*, 0) - r(0, d_c^*))[\rho + u(t_l, t_l) - (1 - \pi_c)u(t_l, t_l) - \pi_c u(t_r, t_l)].$$

Now notice that

$$u(t_l, t_l) - u(x^*, t_l) = [u(t_l, t_l) - u(t_r, t_l)] + [u(t_r, t_l) - u(x^*, t_l)].$$

Thus,

$$[u(t_l, t_l) - u(t_r, t_l)] + [u(t_r, t_l) - u(x^*, t_l)] = (r(d_i^*, 0) - r(0, d_c^*))[\rho + u(t_l, t_l) - (1 - \pi_c)u(t_l, t_l) - \pi_c u(t_r, t_l)].$$

Using the fact that $u(t_l, t_l) = 0$ and rearranging terms, we have

$$u(t_r, t_l) - u(x^*, t_l) = (r(d_i^*, 0) - r(0, d_c^*))\rho + u(t_r, t_l)[1 - (r(d_i^*, 0) - r(0, d_c^*))\pi_c]. \tag{*}$$

Since u is quadratic and $t_l < t_r < x^*$, $u(t_r, t_r) - u(x^*, t_r) < u(t_r, t_l) - u(x^*, t_l)$. Further, $(r(d_i^*, 0) - r(0, d_c^*))\rho + u(t_r, t_l)[1 - (r(d_i^*, 0) - r(0, d_c^*))\pi_c] < (r(d_i^*, 0) - r(0, d_c^*))\rho - u(t_r, t_l)(r(d_i^*, 0) - r(0, d_c^*))(1 - \pi_c)$. These two facts, take together with (*), imply that

$$u(t_r, t_r) - u(x^*, t_r) < (r(d_i^*, 0) - r(0, d_c^*))(\rho - (1 - \pi_c)u(t_r, t_l)).$$

Rearranging terms once again, and using the fact that $u(t_r, t_l) = u(t_l, t_r)$, it thus follows that (A26) is greater than (A25).

We now argue that when $\pi_c \leq \frac{1}{2}$, the specified equilibrium satisfies criterion D1. As mentioned, due to the construction of x^*, type t_l's equilibrium payoff from selecting t_l is identical to the payoff that she would receive from selecting x^*. Hence,

$$U^*(t_l) = u(x^*, t_l) + r(d_i^*, 0)[\rho + u(t_l, t_l)] + (1 - r(d_i^*, 0))[(1 - \pi_c)u(t_l, t_l) + \pi_c u(t_r, t_l)].$$

Further,

$$U^*(t_r) = u(x^*, t_r) + r(d_i^*, 0)[\rho + u(t_r, t_r)] + (1 - r(d_i^*, 0))[(1 - \pi_c)u(t_l, t_r) + \pi_c u(t_r, t_r)].$$

First consider off-path proposals in which $x > x^*$. For such proposals, $\Delta(t_l, x) = \Delta(t_r, x) = \emptyset$. That this is so follows from the following two facts: First, neither incumbent type can enhance her first-period policy payoff relative to her equilibrium first-period policy payoff by selecting a policy to the right of x^*. Second, neither type can enhance her probability of reelection by selecting $x > x^*$, as by selecting x^* the incumbent achieves the maximal reelection probability, $r(d_i^*, 0)$. These facts, taken together, imply that selecting $x > x^*$ is equilibrium dominated for both incumbent types. Hence, criterion D1 imposes no restrictions on beliefs when $x > x^*$.

Now consider an off-path policy proposal in which $x < x^*$. For such policies, $\Delta(t_r, x) \subsetneq \Delta(t_l, x)$. Intuitively, this is a consequence of two facts: First, as $t_l < t_r < x^*$, type t_l gains more policy-wise by breaking equilibrium and selecting $x < x^*$. Second, because $\pi_c \leq \frac{1}{2}$, type t_r's policy benefit from being reelected is at least as large as that of type t_l. Hence, for any $x < x^*$, the probability of reelection necessary to induce type t_l to break equilibrium and select x is less than that necessary to induce type t_r to do so.

Formalizing this argument, selecting policy $x < x^*$ provides type t_l a greater payoff to the game than her equilibrium payoff provided that she is reelected with probability

$$\delta \geq \bar{\delta}(t_l, x) \equiv \frac{u(x^*, t_l) - u(x, t_l)}{\rho - \pi_c u(t_r, t_l)} + r(d_i^*, 0). \tag{A27}$$

Selecting policy $x < x^*$ provides type t_r a greater payoff to the game than her equilibrium payoff provided that she is reelected with probability

$$\delta \geq \bar{\delta}(t_r, x) \equiv \frac{u(x^*, t_r) - u(x, t_r)}{\rho - (1 - \pi_c)u(t_l, t_r)} + r(d_i^*, 0). \quad \text{(A28)}$$

We now prove that $\bar{\delta}(t_l, x) < \bar{\delta}(t_r, x)$. As $t_l < t_r < x^*$, a sufficient condition for $\bar{\delta}(t_l, x) < \bar{\delta}(t_r, x)$ is that

$$\rho - \pi_c u(t_r, t_l) \leq \rho - (1 - \pi_c)u(t_l, t_r) \quad \text{(A29)}$$

and

$$u(x, t_r) - u(x^*, t_r) < u(x, t_l) - u(x^*, t_l). \quad \text{(A30)}$$

That inequality (A29) holds follows from our supposition that $\pi_c \leq \frac{1}{2}$ and the fact that $u(t_l, t_r) = u(t_r, t_l)$. Inequality (A30) holds if and only if $(t_r - t_l)(x^* - x) > 0$. As $t_r > t_l$ and $x^* > x$, it thus follows that (A30) holds. Hence, $\bar{\delta}(t_l, x) < \bar{\delta}(t_r, x)$. But this implies that for all off-path policies $x < x^*$, $\Delta(t_r, x) \subsetneq \Delta(t_l, x)$. Hence, criterion D1 requires that the interest group assign probability one to the incumbent being type t_l upon observing an off-path policy $x < x^*$ proposed. Thus, the beliefs in our specified equilibrium satisfy the requirements of criterion D1. ■

One can also easily verify that when $\pi_c \leq \frac{1}{2}$ and $\rho > \bar{\rho}$, pooling equilibria fail to satisfy criterion D1. However, if one bounds the policy space above (i.e., $X = [0, \bar{x}]$) then pooling equilibria can be sustained when the weight attached to reelection is sufficiently large. Such equilibria take the form of both incumbent types selecting policy $\bar{x}$. Finally, we can show via numerical example that $\pi_c \leq \frac{1}{2}$ is not a necessary condition for the equilibrium specified in Proposition 2 to satisfy criterion D1.

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Figure A.1

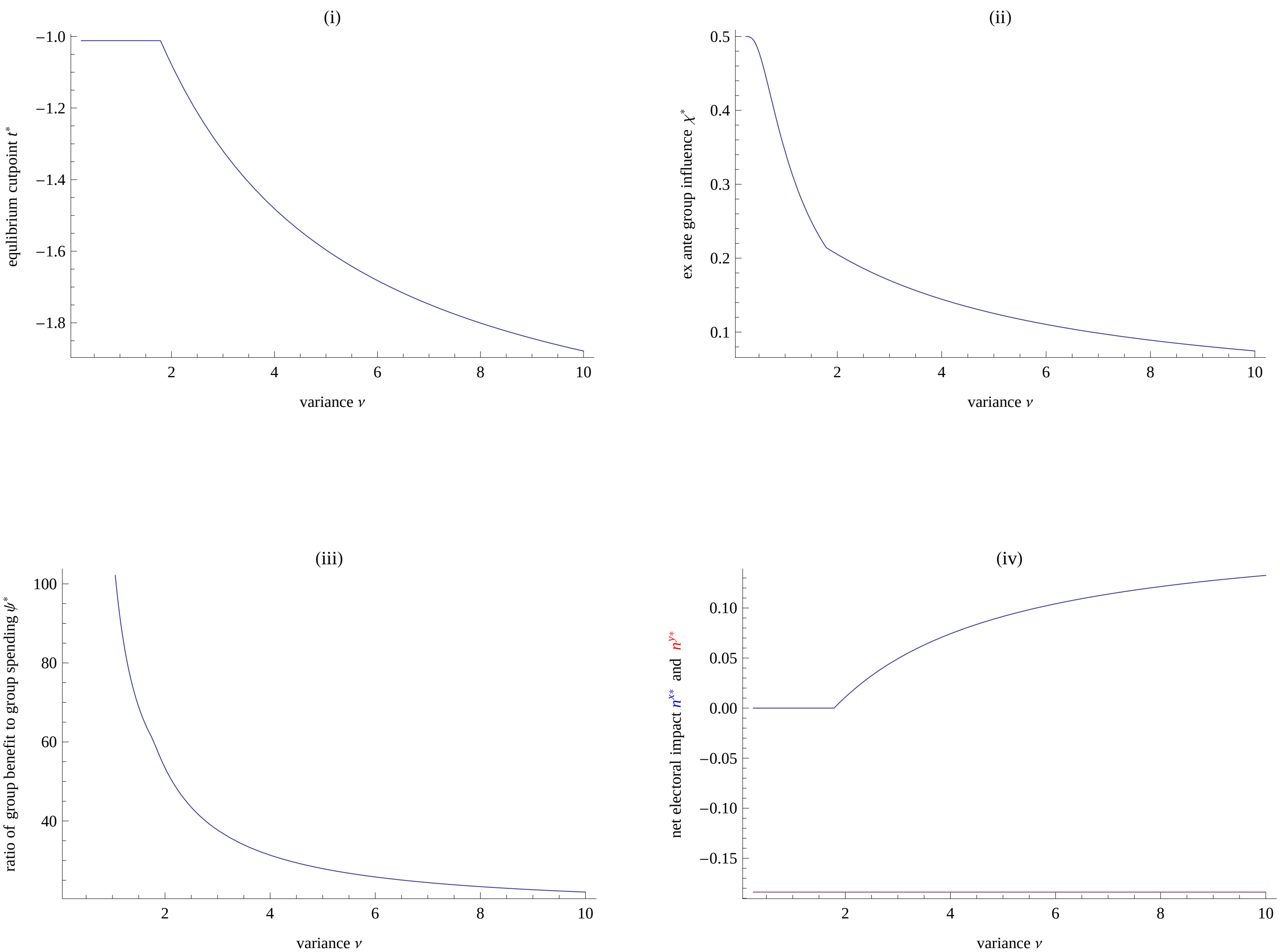
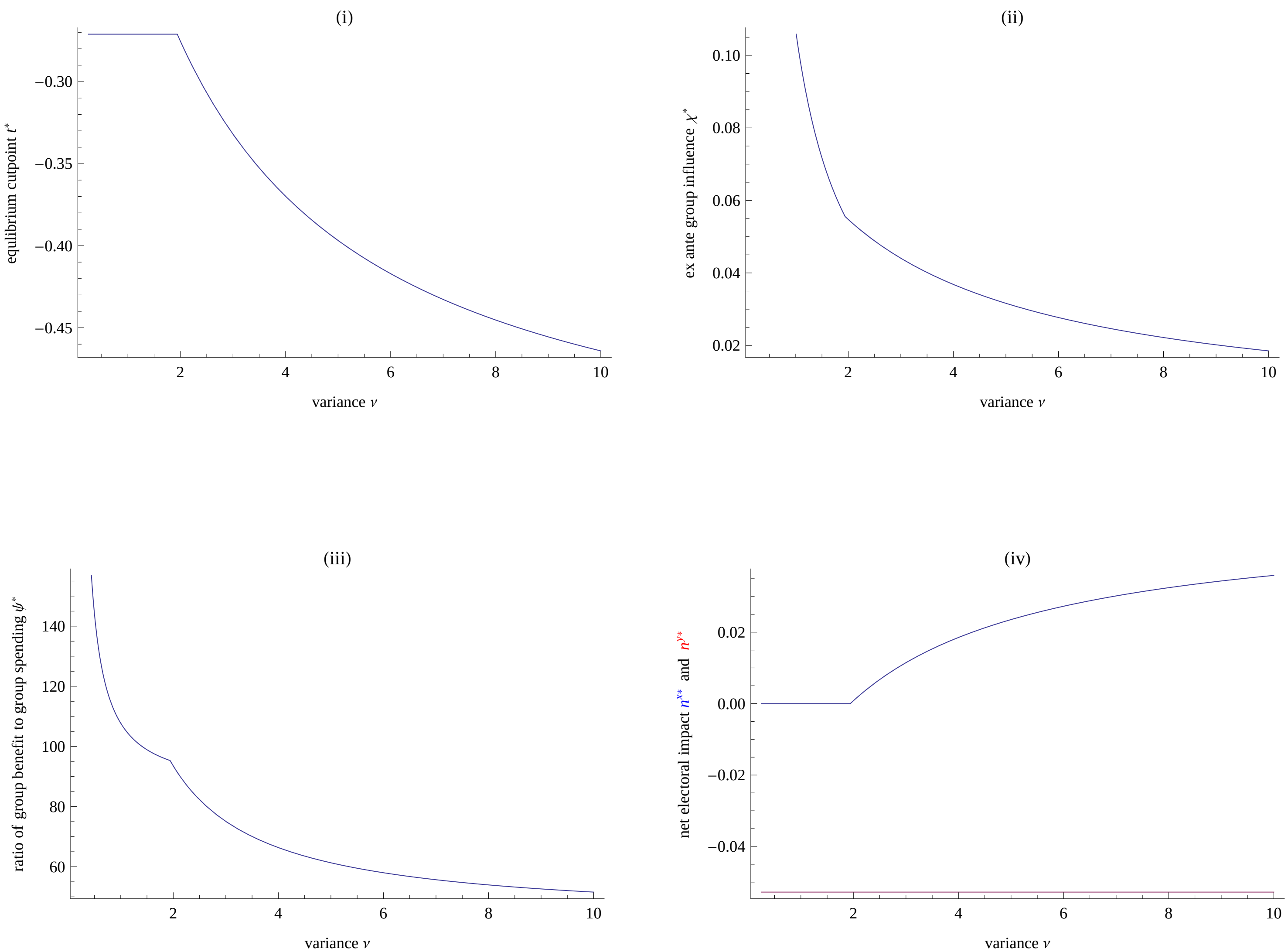
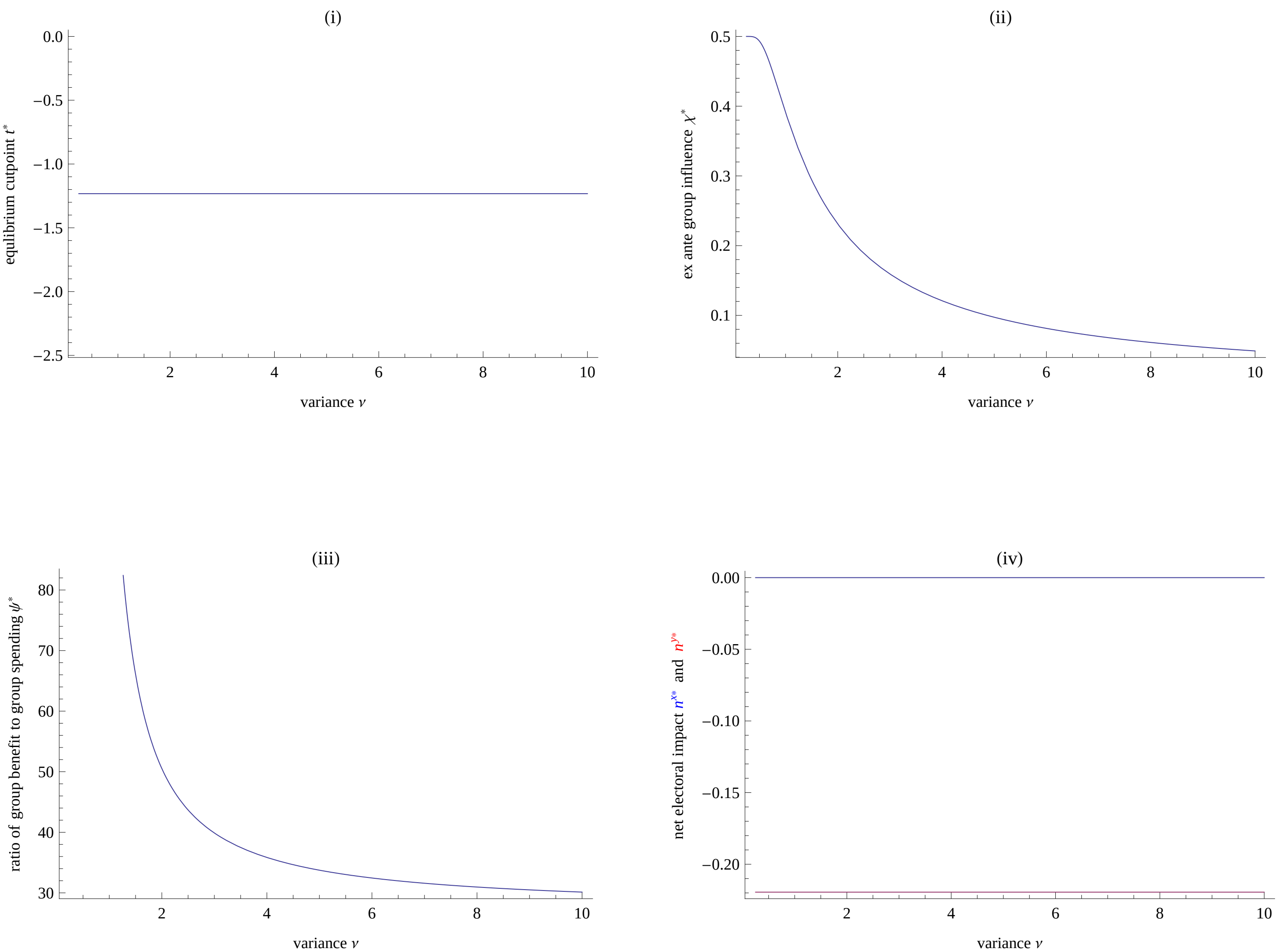

(i)
equlibrium cutpoint t^*
−1.0
−1.2
−1.4
−1.6
−1.8
2
4
6
8
10
variance v
(ii)
ex ante group influence χ^*
0.5
0.4
0.3
0.2
0.1
2
4
6
8
10
variance v
(iii)
ratio of group benefit to group spending ψ^*
100
80
60
40
2
4
6
8
10
variance v
(iv)
net electoral impact n^{x*} and n^{y*}
0.10
0.05
0.00
−0.05
−0.10
−0.15
2
4
6
8
10
variance v

Figure A.2



(i)
equlibrium cutpoint t^*
−0.30
−0.35
−0.40
−0.45
2
4
6
8
10
variance ν
(ii)
ex ante group influence χ^*
0.10
0.08
0.06
0.04
0.02
2
4
6
8
10
variance ν
(iii)
ratio of group benefit to group spending ψ^*
140
120
100
80
60
2
4
6
8
10
variance ν
(iv)
net electoral impact n^{x*} and n^{y*}
0.02
0.00
−0.02
−0.04
2
4
6
8
10
variance ν

Figure A.3


(i)
equilibrium cutpoint t^*
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
2
4
6
8
10
variance ν
(ii)
ex ante group influence χ^*
0.5
0.4
0.3
0.2
0.1
2
4
6
8
10
variance ν
(iii)
ratio of group benefit to group spending ψ^*
80
70
60
50
40
30
2
4
6
8
10
variance ν
(iv)
net electoral impact n^{x*} and n^{y*}
0.00
−0.05
−0.10
−0.15
−0.20
2
4
6
8
10
variance ν