Supplementary Materials for "Retrospective Causal Inference with Machine Learning Ensembles: An Application to Anti-Recidivism Policies in Colombia"

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A. ESTIMATION AND INFERENCE DETAILS

Proposition 1 (Consistency)**.** *Suppose we have*

- *a random sample of size N of observations of O,*
- *bounded support for O,*
- *Assumptions 1-3, and*
- $\hat{g}_j(\underline{a}_j|W_i,A_{-ji})$ *a consistent estimator of* $\Pr[A_j=\underline{a}_j|W_i,A_{-ji}]$.

Under such conditions, $\hat{\psi}_j^{IPW}-\psi_j \xrightarrow{p} 0$ *as* $N\to\infty$.

Proof. By Chebychev's inequality, consistency follows from asymptotic unbiasedness and variance converging to zero for the estimator (Lehmann, 1999, Thm. 2.1.1). By random sampling, Slutsky's theorem, consistency for $\hat{g}_j(\underline{a}_j|W_i,A_{-ji})$, and Assumption 1, as $N\to\infty$, $\hat{\psi}_j^{IPW}$ has the same convergence limit as

$$\bar{\psi}_j^{IPW}=\frac{1}{N}\sum_{i=1}^{N}\frac{I(A_{ji}=\underline{a}_j)}{\Pr[A_j=\underline{a}_j|W_i,A_{-ji}]}Y_i(\underline{a}_j,A_{-j})-\mathrm{E}[Y].$$

Then,

$$\begin{aligned}\mathrm{E}[\bar{\psi}_j^{IPW}]&=\frac{1}{N}\sum_{i=1}^{N}\mathrm{E}\left[\frac{\mathrm{E}[I(A_{ji}=\underline{a}_j)|W_i,A_{-ji}]}{\Pr[A_j=\underline{a}_j|W_i,A_{-ji}]}\mathrm{E}[Y_i(\underline{a}_j,A_{-j})|W_i,A_{-ji}]\right]-\mathrm{E}[Y]\\&=\mathrm{E}[Y(\underline{a}_j,A_{-j})]-\mathrm{E}[Y]=\psi_j,\end{aligned}$$

and so $\mathrm{E}[\hat{\psi}_j^{IPW}-\psi_j]\to 0$ as $N\to\infty$, establishing asymptotic unbiasedness. Next, by consistency for $\hat{g}_j(\underline{a}_j|W_i,A_{-ji})$ and Slutsky's Theorem, $\mathrm{Var}[N\hat{\psi}_j^{IPW}]$ has the same limit as $\mathrm{Var}[N\bar{\psi}_j^{IPW}]$, and by random sampling and bounded support,

$$\frac{1}{N^2}\mathrm{Var}[N\bar{\psi}_j^{IPW}]=\frac{1}{N^2}\sum_{i=1}^{N}\mathrm{Var}\left[\frac{I(A_{ji}=\underline{a}_j)}{\Pr[A_j=\underline{a}_j|W_i,A_{-ji}]}Y_i(\underline{a}_j,A_{-j})\right]\leq\frac{c^2}{N}$$

for some constant c, in which case $\text{Var}[\hat{\psi}_j^{IPW}] \to 0$ as $N \to \infty$, establishing that the variance converges to zero. $\square$

To construct confidence intervals, we rely on well-known results for sieve-type IPW estimators (Hirano, Imbens and Ridder, 2003; Hubbard and Van der Laan, 2008). Define

$$D_{i,IPW} = \left(\frac{I(A_{ji} = \underline{a}_j)}{\hat{g}_j(\underline{a}_j | W_i, A_{-ji})} - 1 \right) Y_i,$$

in which case $\hat{\psi}_j^{IPW} = \frac{1}{N} \sum_{i=1}^{N} D_{i,IPW}$.

Suppose that $g_j(\underline{a}_j | W_i, A_{-ji})$ parameterizes the true distribution for A_j, and $\hat{g}_j(\underline{a}_j | W_i, A_{-ji})$ approaches the maximum likelihood estimate for $g_j(\underline{a}_j | W_i, A_{-ji})$. Then, $\hat{\psi}_{j,k}^{IPW}$ is asymptotically normal and the following estimator is conservative in expectation for the asymptotic variance:

$$\hat{V}(\hat{\psi}_{j,k}^{IPW}) = \frac{v(D_{ki,IPW})}{N},$$

where the $v(.)$ operator computes the sample variance. Define $\hat{S}_{IPW} = \sqrt{\hat{V}(\hat{\psi}_{j,k}^{IPW})}$. Then we have the following approximate $100\% * (1 - \alpha)$ Wald-type confidence interval for our estimate:

$$\hat{\psi}_{j,k}^{IPW} \pm z_{\alpha/2} \hat{S}_{IPW}.$$

We can modify the estimation and inference procedure to account for non-i.i.d. data. We have assumed that (W, A_{-ji}) is a sufficient conditioning set for causal identification and that the model for $g_j(.)$ is sufficient for characterizing counter-factual intervention probabilities conditional on (W, A_{-ji}). For this reason, non-i.i.d. data on O do not require that we change anything about how we go about estimating $\hat{g}_j$. However, we will have to account for any systematic differences between our sample and target population in the distribution of (W, A_{-ji}) when computing $\hat{\psi}_{j,k}^{IPW}$. This estimator is consistent for $\psi_{j,k}^{IPW}$ only if it marginalizes over the (W, A_{-ji}) distribution in the population. The solution is to apply sampling weights that account for sample units' selection probabilities (Thompson, 2012, Ch. 6). When units' selection probabilities are known exactly based on a sampling design (as is the case in our application), we merely need to modify the expression for $\hat{\psi}_{j,k}^{IPW}$ to take the form of a survey weighted mean rather than a simple arithmetic mean. Our standard error and confidence interval estimates apply the usual survey corrections for clustering and stratification in sampling design (Thompson, 2012, Ch. 11-12).

B. DETAILS ON THE APPLICATION

Table 1: Risk factors and hypothetical interventions, details

Risk factor	**Target variable in dataset**	**Target variable description**	**Target variable coding**	**New variable definition**	**Hypothetical intervention**	**Operationalization**
Economic welfare	p136_emp_REC3	Employed 1 year after demobilization	0=unemployed, 1=employed	int_emp: = p136_emp_REC3	Unemployed are made employed.	int_emp: 0 to 1
Sense of security	p145_atrisk_REC2	Felt secure 1 year after demobilization	0=no, 1=yes	int_secure: 0 if 1, 1 if 0	Insecure are made to feel secure.	int_notatrisk: 0 to 1
Confidence in government	p111_gov_promises_1year_REC1	Confident 1 year after demobilization that government would keep promises	1-10 scale, lower means less confident	int_confident: 0 if <=5, 1 if >5	Not confident are made to feel confident.	int_confident: 0 to 1
Emotional wellbeing	index_reint_psych_neg	Scale constructed from variables measuring how psychologically upbeat 1 year after demobilization	Standardized index (mean=0, sd=1)	int_upbeat: 0 if >= .5723912, 1 if <.5723912 (75th pctile)	Psychologically depressed are made to feel upbeat.	int_upbeat: 0 to 1
Horizontal network relations with excombatants	p150_know_excom_REC1b	Of five closest acquaintances, how many were excombatants 1 year after demobilization	Count of 0 to 5	int_excompeers: 0 if 3 or 4, 1 if 1 or 2	Those with more than half excombatant peers are made to have less than half.	int_excompeers: 0 to 1
Vertical network relations with commanders	p66_sup1_talk_REC1	How regularly respondent spoke to commander 1 year after demobilization	1-4 scale, with 1 meaning rarely, and 4 often	int_commander: 0 if 2, 3, or 4; 1 if 1	Those who spoke to commander are made to rarely speak to commander.	int_commander: 0 to 1

Table 2: Workflow for estimating RIEs with ensemble

Step	**Description**	**Files**
1	Define hypothetical interventions and construct intervention indicator variables; can be done in any software package. (Done on each imputation-completed dataset.)	`Hypothetical-Interventions.xlsx` `COLOMBIA-STEP9-interventions.do`
2a	Fit propensity score models for each intervention with the ensemble, using cross-validated risk to generate optimal weights for the different model predictions; steps are automated with the SuperLearner functions for R. (Done on each imputation-completed dataset.)	`interv-pscore-1.R` through `interv-pscore-6.R`
2b	Generate predictions from propensity score models and attach to dataset. Done using prediction functions in the SuperLearner package for R. (Done on each imputation-completed dataset.)	`interv-pscore-1.R` through `interv-pscore-6.R`
2c	Produce estimates of intervention effects, incorporating survey sampling adjustments; can be done with any survey estimation software, such as the survey package in R. (Done on each imputation-completed dataset, and then RIE estimates from the imputation-completed datasets were combined to obtain the final estimates.)	`interv-pscore-1.R` through `interv-pscore-6.R`
3	Summarize results.	`int-results-graph.R` `int-results-balance-tables.R` `int-results-performance-metrics.R`

REFERENCES

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